IAU PUBLISHING JOURNAL OF THEORETICAL PHYSICS

♠ J. Theor. Phys. **2** (2013) 23-38 www.jtpc.ir

Octonion model of dark matter

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Abstract. In this paper, we have made an attempt to discuss the role of division algebras (octonions) in gravity and dark matter where, we have described the octonion space as the combination of two quaternionic spaces namely gravitational G-space and electromagnetic EM-space. We have discussed the standard model and GUTs in terms of split octonion formulation. At last, we have formulated the theory of dark matter in terms of octonion variables i.e. hot dark matter and cold dark matter.

PACS numbers: 03.50.De, 03.65.Fd, 98.80.Cq, 95.35.+d

Keywords: Octonions, gravity, GUTs, gravitational space and electromagnetic space, dark matter, octonionic hot and cold dark matter

ISSN 2251-855X (Print), 2251-9998 (Online)

1. Introduction

The Standard Model (SM) [1, 2, 3, 4, 5] of particle physics summarizes all [6, 7, 8, 9, 10, 11] we know about the fundamental forces of electromagnetism, as well as the weak and strong interactions [12] (without gravity). The Standard Model consists of elementary particles grouped into two classes [12]: bosons (particles that transmit forces) and fermions (particles that make up matter). The bosons have particle spin that is either 0, 1 or 2. The fermions have spin 1/2. On the other hand, particle physics strives to identify the building blocks of matter and describe the interactions that bind them: the set of instructions needed to create a universe. Our most succinct and (we believe) accurate set of instructions is encapsulated in a quantum field theory [3, 4] called the Standard Model, which describes a universe [13] made up of six types of quarks and six types of leptons, bound together by three fundamental forces: strong, weak, and electromagnetic. The standard model is a relativistic quantum field theory [1, 2, 3] that incorporates the basic principles of quantum mechanics and special relativity. Like quantum electrodynamics (QED) the standard model is a gauge theory [14]. However, with the non-Abelian gauge group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ instead of the simple Abelian $U(1)_{em}$ gauge group of QED. The gauge bosons are the photons mediating the electromagnetic interactions, the $W^{\pm}$ and Z^0 bosons mediating the weak interactions [12], as well as the gluons mediating the strong interactions [2, 3]. Gauge theories can exist in several phases: in the Coulomb phase with massless gauge bosons (like in QED), in the Higgs-phase with spontaneously broken gauge symmetry [14] and with massive gauge bosons (*e.g.* the $W^{\pm}$ and Z^0 bosons), and in the confinement phase, in which the gauge bosons do not appear in the spectrum (like the gluons in quantum chromodynamics (QCD)). Thus, the Standard Model of particle physics is the most successful theory of nature in history, but increasingly there are signs that it must be extended by adding new particles that play roles in high-energy reactions [15, 16].

Despite being the most successful theory of particle physics to date, the Standard Model is not perfect [17, 18]. The deficiencies of the Standard Model on the bans of experimental observations which are not yet explain, are described as

• The standard model does not provide an explanation of gravity [19]. Moreover it is incompatible with the most successful theory of gravity to date, general relativity.

• According to the standard model the neutrinos are massless particles [20, 21]. However, neutrino oscillation experiments have shown that neutrinos do have mass. Mass terms for the neutrinos can be added to the standard model by hand, but these lead to new theoretical problems [21]. (For example, the mass terms need to be extraordinarily small).

• The universe is made out of mostly matter. However, the standard model predicts that matter and anti-matter [22, 23, 24] should have been created in (almost) equal amounts, which would have annihilated each other as the universe cooled.

Thus, in astronomy and cosmology, dark matter is matter that is inferred to exist from gravitational effects on visible matter and background radiation, but is undetectable by emitted or scattered electromagnetic radiation. Its existence was hypothesized to account for discrepancies between measurements of the mass of galaxies, clusters of galaxies and the entire universe made through dynamical and general relativistic means, and accounting for matter based on counting atoms in stars and the gas and dust of the interstellar and intergalactic media.

The vast majority of the dark matter in the universe is believed to be nonbaryonic, which means that it contains no atoms and does not interact with ordinary matter via electromagnetic forces. The nonbaryonic dark matter [20, 21, 22] includes neutrinos, and possibly hypothetical entities such as axions, or supersymmetric particles. Unlike baryonic dark matter, nonbaryonic dark matter does not contribute to the formation of the elements in the early universe and so its presence is revealed only via its gravitational attraction. There are two type of nonbaryonic dark matter respectively defined as hot dark matter and cold dark matter. In this paper, we have analyzed the role of division algebras (octonions) in gravity and nonbaryonic dark matter. The two fundamental mathematical structures (division algebras) a physicist uses in his everyday life are the real ($\mathbf{R}$) and the complex ($\mathbf{C}$) numbers. Complex numbers are described as pairs of real numbers with a specific multiplication laws. One can however go even further and build two other sets of numbers, known in mathematics as quaternions ($\mathbf{H}$) [25, 26, 27, 28] and octonions ($\mathcal{O}$) [29, 30]. The quaternions, formed as pairs of complex numbers are non-commutative whereas the octonions, formed as pairs of quaternion numbers are both non-commutative and non-associative. The four sets of numbers are mathematically known as division algebras. The octonions are the last division algebra, no further generalization being consistent with the laws of mathematics. So, there exists four normed division algebras [27]: the real numbers, complex numbers, quaternions [25, 26, 27, 29], and octonions [29, 30, 31]. Here, we have discussed the role of octonions in grand unified theory (GUT) gauge group of which is describes $SU(3) \times SU(2) \times U(1)$. Thus, we have established the covariant derivative, gauge field strength and field equation for the case of grand unified theory in terms of 2×2 Zorn vector matrix realization of split octonion. As such, the octonionic formulation regardless a generalization of GUTs for the mixing of gauge current used for U(1), SU(2) and SU(3) sectors associated respectively with the electromagnetic, weak and strong interactions in presence of dyons. We have described the octonion space as the combination of two quaternionic spaces namely gravitational G-space and electromagnetic EM-space. Consequently, we have formulated the theory of dark matter in terms of octonion variables. It is emphasized that the dark matter neither emits nor absorbs light or electromagnetic radiation at any significant level. Thus, the dark matter (nonbaryonic) has been investigated in terms of octonion hot-dark matter and octonion cold-matter. As such, we have derived the various quantum equations for octonionic hot dark matter and cold dark matter.

2. Preliminaries

The octonions [30, 31] are the largest algebra, with eight dimensions, double the number of the quaternions from which they are an extension. The octonions can be thought of as octets (or 8-tuples) of real numbers. An octonion $x \in \mathcal{O}$ is expressed [34, 35, 36, 37] as a real linear combination of the unit octonions $(e_0, e_1, e_2, e_3, e_4, e_5, e_6, e_7)$ i.e.

$$x = (x_0,\, x_1,\, x_2,\, x_3,\, x_4,\, x_5,\, x_6,\, x_7) = x_0 e_0 + \sum_{A=1}^{7} x_A e_A, \tag{1}$$

where $e_A (A = 1, ..., 7)$ are imaginary octonion units and e_0 is the multiplicative unit element. The octet $(e_0, e_1, e_2, e_3, e_4, e_5, e_6, e_7)$ is known as the octonion basis and its elements satisfy the following multiplication rules

$$\begin{aligned} & e_0 = 1, \;\; e_0 e_A = e_A e_0 = e_A, \\ & e_A e_B = -\delta_{AB} e_0 + f_{ABC}\, e_C. \;\; (A, B, C = 1, ..., 7) \end{aligned} \tag{2}$$

The structure constants f_{ABC} are completely antisymmetric and take the value 1 i.e. $f_{ABC} = +1 = (123), (471), (257), (165), (624), (543), (736)$. Here the octonion algebra $\mathcal{O}$ is described over the algebra of rational numbers having the vector space of dimension 8. Octonion algebra is non associative and multiplication rules for its basis elements given by equation (2) is then generalized in Table 1 [34].

Hence, we get the following relations among octonion basis elements i.e.

$$[e_A,\; e_B] = 2 f_{ABC} e_C, \quad \{e_A,\; e_B\} = -\delta_{AB} e_0, \quad e_A(e_B e_C) \neq (e_A e_B) e_C, \tag{3}$$

where brackets [ ] and { } are used respectively for commutation and the anti commutation relations while δ_{AB} is the usual Kronecker delta symbol. Octonion conjugate is thus defined as

$$\bar{x} = x_0 e_0 - \sum_{A=1}^{7} x_A e_A. \tag{4}$$

An octonion can be decomposed in terms of its scalar ($\mathbf{Sc}(x)$) and vector ($\mathbf{Vec}(x)$) parts as

$$\mathbf{Sc}(x) = \frac{1}{2}(x + \bar{x}) = x_0, \quad \mathbf{Vec}(x) = \frac{1}{2}(x - \bar{x}) = \sum_{A=1}^{7} x_A e_A. \tag{5}$$

Conjugates of product of two octonions and its own are described as

$$(\overline{xy}) = \bar{y}\,\bar{x}, \qquad \overline{(\bar{x})} \;= x, \tag{6}$$

while the scalar product of two octonions is defined as

$$\langle x\,,\, y\,\rangle \;= \sum_{\alpha=0}^{7} x_\alpha y_\alpha = \frac{1}{2}(x\,\bar{y} + y\,\bar{x}) = \frac{1}{2}(\bar{x}\,y + \bar{y}\,x), \tag{7}$$

which can be written in terms of octonion units as

$$\langle e_A\,,\, e_B\,\rangle \;= \frac{1}{2}(e_A \bar{e}_B + e_B \bar{e}_A) = \frac{1}{2}(\bar{e}_A e_B + \bar{e}_B e_A) = \delta_{AB}. \tag{8}$$

The norm of the octonion $N(x)$ is defined as

$$N(x)=\bar{x}\,x=x\,\bar{x}=\sum_{\alpha=0}^{7}x_{\alpha}^{2}e_{0}, \tag{9}$$

which is zero if $x=0$, and is always positive otherwise. It also satisfies the following property of normed algebra

$$N(xy)=N(x)\,N(y)=N(y)\,N(x). \tag{10}$$

As such, for a nonzero octonion x, we define its inverse as

$$x^{-1}=\frac{\bar{x}}{N(x)}, \tag{11}$$

which shows that

$$x^{-1}x=x\,x^{-1}=1.e_{0};\quad (xy)^{-1}=y^{-1}x^{-1}. \tag{12}$$

3. Octonion Generalized Dirac-Maxwell's (GDM) equations

In order to consider the generalized electromagnetic fields of dyon [34, 38], we may write the various quantum equations of dyons in octonion formulation. Thus the octonion valued potential, in eight dimensional formalism as the combinations of two four dimensional spaces, is defined as

$$\mathbf{V}\left\{V_{0},V_{1},V_{2},V_{3},V_{4},V_{5},V_{6},V_{7}\right\}=\sum_{\mu=0}^{7}e_{\mu}V_{\mu}. \tag{13}$$

We may now identify the components of generalized potential of dyons as

$$\begin{aligned}&V_{0}\longmapsto\varphi,\quad V_{1}\longmapsto A_{x},\qquad V_{2}\longmapsto A_{y},\quad V_{3}\longmapsto A_{z},\\&V_{4}\longmapsto iB_{x},\quad V_{5}\longmapsto iB_{y},\ \ V_{6}\longmapsto iB_{z},\quad V_{7}\longmapsto i\phi,\quad (i=\sqrt{-1})\end{aligned} \tag{14}$$

where $(\phi,\,A_{x},\,A_{y},\,A_{z})=(\phi,\overrightarrow{A})\equiv\{A^{\mu}\}$ and $(\varphi,\,B_{x},\,B_{y},\,B_{z})=(\varphi,\overrightarrow{B})\equiv\{B^{\mu}\}$ are respectively described as the components of electric $\{A_{\mu}\}$ and magnetic $\{B_{\mu}\}$ four potentials of dyons. In order to obtain the generalized field equations of dyons in four dimensional space time, we may identify differential operator to be four dimensional, so that the differential operator be written as

$$\mathcal{D}\left\{e_{1},e_{2},e_{3},e_{7}\right\}=e_{j}\partial_{j}+e_{7}\partial_{7}, \tag{15}$$

where $\partial_{7}=-i\frac{\partial}{\partial t}\quad(i=\sqrt{-1})$, $\partial_{j}=\frac{\partial}{\partial x_{j}}\quad(j=1,2,3)$ and other components ∂_{j+3} may be taken vanishing as we are concerned with classical electrodynamics of dyons in four dimensional space-time world [34, 35]. Octonion conjugate of equation (15) may then be written as

$$\bar{\mathcal{D}}\left\{e_{1},e_{2},e_{3},e_{7}\right\}=\,-\,e_{j}\partial_{j}-e_{7}\partial_{7}. \tag{16}$$

Now operating $\bar{\mathcal{D}}$ given by equation (16) to octonion potential $\mathbf{V}$ of equation (13) for the octonionic potential wave equations, we get

$$\begin{aligned}&\bar{\mathcal{D}}\,\mathbf{V}=\mathbf{F}\left\{F_{0},\,F_{1},\,F_{2},\,F_{3},\,F_{4},\,F_{5},\,F_{6},\,F_{7}\right\}\\&=\sum_{\mu=0}^{7}e_{\mu}F_{\mu},\end{aligned} \tag{17}$$

where **F** is an octonion reproduces the generalized electromagnetic fields of dyons, may be express as

$$\begin{aligned}
F_1 &= (-\partial_1\varphi + \partial_2 A_z - \partial_3 A_y - i\partial_7 B_x) \mapsto H_x, \\
F_2 &= (-\partial_2\varphi + \partial_3 A_x - \partial_1 A_z - i\partial_7 B_y) \mapsto H_y, \\
F_3 &= (-\partial_3\varphi + \partial_1 A_y - \partial_2 A_x - i\partial_7 B_z) \mapsto H_z, \\
F_4 &= i\,(-\partial_1\phi - \partial_2 B_z + \partial_3 B_y - i\partial_7 A_x) \mapsto i\,E_x, \\
F_5 &= i\,(-\partial_2\phi - \partial_3 B_x + \partial_1 B_z - i\partial_7 A_y) \mapsto i\,E_y, \\
F_6 &= i\,(-\partial_3\phi - \partial_1 B_y + \partial_2 B_x - i\partial_7 A_z) \mapsto i\,E_z, \\
F_0 &= -(\overrightarrow{\nabla} \cdot \overrightarrow{A} + i\partial_7\phi) \mapsto 0, \quad F_7 = i\,(\overrightarrow{\nabla} \cdot \overrightarrow{B} + i\partial_7\varphi) \mapsto 0.
\end{aligned} \tag{18}$$

Here $\overrightarrow{E}\,(E_x, E_y, E_z)$ and $\overrightarrow{H}\,(H_x, H_y, H_z)$ are the components of electric and magnetic four potentials, and $F_0 = F_7 = 0$ (Lorentz gauge conditions). The generalized electric $(\overrightarrow{E})$ and magnetic $(\overrightarrow{H})$ fields of dyons in terms of components of electric and magnetic four potentials

$$\begin{aligned}
\overrightarrow{E} &= -\frac{\partial \overrightarrow{A}}{\partial t} - \overrightarrow{\nabla}\phi - \overrightarrow{\nabla} \times \overrightarrow{B}, \\
\overrightarrow{H} &= -\frac{\partial \overrightarrow{B}}{\partial t} - \overrightarrow{\nabla}\varphi + \overrightarrow{\nabla} \times \overrightarrow{A}.
\end{aligned} \tag{19}$$

Now applying the differential operator (15) to equation (17), we get

$$\mathbf{DF} = \mathbf{J}\,\{J_0,\, J_1,\, J_2,\, J_3,\, J_4,\, J_5,\, J_6,\, J_7\} = \sum_{\mu=0}^{7} e_\mu J_\mu. \tag{20}$$

We may now identify the components of generalized current of dyons as

$$\begin{aligned}
&J_0 \longmapsto -\varrho, \quad J_1 \longmapsto \mathrm{j_x}, \qquad J_2 \longmapsto \mathrm{j_y}, \quad J_3 \longmapsto \mathrm{j_z}, \\
&J_4 \longmapsto -ik_x, \quad J_5 \longmapsto -ik_y, \quad J_6 \longmapsto -ik_z, \quad J_7 \longmapsto i\rho.
\end{aligned} \tag{21}$$

Here $(\rho,\, \overrightarrow{j}) = \{\mathrm{j}_\mu\}$ and $(\varrho,\, \overrightarrow{j}) = \{k_\mu\}$ are respectively the four currents associated with electric charge and magnetic monopole of dyons. Generalized Dirac-Maxwell's equations represent one of the most elegant and concise ways to state the fundamentals of electricity and magnetism. Thus, equations (20) and (21) thus lead to following differential equations

$$\begin{aligned}
(\overrightarrow{\nabla} \cdot \overrightarrow{E}) \; &= \rho, \\
(\overrightarrow{\nabla} \times \overrightarrow{E}) &= -\frac{\partial \overrightarrow{H}}{\partial t} - \vec{k}, \\
(\overrightarrow{\nabla} \times \overrightarrow{H}) &= \frac{\partial \overrightarrow{E}}{\partial t} + \overrightarrow{j}, \\
(\overrightarrow{\nabla} \cdot \overrightarrow{H}) \; &= \varrho,
\end{aligned} \tag{22}$$

which are the generalized Dirac-Maxwell's (GDM) equations of generalized fields of dyons. Octonion formulation is compact and simpler.

4. Role of octonion in Standard Model and GUTs

Let us start with the local $SU(3)\times SU(2)\times U(1)$ gauge symmetry which is an internal symmetry that Standard Model. The smallest simple Lie group which contains the standard model, and upon which the first Grand Unified Theory (GUT) was based, is $SU(5)\supset SU(3)\times SU(2)\times U(1)$. Here we may extend $SU(2)\times U(1)$ gauge theory [37] to the $SU(3)\times SU(2)\times U(1)$ gauge theory in terms of split octonion formulation [35]. We may described the $SU(3)\times SU(2)\times U(1)$ gauge potential as

$$\begin{aligned} A_{\mu} &\mapsto A_{\mu}^{0}+A_{\mu}^{a}e_{a}+A_{\mu}^{\alpha}e_{\alpha}, \\ B_{\mu} &\mapsto B_{\mu}^{0}+B_{\mu}^{a}e_{a}+B_{\mu}^{\alpha}e_{\alpha}, (\forall\mu=0,1,2,3;\ a=1,2,3;\ \alpha=1,...,8.) \end{aligned} \tag{23}$$

where the components of electric A_{μ}^{0} and magnetic B_{μ}^{0} are the four potentials of dyons in case of $U(1)$ while respectively the A_{μ}^{a},B_{μ}^{a} and A_{μ}^{α},B_{μ}^{α} describe of the $SU(2)$ and $SU(3)$ gauge field theory. So, the covariant derivative in the case of $SU(3)\times SU(2)\times U(1)$ octonion gauge field in the split octonion form (2×2 Zorn's matrix) may be expressed as [35, 37, 38]

$$D_{\mu}=\left(\begin{array}{cc} \partial_{\mu}+A_{\mu}^{0}+A_{\mu}^{a}e_{a}+A_{\mu}^{\alpha}e_{\alpha} & 0\\ 0 & \partial_{\mu}+B_{\mu}^{0}+B_{\mu}^{a}e_{a}+B_{\mu}^{\alpha}e_{\alpha} \end{array}\right). \tag{24}$$

Similarly

$$D_{\nu}=\left(\begin{array}{cc} \partial_{\nu}+A_{\nu}^{0}+A_{\nu}^{a}e_{a}+A_{\nu}^{\alpha}e_{\alpha} & 0\\ 0 & \partial_{\nu}+B_{\nu}^{0}+B_{\nu}^{a}e_{a}+B_{\nu}^{\alpha}e_{\alpha} \end{array}\right). \tag{25}$$

On subtraction, i.e. $[D_{\mu},D_{\nu}]=D_{\mu}D_{\nu}-D_{\nu}D_{\mu}$, these equations reduce to

$$[D_{\mu},D_{\nu}]=\left(\begin{array}{cc} G_{\mu\nu}^{0}+G_{\mu\nu}^{a}e_{a}+G_{\mu\nu}^{\alpha}e_{\alpha} & 0\\ 0 & \mathsf{G}_{\mu\nu}^{0}+\mathsf{G}_{\mu\nu}^{\mathsf{a}}e_{a}+\mathsf{G}_{\mu\nu}^{\alpha}e_{\alpha} \end{array}\right)\longmapsto\mathbf{G}_{\mu\nu}^{\alpha}, \tag{26}$$

which is $SU(3)\times SU(2)\times U(1)$ octonion gauge field strength for dyons in 2×2 Zorn matrix realization. In equation (26)

$$\begin{aligned} G_{\mu\nu}^{0} &= \partial_{\mu}A_{\nu}^{0}-\partial_{\nu}A_{\mu}^{0}+\left[A_{\mu}^{0},A_{\nu}^{0}\right]\longmapsto E_{\mu\nu}^{0}, \quad (U(1)_{e}\,\text{gauge})\\ G_{\mu\nu}^{a} &= \partial_{\mu}A_{\nu}^{a}-\partial_{\nu}A_{\mu}^{a}+e_{a}\left[A_{\mu}^{a},A_{\nu}^{a}\right]\longmapsto E_{\mu\nu}^{a}, \quad (SU(2)_{e}\,\text{gauge})\\ G_{\mu\nu}^{\alpha} &= \partial_{\mu}A_{\nu}^{\alpha}-\partial_{\nu}A_{\mu}^{\alpha}+e_{a}\left[A_{\mu}^{\alpha},A_{\nu}^{\alpha}\right]\longmapsto E_{\mu\nu}^{\alpha}, \quad (SU(3)_{e}\,\text{gauge}) \end{aligned} \tag{27}$$

are the constituents of $U(1)_{e}\times SU(2)_{e}\times SU(3)_{e}$ gauge structures in presence of electric charge. Similarly

$$\begin{aligned} \mathsf{G}_{\mu\nu}^{0} &= \partial_{\mu}B_{\nu}^{0}-\partial_{\nu}B_{\mu}^{0}+\left[B_{\mu}^{0},B_{\nu}^{0}\right]\longmapsto H_{\mu\nu}^{0}, \quad (U(1)_{m}\,\text{gauge})\\ \mathsf{G}_{\mu\nu}^{\mathsf{a}} &= \partial_{\mu}B_{\nu}^{a}-\partial_{\nu}B_{\mu}^{a}+e_{a}\left[B_{\mu}^{a},B_{\nu}^{a}\right]\longmapsto H_{\mu\nu}^{a}, \quad (SU(2)_{m}\,\text{gauge})\\ \mathsf{G}_{\mu\nu}^{\alpha} &= \partial_{\mu}B_{\nu}^{\alpha}-\partial_{\nu}B_{\mu}^{\alpha}+e_{a}\left[B_{\mu}^{\alpha},B_{\nu}^{\alpha}\right]\longmapsto H_{\mu\nu}^{\alpha}, \quad (SU(3)_{m}\,\text{gauge}) \end{aligned} \tag{28}$$

are the constituents of $U(1)_{m}\times SU(2)_{m}\times SU(3)_{m}$ gauge structure in presence of magnetic monopole. Now operating D_{μ} given by the equation (24) to the octonion gauge field strength $\mathbf{G}_{\mu\nu}^{\alpha}$ (26), we get

$$D_{\mu}\mathbf{G}_{\mu\nu}^{\alpha}=\mathbf{J}_{\nu}^{\alpha}. \tag{29}$$

Here $\mathbf{J}_\nu^\alpha$ is $U(1)\times SU(2)\times SU(3)$ form of octonion gauge current for dyons which may be expressed in term of 2×2 Zorn matrix as

$$\mathbf{J}_\nu^\alpha=\begin{pmatrix} j_\nu^0+j_\nu^a e_a+j_\nu^\alpha e_\alpha & 0\\ 0 & k_\nu^0+k_\nu^a e_a+k_\nu^\alpha e_\alpha\end{pmatrix}, \tag{30}$$

from which we may get following field equations of dyons

$$\begin{aligned} j_\nu^0 &= \partial_\mu G_{\mu\nu}^0; \quad (\forall\,\mu,\nu=0,1,2,3)\\ j_\nu^a &= \partial_\mu G_{\mu\nu}^a; \quad (\forall\,a=1,2,3)\\ j_\nu^\alpha &= \partial_\mu G_{\mu\nu}^\alpha; \quad (\forall\,\alpha=1,...,8)\\ k_\nu^0 &= \partial_\mu G_{\mu\nu}^0; \quad (\forall\,\mu,\nu=0,1,2,3)\\ k_\nu^a &= \partial_\mu G_{\mu\nu}^a; \quad (\forall\,a=1,2,3)\\ k_\nu^\alpha &= \partial_\mu G_{\mu\nu}^\alpha, \quad (\forall\,\alpha=1,...,8) \end{aligned} \tag{31}$$

were j_ν^0 is the $U(1)$ current for electric charge, j_ν^a is the $SU(2)$ weak current associated with electric charge and j_ν^α is the current associated with $SU(3)_c$ used for chromo electric charge. On the other hand k_ν^0 is $U(1)$ the counterpart of the four current, k_ν^a is the $SU(2)$ weak current while the k_ν^α is $SU(3)_c$ gluonic current due to the presence of magnetic monopole.

As such, the octonionic formulation regardless a generalization of GUTs for the mixing of gauge currents used for $U(1)$, $SU(2)$ and $SU(3)_c$ sectors associated respectively with the electromagnetic, weak and strong interactions (except the gravitational interaction) in presence of dyons showing the duality invariance as well. Consequently, the continuity equation is generalized as

$$D_\mu\mathbf{J}_\mu^\alpha=\begin{pmatrix} \partial_\mu j_\mu^0+\partial_\mu j_\mu^a e_a+\partial_\mu j_\mu^\alpha e_\alpha & 0\\ 0 & \partial_\mu k_\nu^0+\partial_\mu k_\nu^a e_a+\partial_\mu k_\nu^\alpha e_\alpha\end{pmatrix}=0. \tag{32}$$

In the next section, we have discussed the role of octonion in presence of gravitational field together with electromagnetic field.

5. Octonionic gravitational and electromagnetic interactions

Let us identify the octonion space (eight dimensional) as the combination of two quaternionic spaces namely associated with the gravitational interaction (G-space) and electromagnetic interaction (EM-space) [32, 33]. So, we may write the octonionic (gravitational-electromagnetic) space as

$$\mathcal{O}=(\mathcal{O}_{g-space}\,,\ \mathcal{O}_{em-space})\Longrightarrow((e_0,e_1,e_2,e_3)\,,\ (e_4,e_5,e_6,e_7))\,, \tag{33}$$

where $(\mathcal{O}_{g-space})$ is octonionic gravitational space consists e_0,e_1,e_2,e_3 octonion basis and $(\mathcal{O}_{em-space})$ is octonionic electromagnetic space consists e_4,e_5,e_6,e_7. So,

$$\mathcal{O}=(e_0,e_1,e_2,e_3,e_4,e_5,e_6,e_7)=(\mathcal{O}_g+\mathcal{O}_{em}). \tag{34}$$

Any physical quantity $X\in\mathcal{O}$ may be written as

$$X=X_g+X_{em}=\sum_{j=0}^{3}\left(X_{g_j}e_j\right)+e_7\sum_{j=0}^{3}\left(X_{em_j}e_j\right). \tag{35}$$

Accordingly, the octonion differential operator $\mathcal{D}$ also may be written as the combination of the two quaternionic space (G-space & EM-space) in the terms of eight dimensional space as

$$\mathcal{D}=\mathcal{D}_{g}+\mathcal{D}_{em}=\sum_{j=0}^{3}\left(\partial_{g_{j}}e_{j}\right)+e_{7}\sum_{k=0}^{3}\left(\partial_{em_{k}}e_{k}\right). \tag{36}$$

Thus, the octonion conjugate of equation (36) may then be written as

$$\overline{\mathcal{D}}=\overline{\mathcal{D}}_{g}+\overline{\mathcal{D}}_{em}=\partial_{g_{0}}e_{0}-\sum_{j=1}^{3}\left(\partial_{g_{j}}e_{j}\right)-e_{7}\sum_{k=0}^{3}\left(\partial_{em_{k}}e_{k}\right). \tag{37}$$

Accordingly, the octonion valued potential, in eight dimensional formalism may also be written as the combinations of two four dimensional quaternionic spaces (i.e. G-space and EM-space) as

$$\mathbf{V}=\left(V_{g}\,,\;V_{em}\right)=\;\left(\left(V_{g_{0}},V_{g_{1}},V_{g_{2}},V_{g_{3}}\right),\;\left(V_{em_{0}},V_{em_{1}},V_{em_{2}},V_{em_{3}}\right)\right), \tag{38}$$

which can further be reduced to

$$\mathbf{V}=\sum_{j=0}^{3}\left(V_{g_{j}}e_{j}\right)+e_{7}\sum_{k=0}^{3}\left(V_{em_{k}}e_{k}\right). \tag{39}$$

As such, we may obtain the octonion potential wave equation for gravitational-electromagnetic space by operating $\overline{\mathcal{D}}$ given by equation (37) to octonion potential $\mathbf{V}$ we get

$$\overline{\mathcal{D}}\,\mathbf{V}=\mathbf{F}=\left(\left(F_{0},F_{1},F_{2},F_{3}\right),\left(F_{4},F_{5},F_{6},F_{7}\right)\right), \tag{40}$$

where $\mathbf{F}(F_{0},F_{1},F_{2},F_{3},F_{4},F_{5},F_{6},F_{7})$ is also an octonion reproduces the field strength [34] of generalized gravitational-electromagnetic fields. Thus, we may be express $\mathbf{F}$ as

$$\mathbf{F}=F_{g}+F_{em}=\sum_{j=0}^{3}\left(F_{g_{j}}e_{j}\right)+e_{7}\sum_{k=0}^{3}\left(F_{em_{k}}e_{k}\right), \tag{41}$$

here the components of $\mathbf{F}\,(F_{g_{0}},\,F_{g_{1}},\,F_{g_{2}},\,F_{g_{3}},\,F_{em_{0}},\,F_{em_{1}},\,F_{em_{2}},\,F_{em_{3}})$ are expressed as

$$\begin{aligned}
F_{g_{0}}&=\{(\partial_{g_{0}}V_{g_{0}}+\partial_{g_{1}}V_{g_{1}}+\partial_{g_{2}}V_{g_{2}}+\partial_{g_{3}}V_{g_{3}})+e_{7}(-\partial_{em_{3}}V_{g_{0}}+\partial_{em_{0}}V_{g_{1}}+\partial_{em_{1}}V_{g_{2}}+\partial_{em_{2}}V_{g_{3}})\},\\
F_{g_{1}}&=\{(\partial_{g_{0}}V_{g_{1}}-\partial_{g_{1}}V_{g_{0}}-\partial_{g_{2}}V_{g_{3}}+\partial_{g_{3}}V_{g_{2}})+e_{7}(-\partial_{em_{0}}V_{g_{0}}+\partial_{em_{1}}V_{g_{3}}-\partial_{em_{2}}V_{g_{2}}-\partial_{em_{3}}V_{g_{1}})\},\\
F_{g_{2}}&=\{(\partial_{g_{0}}V_{g_{2}}-\partial_{g_{2}}V_{g_{0}}+\partial_{g_{1}}V_{g_{3}}-\partial_{g_{3}}V_{g_{1}})+e_{7}(-\partial_{em_{1}}V_{g_{0}}-\partial_{em_{0}}V_{g_{3}}+\partial_{em_{2}}V_{g_{1}}-\partial_{em_{3}}V_{g_{2}})\},\\
F_{g_{3}}&=\{(\partial_{g_{0}}V_{g_{3}}-\partial_{g_{3}}V_{g_{0}}-\partial_{g_{1}}V_{g_{2}}+\partial_{g_{2}}V_{g_{1}})+e_{7}(-\partial_{em_{2}}V_{g_{0}}+\partial_{em_{0}}V_{g_{2}}-\partial_{em_{1}}V_{g_{1}}-\partial_{em_{3}}V_{g_{3}})\},\\
F_{em_{0}}&=\{(\partial_{g_{0}}V_{em_{0}}+\partial_{g_{1}}V_{em_{3}}+\partial_{g_{2}}V_{em_{2}}-\partial_{g_{3}}V_{em_{1}})+e_{7}(-\partial_{em_{0}}V_{em_{3}}+\partial_{em_{1}}V_{em_{2}}-\partial_{em_{2}}V_{em_{1}}+\partial_{em_{3}}V_{em_{0}})\},\\
F_{em_{1}}&=\{(\partial_{g_{0}}V_{em_{1}}-\partial_{g_{1}}V_{em_{2}}+\partial_{g_{2}}V_{em_{3}}+\partial_{g_{3}}V_{em_{0}})+e_{7}(-\partial_{em_{0}}V_{em_{2}}-\partial_{em_{1}}V_{em_{3}}+\partial_{em_{2}}V_{em_{0}}+\partial_{em_{3}}V_{em_{1}})\},\\
F_{em_{2}}&=\{(\partial_{g_{0}}V_{em_{2}}+\partial_{g_{1}}V_{em_{1}}-\partial_{g_{2}}V_{em_{0}}+\partial_{g_{3}}V_{em_{3}})+e_{7}(-\partial_{em_{1}}V_{em_{0}}+\partial_{em_{0}}V_{em_{1}}-\partial_{em_{2}}V_{em_{3}}+\partial_{em_{3}}V_{em_{2}})\},\\
F_{em_{3}}&=\{(\partial_{g_{0}}V_{em_{3}}-\partial_{g_{1}}V_{em_{0}}-\partial_{g_{2}}V_{em_{1}}-\partial_{g_{3}}V_{em_{2}})+e_{7}(\partial_{em_{0}}V_{em_{0}}+\partial_{em_{1}}V_{em_{1}}+\partial_{em_{2}}V_{em_{2}}+\partial_{em_{3}}V_{em_{3}})\},
\end{aligned} \tag{42}$$

using the Lorentz Gauge conditions in the equation (42), i.e. $F_{g_{0}}=F_{em_{3}}=0$. Thus, equation (41) may be written as

$$\mathbf{F}=F_{g}+F_{em}=(F_{g_{1}}e_{1}+F_{g_{2}}e_{2}+F_{g_{3}}e_{3})+(F_{em_{0}}e_{4}+F_{em_{1}}e_{5}+F_{em_{2}}e_{6}). \tag{43}$$

Here, the first term ($F_{g}=F_{g_{1}},F_{g_{2}},F_{g_{3}}$) is defined as the field strength of the gravitational interaction in G-space while the second term ($F_{em}=F_{em_{0}},F_{em_{1}},F_{em_{2}}$) is associated with the field strength of the electromagnetic interaction in EM-space. Hence, we may obtain the octonionic field equation in gravitational-electromagnetic space on applying the differential operator (36) to equation (43) we get the compact notation in terms of an octonionic gravitational-electromagnetic space as

$$\mathcal{D}\mathbf{F}=\mathbf{J}=\left(\left(J_{0},\,J_{1},\,J_{2},J_{3}\right),\left(J_{4},\,J_{5},\,J_{6},\,J_{7}\right)\right),\tag{44}$$

where $\mathbf{J}(J_0,J_1,J_2,J_3,J_4,J_5,J_6,J_7)$ is also an octonion reproduces the field current source. So, it may be expressed as

$$\mathbf{J}=\sum_{j=0}^{3}\left(J_{(g-g)_{j}}+J_{(em-em)_{k}}\right)e_{j}+e_{7}\sum_{k=0}^{3}\left(J_{(em-g)_{k}}+J_{(g-em)_{k}}\right)e_{k}.\tag{45}$$

Here $J_{(g-g)}$, $J_{(em-em)}$, $J_{(em-g)}$, $J_{(g-em)}$ are defined for the octonionic current source [34, 35, 36] respectively for gravitational-gravitational, electromagnetic-electromagnetic, electromagnetic-gravitational, gravitational-electromagnetic interaction. As such, the components of octonionic current source $\mathbf{J}$ are described as

$$\begin{aligned}
J_{(g-g)_{0}}&=(\partial_{g1}F_{g1}+\partial_{g2}F_{g2}+\partial_{g3}F_{g3}), & J_{(em-em)_{0}}&=(\partial_{em_{0}}F_{em_{0}}+\partial_{em_{1}}F_{em_{1}}+\partial_{em_{2}}F_{em_{2}}),\\
J_{(g-g)_{1}}&=(\partial_{g0}F_{g1}-\partial_{g2}F_{g3}+\partial_{g3}F_{g2}), & J_{(em-em)_{1}}&=(-\partial_{em_{1}}F_{em_{2}}+\partial_{em_{2}}F_{em_{1}}-\partial_{em_{3}}F_{em_{0}}),\\
J_{(g-g)_{2}}&=(\partial_{g0}F_{g2}-\partial_{g1}F_{g3}+\partial_{g3}F_{g1}), & J_{(em-em)_{2}}&=(-\partial_{em_{2}}F_{em_{0}}+\partial_{em_{0}}F_{em_{2}}-\partial_{em_{3}}F_{em_{1}}),\\
J_{(g-g)_{3}}&=(\partial_{g0}F_{g3}+\partial_{g1}F_{g2}-\partial_{g2}F_{g1}), & J_{(em-em)_{3}}&=(-\partial_{em_{0}}F_{em_{1}}+\partial_{em_{1}}F_{em_{0}}-\partial_{em_{3}}F_{em_{2}}),\\
J_{(em-g)_{0}}&=(\partial_{g0}F_{em_{0}}-\partial_{g2}F_{em_{2}}+\partial_{g3}F_{em_{1}}), & J_{(g-em)_{0}}&=(-\partial_{em_{1}}F_{g3}+\partial_{em_{2}}F_{g2}+\partial_{em_{3}}F_{g1}),\\
J_{(em-g)_{1}}&=(\partial_{g0}F_{em_{1}}+\partial_{g1}F_{em_{2}}-\partial_{g3}F_{em_{0}}), & J_{(g-em)_{1}}&=(-\partial_{em_{2}}F_{g1}+\partial_{em_{0}}F_{g3}+\partial_{em_{3}}F_{g2}),\\
J_{(em-g)_{2}}&=(\partial_{g0}F_{em_{2}}-\partial_{g1}F_{em_{1}}+\partial_{g2}F_{em_{0}}), & J_{(g-em)_{2}}&=(-\partial_{em_{0}}F_{g2}+\partial_{em_{1}}F_{g1}+\partial_{em_{3}}F_{g3}),\\
J_{(em-g)_{3}}&=(\partial_{g1}F_{em_{0}}+\partial_{g2}F_{em_{1}}+\partial_{g3}F_{em_{2}}), & J_{(g-em)_{3}}&=(-\partial_{em_{0}}F_{g1}-\partial_{em_{1}}F_{g2}-\partial_{em_{2}}F_{g3}),
\end{aligned}\tag{46}$$

which are analogous to the generalized Dirac-Maxwell's (GDM) equations (22) in presence of gravitational-gravitational (G-G), electromagnetic-electromagnetic (EM-EM), electromagnetic-gravitational (EM-G), gravitational-electromagnetic (G-EM) interaction.

Consequently, the octonionic radius vector ($\mathbf{R}=R_0,R_1,R_2,R_3,R_4,R_5,R_6,R_7$) is defined the combination of two quaternionic space in the following manner

$$\mathbf{R}=\left(R_{0},R_{1},R_{2},R_{3}\right),\left(R_{4},R_{5},R_{6},R_{7}\right)=\sum_{j=0}^{3}R_{j}e_{j}+\sum_{k=4}^{7}R_{k}e_{k},\tag{47}$$

which yields the velocity ($\mathbf{v}$) in the octonionic (gravitational-electromagnetic) representation as

$$\mathbf{v}=\frac{\partial\mathbf{R}}{\partial t}=\frac{\partial}{\partial t}\left(\sum_{j=0}^{3}R_{j}e_{j}+\sum_{k=4}^{7}R_{k}e_{k}\right)=\sum_{j=0}^{3}v_{j}e_{j}+\sum_{k=4}^{7}v_{k}e_{k}.\tag{48}$$

We may also described the charge and mass of the particle in octonionic space as

$$\begin{aligned}
J_{(g-g)_{j}} &\cong\sum_{j=0}^{3}\left(Q_{(g-g)_{j}}v_{j}\right), & J_{(em-g)_{k}} &\cong\sum_{j,k=0}^{3}\left(Q_{(em-g)_{k}}v_{j+3}\right),\\
J_{(em-em)_{j}} &\cong\sum_{j=0}^{3}\left(Q_{(em-em)_{j}}v_{j}\right), & J_{(g-em)_{k}} &\cong\sum_{j,k=0}^{3}\left(Q_{(g-em)_{k}}v_{j+3}\right),
\end{aligned}\tag{49}$$

where $Q_{(g-g)},Q_{(em-g)},Q_{(g-em)}$ are respectively denoted the "Mass" of the gravitational - gravitational (G-G), electromagnetic - gravitational (EM-G), gravitational - electromagnetic (G-EM) interactions while $Q_{(em-em)}$ represent the "Charge" of the electromagnetic -

electromagnetic (EM-EM) interaction. So, the $Q_{(g-g)}$,$Q_{(em-g)}$, $Q_{(g-em)}$ and $Q_{(em-em)}$ respectively describe the "Generalized mass" and "Generalized charge". From the equations (45), (46) and (49), we may obtain the four-type of subfields in the octonionic electromagnetic-gravitational fields as

- Gravitational-Gravitational (G-G) subfield.
- Electromagnetic-Gravitational (EM-G) subfield.
- Electromagnetic-Electromagnetic (EM-EM) subfield.
- Gravitational-Electromagnetic (G-EM) subfield.

Thus, from above four subfields, we have describe the octonion dark matter in the following sections. The octonionic model of dark matter is distinct with respect to the standard model and GUTs.

6. Dark matter

The *Dark Matter* [20, 21, 22] is a type of matter hypothesized to account for a large part of the total mass in the universe. Although dark matter had historically been inferred by many astronomical observations, its composition long remained speculative. Early theories of dark matter concentrated on hidden heavy normal objects, such as black holes, neutron stars, faint old white dwarfs, brown dwarfs, as the possible candidates for dark matter, collectively known as massive compact halo objects or MACHOs. Furthermore, data from a number of lines of other evidence, including galaxy rotation curves, gravitational lensing, structure formation, and the fraction of baryons in clusters and the cluster abundance combined with independent evidence for the baryon density, indicated that 85–90% of the mass in the universe does not interact with the electromagnetic force. The majority of dark matter in the universe cannot be baryons, and thus does not form atoms. This nonbaryonic dark matter is evident through its gravitational effect. Consequently, the most commonly held view was that dark matter is primarily non-baryonic, made of one or more elementary particles other than the usual electrons, protons, and neutrons. The most commonly proposed particles [20, 21, 22] then became WIMPs (Weakly Interacting Massive Particles, including neutralinos), or axions, or sterile neutrinos, though many other possible candidates have been proposed. Accordingly the nonbaryonic dark matter also may include the photon, graviton, intermediate bosons and neutrinos, or supersymmetric particles etc. Nonbaryonic dark matter is classified in terms of the mass of the particles that is assumed to make it up, or the typical velocity dispersion of those particles (since more massive particles move more slowly). There are mainly two prominent hypotheses on nonbaryonic dark matter, called cold dark matter (CDM) and hot dark matter (HDM). Determining the nature of this dark matter is one of the most important problems in modern cosmology and particle physics. So, in terms of higher dimensional theory, we have made an attempt to express the nonbaryonic dark matter in terms of octonion representation in the following manner.

7. Octonionic hot dark matter (OHDM)

Octonionic hot dark matter assumed to compose of particles that have zero or near-zero mass. The special theory of relativity requires that massless particles move at the speed of light while near-zero mass particles move at nearly the speed of light. Thus, the octonionic hot dark matter may be associated with the gravitational-gravitational (G-G) and electromagnetic-electromagnetic (EM-EM) subfields. Thus, we may write the quantum equation for octonionic hot dark matter in terms of potential, field and current equations. So, the potential wave equations from (35) and (36), may be written in the quaternionic (G-G) space as

$$\mathcal{D}_g X_g = \sum_{j=0}^{3}\left(\partial_{g_j} e_j\right)\cdot\sum_{j=0}^{3}\left(X_{g_j} e_j\right) = \sum_{j=0}^{3}\left(V_{(g-g)_j} e_j\right), \tag{50}$$

which may further be written for EM-EM sector as

$$\mathcal{D}_{em} X_{em} = \sum_{j=0}^{3}\left(\partial_{em_j} e_{j+4}\right)\cdot\sum_{j=0}^{3}\left(X_{em_j} e_{j+4}\right) = \sum_{j=0}^{3}\left(V_{(em-em)_j} e_j\right). \tag{51}$$

Thus, equations (37) and (39) reduces to

$$\overline{\mathcal{D}}_g V_g = \partial_{g_0} e_0 - \sum_{j=1}^{3}\left(\partial_{g_j} e_j\right)\cdot\sum_{j=0}^{3}\left(V_{g_j} e_j\right) = \sum_{j=0}^{3}\left(F_{(g-g)_j} e_j\right), \tag{52}$$

and

$$\overline{\mathcal{D}}_{em} V_{em} = -\sum_{j=0}^{3}\left(\partial_{em_j} e_{j+4}\right)\cdot\sum_{j=0}^{3}\left(V_{em_j} e_{j+4}\right) = \sum_{j=0}^{3}\left(F_{(em-em)_j} e_j\right). \tag{53}$$

Accordingly, the field source equations from (35) and (43), are respectively described as

$$\mathcal{D}_g F_g = \sum_{j=0}^{3}\left(\partial_{g_j} e_j\right)\cdot\sum_{j=0}^{3}\left(F_{g_j} e_j\right) = \sum_{j=0}^{3}\left(J_{(g-g)_j} e_j\right), \tag{54}$$

and

$$\mathcal{D}_{em} F_{em} = \sum_{j=0}^{3}\left(\partial_{em_j} e_{j+4}\right)\cdot\sum_{j=0}^{3}\left(F_{em_j} e_{j+4}\right) = \sum_{j=0}^{3}\left(J_{(em-em)_j} e_j\right). \tag{55}$$

These two equations (54), (55) describe the generalized fields equations (22) in terms of octonionic hot dark matter comparizing gravitational-gravitational (G-G) and electromagnetic-electromagnetic (EM-EM) interactions. Hence, we may conclude that the quantum equations for octonionic hot dark matter (i.e. like as photon and graviton) are expressed in the terms of quaternionic representations of octonions.

8. Octonionic cold dark matter (OCDM)

Like wise, the octonionic cold dark matter may be described as the composition of the massive objects moving at sub-relativistic velocities. So, the difference between the octonionic cold dark matter (OCDM) and the octonionic hot dark matter (OHDM) is significant in the formulation of structure, because the velocities of octonions hot dark matter cause it to wipe out structure on small scales. Thus, the octonionic cold dark matter is associated with the electromagnetic-gravitational (EM-G) and gravitational-electromagnetic (G-EM) subfields. Hence, the octonionic cold dark matter (OCDM) is assumed to include other intermediate massive particles (non baryonic). So, we may write the quantum equations for octonions cold dark matter in terms of potential, field and current equations. The potential wave equations from (35) and (36) may then be written respectively as

$$\mathcal{D}_{em} X_g = \sum_{j=0}^{3}\left(\partial_{em_j} e_{j+4}\right)\cdot\sum_{j=0}^{3}\left(X_{g_j} e_j\right) = \sum_{j=0}^{7}\left(V_{(em-g)_j} e_j\right), \tag{56}$$

and

$$\mathcal{D}_g X_{em} = \sum_{j=0}^{3}\left(\partial_{g_j} e_j\right)\cdot\sum_{j=0}^{3}\left(X_{em_j} e_{j+4}\right) = \sum_{j=0}^{7}\left(V_{(g-em)_j} e_j\right). \tag{57}$$

Accordingly, the field equations from (37) and (39) are respectively described as

$$\overline{\mathcal{D}}_{em} V_g = -\sum_{j=0}^{3}\left(\partial_{em_j} e_{j+4}\right)\cdot\sum_{j=0}^{3}\left(V_{g_j} e_j\right) = \sum_{j=0}^{7}\left(F_{(em-g)_j} e_j\right), \tag{58}$$

and

$$\overline{\mathcal{D}}_g V_{em} = \partial_{g0} e_0 - \sum_{j=1}^{3}\left(\partial_{g_j} e_j\right)\cdot\sum_{j=0}^{3}\left(V_{em_j} e_{j+4}\right) = \sum_{j=0}^{7}\left(F_{(g-em)_j} e_j\right). \tag{59}$$

On the other hand the field source equations (35) and (43) are expressed as

$$\mathcal{D}_{em} F_g = \sum_{j=0}^{3}\left(\partial_{em_j} e_{j+4}\right)\cdot\sum_{j=0}^{3}\left(F_{g_j} e_j\right) = \sum_{j=0}^{7}\left(J_{(em-g)_j} e_j\right), \tag{60}$$

and

$$\mathcal{D}_g F_{em} = \sum_{j=0}^{3}\left(\partial_{g_j} e_{j+4}\right)\cdot\sum_{j=0}^{3}\left(F_{em_j} e_j\right) = \sum_{j=0}^{7}\left(J_{(g-em)_j} e_j\right). \tag{61}$$

These equation on simplification, describe the fields equations and other quantum equations for octonionic cold dark matter in the presence of electromagnetic-gravitational (EM-G) and gravitational-electromagnetic (G-EM) interactions. So, the quantum equations for octonionic cold dark matter may easily be expressed in the terms of simpler and compact notation of octonions representations.

9. Results and discussion

The smallest simple Lie group which contains the standard model, and upon which the first Grand Unified Theory was based, is $SU(5) \supset SU(3) \times SU(2) \times U(1)$. Hence, we have obtained the field equations (27), (28) for the grand unified theory i.e. $SU(3) \times SU(2) \times U(1)$ gauge for the fields associated with dyons. Equation (31) represents the various gauge currents of GUTs namely the U(1) gauge current, SU(2) gauge current and SU(3) gauge current in presence of electric (magnetic) and magnetic (electric) charges. As such, the octonionic formulation regardless a generalization of GUTs for the mixing of gauge currents used for $U(1)$, $SU(2)$ and $SU(3)_c$ sectors associated respectively with the electromagnetic, weak and strong interactions in presence of dyons showing the duality invariance as well in terms of the continuity equation obtained in equation (32).

The dark matter has been considered as a type of matter hypothesized to account for a large part of the total mass in the universe. Dark matter cannot be seen directly with telescopes which is neither emits nor absorbs light or other electromagnetic radiation at any significant level. Instead, its existence and properties are inferred from its gravitational effects on visible matter, radiation and the large scale structure of the universe. Here matter and energy (which special relativity tells us are equivalent) are distinguished by their different dependence on the cosmic volume: matter density decreases with the inverse of the volume, while energy density remains (approximately) constant. Only about 4.6% of the mass-energy of the universe is ordinary matter, about 23% is thought to be composed of dark matter. The remaining 72% is thought to consist of dark energy, an even stranger component, distributed almost uniformly in space and with energy density non-evolving or slowly evolving with time.

On the other hand, in experiment point of views the summary of the current measurements of the matter density Ω_{matter}, the dark matter density Ω_{dark} and the energy density Ω_{energy} are given below [40, 41]. Each are in units of the critical density $\rho_{critical} \sim 3h^2/(8\pi G)$, where G is the Newton's gravitational constant, and h is the present value of the Hubble constant. Moreover, three types of observations-supernova measurements of the recent expansion history of the Universe, cosmic microwave background measurements of the degree of spatial flatness, and measurements of the amount of matter in galaxy structures obtained through big galaxy redshift surveys agree with each other in a region around the best current values of the matter and energy densities $\Omega_{matter} \sim 0.27$ and $\Omega_{energy} \sim 0.73$. Measurements of the baryon density [42, 43, 44] in the universe using the cosmic microwave background spectrum and primordial nucleosynthesis constrain the baryon density Ω_{baryon} to a value less than ~ 0.03. The difference $\Omega_{matter} - \Omega_{baryon} \sim 0.24$ must be in the form of non-baryonic dark matter.

In the Table 2, the matter density:

$$\Omega_{matter} \cong \Omega_{baryon} + \Omega_{Hot-dark-matter} + \Omega_{Cold-dark-matter},$$

includes both baryonic matter and dark matter, and moreover the dark matter can be classed either as *hot* or *cold* depending on whether it was relativistic or not in the early universe, which are already expressed in octonionic form. The total dark matter density is $\Omega_{dark} \cong \Omega_{Hot-dark-matter} + \Omega_{Cold-dark-matter}$.

It should be noted that the cold dark matter density is non-zero and grater than hot dark matter density. Thus cold dark matter may be expressed in octonionic eight dimensional form, whereas the hot dark matter density can only be a minor component and it can be easily expressed in quaternionic units. So, in higher dimensional octonion theory, the octonion dark model describes the both non-baryonic dark matter i.e. octonion hot dark matter and octonion cold dark matter. Octonionic hot dark matter is composed of particles that have zero or near-zero mass. As such, we have established the various quantum equation for octonionic hot dark matter in terms of potential, field and current equations given by equations (50)-(55). It is concluded that the quantum equations for octonionic hot dark matter are expressed in the terms of quaternionic representations of octonions. Accordingly, the octonionic cold dark matter has been described as the composition of the massive objects moving at sub-relativistic velocities. Hence, the octonionic cold dark matter (OCDM) includes the massive particles, i.e. the other intermediate non-baryonic particles. So, we have established the quantum equations for octonionic cold dark matter in terms of potential, field and current equations given by equations (56)-(61).

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$\cdot$	e_1	e_2	e_3	e_4	e_5	e_6	e_7
e_1	-1	e_3	$-e_2$	e_7	$-e_6$	e_5	$-e_4$
e_2	$-e_3$	-1	e_1	e_6	e_7	$-e_4$	$-e_5$
e_3	e_2	$-e_1$	-1	$-e_5$	e_4	e_7	$-e_6$
e_4	$-e_7$	$-e_6$	e_5	-1	$-e_3$	e_2	e_1
e_5	e_6	$-e_7$	$-e_4$	e_3	-1	$-e_1$	e_2
e_6	$-e_5$	e_4	$-e_7$	$-e_2$	e_1	-1	e_3
e_7	e_4	e_5	e_6	$-e_1$	$-e_2$	$-e_3$	-1

Table 1. Octonion multiplication table.

$0.49 \leq \Omega_{energy} \leq 0.74$	$0.49 \leq \Omega_{energy} \leq 0.76$
$0.20 \leq \Omega_{matter} \leq 0.50$	
$0.11 \leq h^2\Omega_{dark} \leq 0.17$	$0.09 \leq h^2\Omega_{dark} \leq 0.17$
$0.00 \leq h^2\Omega_{Hot-dark-matter} \leq 0.12$	
$0.10 \leq h^2\Omega_{Cold-dark-matter} \leq 0.32$	
$0.02 \leq h^2\Omega_{baryon} \leq 0.03$	$0.01 \leq h^2\Omega_{baryon} \leq 0.03$

Table 2. The ranges of density parameters within the standard cosmological model derived from the first release data (left-hand column)[46]) with corresponding values newest data (right-hand column) [47] and h is the Hubble parameter and values of 0.74 ± 0.08 and 0.72 ± 0.08 were used in [46] and [47], respectively.