MIBC AND THE DIRAC SPIN EFFECT IN TORSION GRAVITY

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The spin precession of a Dirac particle in monotonically increasingly boosted coordinates is calculated using torsion gravity (teleparallel theory of gravity). Also, we find the vector and the axial-vector parts of the torsion tensor.

Key words: MIB Coordinates, Dirac Spin Effect, teleparallel gravity..

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1. INTRODUCTION

The inertia of intrinsic spin has been introduced for the first time by Mashoon [1, 2] and was illustrated by the rotation-spin coupling. The theoretical investigations was performed by Hehl and Ni straightforwardly [3]. After these interesting works, some researchers extended the calculations [4, 5]. Nonetheless, relativistic treatment has not been discussed by these authors. In order to test the existence of this term an experiment was carried out by Mashoon et al. and the others [6–9]. Previously, Zhang [10] calculated Dirac-Spin effect in rotating frame with a relativistic factor. Here, we carry out the calculations for monotonically increasingly boosted coordinates (MIBc). As a matter of fact, we want to construct a connection between the torsion-spin effect and the rotation-spin effect associated with the MIBc.

The dynamics of the gravitational field can be investigated with the help of torsion gravity which is characterized by the zero curvature identically [11]. In this theory, the basic entity is the non-trivial tetrad field while in Einstein's theory of general relativity the metric tensor plays the role of the basic entity. The torsion gravity corresponds to a gauge theory for the translation group [12, 13] based on Weitzenböck geometry [14]. Although there are some fundamental differences, these two theories give equivalent descriptions of the gravitational interaction [15]. Thus, this conclusion implies that curvature and torsion might be simply alternative ways of describing the gravitational field. In some other theories [16, 17], torsion is the only relevant when spins are important [18], thence it represents additional degrees

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of freedom as compared to curvature and some new physics may be associated with it.

The dynamical spacetime effects on the spin is brought into the Dirac equation through the spin connection coming into sight in the Dirac equation including gravitation [19, 20]. The covariant Lagrangian of the Dirac spinor field is

$$Ł_{Dirac} = -m\overline{\Psi}\Psi + \frac{1}{2}\xi_i{}^{\alpha}[\Psi\widetilde{\gamma}^i\nabla_\alpha\overline{\Psi} - \nabla_\alpha\overline{\Psi}\widetilde{\gamma}^i\Psi], \tag{1}$$

where $\widetilde{\gamma}^i$ are the Dirac matrices in flat spacetime which are given exactly as

$$\overrightarrow{\widetilde{\gamma}} = \begin{pmatrix} 0 & \overrightarrow{\sigma} \\ -\overrightarrow{\sigma} & 0 \end{pmatrix}, \qquad \widetilde{\gamma}^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \tag{2}$$

here, I and 0 mean 2×2 identity and null matrices, respectively. $\overrightarrow{\sigma}$ matrices are given by

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{3}$$

and $\xi_i{}^{\alpha}$ is the vierbein field [21]. If one variates this Lagrangian with respect to $\overline{\Psi}$, one can find the Dirac equation in Weitzenböck spacetime as below

$$[\xi_i{}^{\alpha}\widetilde{\gamma}^i\{\partial_\mu + \Gamma_\mu\} + m]\Psi = 0. \tag{4}$$

where Ψ is the four-component Dirac wave-function. The spin connection Γ_μ in explicit form is

$$\Gamma_\mu \equiv \frac{1}{8}[\widetilde{\gamma}^j, \widetilde{\gamma}^k]\xi_j{}^{\alpha}\xi_{k\alpha;\mu}. \tag{5}$$

One can show that [4]

$$\widetilde{\gamma}^i[\widetilde{\gamma}^j, \widetilde{\gamma}^k] = 2\eta^{ij}\widetilde{\gamma}^k - 2\eta^{ik}\widetilde{\gamma}^j - 2i\varepsilon^{nijk}\widetilde{\gamma}_5\widetilde{\gamma}_n. \tag{6}$$

Here η^{ij} is the Minkowski metric, ε^{nijk} is the totally antisymmetric Levi-Civita tensor ($\varepsilon^{0123} = 1$), and $\widetilde{\gamma}_5 = i\widetilde{\gamma}_0\widetilde{\gamma}_1\widetilde{\gamma}_2\widetilde{\gamma}_3$. Then, the spin connection contributes the following [20]:

$$\Gamma_\mu = \frac{1}{2}V_\mu - \frac{3i}{4}A_\mu\widetilde{\gamma}_5. \tag{7}$$

Here, V_μ and A_μ are the vector part and the Axial-vector part of the torsion tensor, respectively, and these quantities will be introduced in the exact form later.

In the general version of torsional gravity in Weitzenböck spacetime, many researchers showed that the spin precession of a Dirac particle is closely related to the axial-vector torsion [10, 22–27], and it will be interesting to notice that the axial-vector torsion describes the deviation of the axial symmetry from the spherical symmetry [22].

$$\frac{d\overrightarrow{S}}{dt} = -\frac{3}{2}\overrightarrow{A} \times \overrightarrow{S} \tag{8}$$

where $\overrightarrow{S}$ is the semi-classical spin vector of a Dirac particle and $\overrightarrow{A}$ is the space-like part of the axial-vector torsion. Hence, the corresponding additional Hamiltonian energy term is

$$\delta H = -\frac{3}{2}\overrightarrow{A}.\overrightarrow{\sigma} \tag{9}$$

where $\overrightarrow{\sigma}$ represents the spin of the particle [1].

The torsion tensor can be divided into three irreducible parts under the global Lorentz transformation group [20]. Hence, the tensor part is

$$t_{\alpha\mu\nu} = \frac{1}{2}(T_{\alpha\mu\nu} + T_{\mu\alpha\nu}) + \frac{1}{6}(g_{\nu\alpha}T^{\delta}{}_{\delta\mu} + g_{\nu\mu}T^{\omega}{}_{\omega\alpha}) - \frac{1}{3}g_{\alpha\mu}T^{\rho}{}_{\rho\nu}, \tag{10}$$

the vector part is

$$V_\mu = T^{\alpha}{}_{\alpha\mu}, \tag{11}$$

and the axial-vector part is

$$A^\mu = \frac{1}{6}\varepsilon^{\mu\nu\alpha\beta}T_{\nu\alpha\beta}. \tag{12}$$

Now, the torsion tensor can be formulated by using these components:

$$T_{\alpha\mu\nu} = \frac{1}{2}(t_{\alpha\mu\nu} - t_{\alpha\nu\mu}) + \frac{1}{3}(g_{\alpha\mu}V_\nu - g_{\alpha\nu}V_\mu) + \varepsilon_{\alpha\mu\nu\sigma}A^\sigma, \tag{13}$$

where

$$\varepsilon^{\alpha\mu\nu\sigma} = \frac{1}{\sqrt{-g}}\delta^{\alpha\mu\nu\sigma}. \tag{14}$$

Here, $\delta = \delta^{\alpha\mu\nu\sigma}$ and $\overline{\delta} = \delta_{\alpha\mu\nu\sigma}$ are completely skew symmetric tensor densities weighted -1 and $+1$, respectively [20]. It is important to mention here that the deviation is described by the axial-vector torsion.

Furthermore, the relation of Weitzenböck connection [14] is

$$\Gamma^{\lambda}{}_{\alpha\beta} = \widetilde{\Gamma}^{\lambda}{}_{\alpha\beta} - \Upsilon^{\lambda}{}_{\alpha\beta} \tag{15}$$

where $\widetilde{\Gamma}^{\lambda}{}_{\alpha\beta}$ is the Levi-Civita connection of the metric $g_{\alpha\beta} = \eta_{ij}\xi^{i}{}_{\alpha}\xi^{j}{}_{\beta}$, and is given by

$$\widetilde{\Gamma}^{\alpha}{}_{\mu\nu} = \frac{1}{2}g^{\alpha\beta}\left(\partial_\mu g_{\beta\nu} + \partial_\nu g_{\beta\mu} - \partial_\beta g_{\mu\nu}\right), \tag{16}$$

and

$$\Upsilon^{\lambda}{}_{\alpha\beta} = \frac{1}{2}\left(T_{\alpha}{}^{\lambda}{}_{\beta} + T_{\beta}{}^{\lambda}{}_{\alpha} - T^{\lambda}{}_{\alpha\beta}\right) \tag{17}$$

defines the contorsion tensor. Here

$$T^{\lambda}{}_{\alpha\beta} = \Gamma^{\lambda}{}_{\beta\alpha} - \Gamma^{\lambda}{}_{\alpha\beta} \tag{18}$$

is the torsion of the Weitzenböck connection. A non-trivial field can be considered to represent the linear Weitzenböck connection [28]

$$\Gamma^{\lambda}{}_{\alpha\beta} = \xi_i{}^{\lambda}\partial_\beta \xi^i{}_\alpha. \tag{19}$$

By using a vierbein field satisfying

$$\xi^i{}_\alpha \xi_i{}^\beta = \delta^\beta_\alpha, \qquad \xi^i{}_\alpha \xi_j{}^\alpha = \delta^i_j \tag{20}$$

the Tensor and Lorentz indices can be interchanged.

In order to denote the tensor indices in relation with spacetime the Greek alphabet will be used and to denote Local Lorentz indices the Latin alphabet will be used. In this work, we use assume that the speed of light is set equal to unit.

2. THE MIBC

We focus on a self-interacting scalar field which is described by the action

$$S = -\int d^4x\sqrt{|g|}\left\{\frac{1}{2}g^{\mu\nu}\nabla_\mu\varphi\nabla_\nu\varphi - V(\varphi)\right\}, \tag{21}$$

here we chose $V(\varphi) = \frac{1}{4}(\varphi^2-1)^2$ to be a symmetric double well potential [29]. This is identical to use $V(\varphi) = \frac{\varrho}{4}(\varphi^2 - \frac{m^2}{\varrho})^2$ and introduce dimensionless variables $r = mr'$, $t = mt'$ and $\chi = \frac{\sqrt{\varrho}}{m}\varphi$. The metric of spherical symmetric flat spacetime in standard spherical polar coordinates (t', r', θ', ϕ') is

$$ds'^2 = dt'^2 - dr'^2 - r'^2(d\theta'^2 + \sin^2\theta' d\phi'^2). \tag{22}$$

Now, we define a new radial coordinate r, which interpolates between the old radial coordinate at small r' and an outgoing null coordinate at large r'. Especially, we consider [30]

$$t' = t, \qquad r' = r + \Delta(r)t, \qquad \theta' = \theta, \qquad \phi' = \phi, \tag{23}$$

where the function $\Delta(r)$ increases monotonically and interpolates between 0 and 1 smoothly at some characteristic cutoff, r_c, so that

$$\begin{aligned} &\Delta(r) \to 0, \qquad r \ll r_c, \\ &\Delta(r) \to 1, \qquad r \gg r_c. \end{aligned} \tag{24}$$

t and r will be called as the MIB coordinates. The MIB system can be reduced to the original spherical coordinates (t', r') for $r \ll r_c$, and both ingoing and outgoing (from $r \gg r_c$) radiation tends to be *frozen* in the transition layer, $r \approx r_c$ [30]. On the other hand, the MIB system will not cover all of the (t', r') half-plane. However, if $\Delta(r)$ increases monotonically, the determinant of the Jacobian of the transformation is obtained as non-zero for all t such that $t > -|\Delta(r)|_{max}$ [30]. From this point of view, for this range of t, the transformation to and from standard spherical coordinate system is well-defined and though a coordinate singularity inevitably forms as past time-like infinity ($t \to -\infty$), this has no effect on the *forward evolution* of initial data given at $t = 0$ [30].

The coordinate system that is chosen results in the following spherically symmetric Arnowitt, Deser and Misner 3+1 Form [30, 31]

$$g_{\mu\nu}dx^{\mu}dx^{\nu} = [w(t,r) - p^2(t,r)h^2(t,r)]dt^2 - p^2(t,r)dr^2 \\ -r^2 f^2(t,r)[d\theta^2 + \sin^2\theta d\phi^2] - 2p^2(t,r)h(t,r)dtdr \tag{25}$$

where

$$p(t,r) = 1 + t\Delta'(r), \tag{26}$$

$$h(t,r) = \frac{\Delta(r)}{1 + t\Delta'(r)}, \tag{27}$$

$$f(t,r) = 1 + \frac{t}{r}\Delta(r), \tag{28}$$

$$w(t,r) = 1. \tag{29}$$

In the nomenclature of the Arnowitt, Deser and Misner formalism, $w(t,r)$ is the lapse function, while $h(t,r)$ is the radial component of the shift vector. In this work, we adopt the following specific form for $\Delta(r)$:

$$\Delta(r) = \frac{1}{2+\varepsilon}\left[1 + \tanh\frac{r - r_c}{\delta}\right], \tag{30}$$

where

$$\varepsilon = -\frac{1}{2}(1 + \tanh\frac{r_c}{\delta}) \tag{31}$$

is chosen to satisfy the regularity condition at $r = 0$.

3. CALCULATION OF THE DIRAC SPIN EFFECT

The surviving components of the metric tensor $g_{\mu\nu}$ for the line-element (25) are defined by

$$g_{\mu\nu} = [w(t,r) - p^2(t,r)h^2(t,r)]\delta^0_\mu\delta^0_\nu - p^2(t,r)\delta^1_\mu\delta^1_\nu \\ -r^2 f^2(t,r)[\delta^2_\mu\delta^2_\nu + \sin^2\theta\delta^3_\mu\delta^3_\nu] \\ -p^2(t,r)h(t,r)[\delta^0_\mu\delta^1_\nu + \delta^1_\mu\delta^0_\nu], \tag{32}$$

and the non-zero components of its inverse form $g^{\mu\nu}$ are given by the following relation

$$\begin{aligned} g^{\mu\nu} &= w^{-2}(t,r)\delta^\mu_0\delta^\nu_0 - [p^{-2}(t,r) - h^2(t,r)w^{-2}(t,r)]\delta^\mu_1\delta^\nu_1 \\ &\quad -r^{-2}f^{-2}(t,r)\left(\delta^\mu_2\delta^\nu_2 + \csc^2\theta\delta^\mu_3\delta^\nu_3\right) \\ &\quad -h^2(t,r)w^{-2}(t,r)\left[\delta^\mu_0\delta^\nu_1 + \delta^\mu_1\delta^\nu_0\right]. \end{aligned} \tag{33}$$

The general form of the vierbein, ξ^μ_i, having spherical symmetry was given by Robertson [32]. In the Cartesian form it can be written as

$$\begin{aligned} \xi^0_0 &= iC_1, \qquad \xi^0_a = C_2x^a, \qquad \xi^\alpha_0 = iC_3x^\alpha, \\ \xi^\alpha_a &= C_4\delta^\alpha_a + C_5x^ax^\alpha + \epsilon_{a\alpha\beta}C_6x^\beta, \end{aligned} \tag{34}$$

here C_i $(i = 1,2,3,4,5,6)$ are functions of t and $r = (x^bx^b)^{1/2}$, and the zeroth vector ${\xi_0}^\mu$ has the factor $i^2 = -1$ to preserve Lorentz signature, and the tetrad of Minkowski space-time is now ${\xi_b}^\mu = \mathtt{diag}(i, \delta^\alpha_b)$ where (b=1,2,3). Using the general coordinate transformation

$$\xi_{a\mu} = \frac{\partial \mathbf{X}^{\nu'}}{\partial \mathbf{X}^\mu} h_{a\nu} \tag{35}$$

where $\{\mathbf{X}^\mu\}$ and $\left\{\mathbf{X}^{\nu'}\right\}$ are, respectively, the isotropic and Schwarzschild coordinates (t,r,θ,ϕ). In the spherical, static and isotropic coordinate system

$$\mathbf{X}^1 = r\sin\theta\cos\phi, \tag{36}$$

$$\mathbf{X}^2 = r\sin\theta\sin\phi, \tag{37}$$

$$\mathbf{X}^3 = r\cos\theta. \tag{38}$$

Hence, we obtain the vierbein components of ${\xi^a}_\mu$ as

$${\xi^a}_\mu = \begin{pmatrix} -\frac{p^2(t,r)h(t,r)}{\aleph} & \aleph & 0 & 0 \\ 0 & p(t,r)s\theta c\phi & rf(t,r)c\theta c\phi & -\Im s\phi \\ 0 & p(t,r)s\theta s\phi & rf(t,r)c\theta s\phi & \Im c\phi \\ 0 & p(t,r)c\theta & -rf(t,r)s\theta & 0 \end{pmatrix}, \tag{39}$$

and the components of inverse matrix ${\xi_a}^\mu$

$${\xi_a}^\mu = \begin{pmatrix} -\frac{\aleph}{p^2(t,r)h(t,r)} & 0 & 0 & 0 \\ -\frac{\aleph\Im}{p^3(t,r)h(t,r)}s\theta c\phi & \frac{s\theta c\phi}{p(t,r)} & \frac{c\theta c\phi}{rf(t,r)} & -\frac{s\phi}{\Im} \\ \frac{\aleph\Im}{p^3(t,r)h(t,r)}s\theta s\phi & \frac{s\theta s\phi}{p(t,r)} & \frac{c\theta s\phi}{rf(t,r)} & \frac{c\phi}{\Im} \\ -\frac{\aleph\Im}{p^3(t,r)h(t,r)}c\theta & \frac{c\theta}{p(t,r)} & \frac{s\theta}{rf(t,r)} & 0 \end{pmatrix}, \tag{40}$$

where

$$\Im^2(t,r) = \aleph^2(t,r) + p^2(t,r), \tag{41}$$

$$\aleph(t,r) = -\frac{p^2(t,r)h(t,r)}{[w(t,r)-p^2(t,r)h^2(t,r)]^{1/2}}, \tag{42}$$

and we have introduced the following notation: $s\theta = \sin\theta$, $c\theta = \cos\theta$, $s\phi = \sin\phi$ and $c\phi = \cos\phi$.

Next, the non-zero components of the Weitzenböck connection are obtained:

$$\Gamma^0{}_{00} = (\ln h)_{,t} + 2(\ln p)_{,t} + (\ln\aleph)_{,t}, \tag{43}$$

$$\Gamma^0{}_{01} = (\ln h)_{,r} + 2(\ln r)_{,t} + (\ln\aleph)_{,r}, \tag{44}$$

$$\Gamma^0{}_{10} = -\frac{\aleph(\Im p_{,t} + p\aleph_{,t})}{hp^3}, \tag{45}$$

$$\Gamma^0{}_{11} = -\frac{\aleph(\Im p_{,r} + p\aleph_{,r})}{hp^3}, \tag{46}$$

$$\Gamma^0{}_{22} = \frac{rf\aleph\Im}{hp^3}, \tag{47}$$

$$\Gamma^0{}_{33} = \frac{\aleph\Im^2}{hp^3}, \tag{48}$$

$$\Gamma^1{}_{10} = (\ln p)_{,t}, \tag{49}$$

$$\Gamma^1{}_{11} = (\ln p)_{,r}, \tag{50}$$

$$\Gamma^1{}_{21} = -\frac{rf}{p}, \tag{51}$$

$$\Gamma^1{}_{33} = -\frac{\Im}{p}\sin\theta, \tag{52}$$

$$\Gamma^2{}_{10} = \frac{p_{,t}}{rf}\sin 2\theta, \tag{53}$$

$$\Gamma^2{}_{11} = \frac{p_{,r}}{rf}\sin 2\theta, \tag{54}$$

$$\Gamma^2{}_{12} = \frac{p}{rf}\cos 2\theta, \tag{55}$$

$$\Gamma^2{}_{20} = (\ln f)_{,t}\cos 2\theta, \tag{56}$$

$$\Gamma^2{}_{21} = \frac{\sin 2\theta}{r}[1 + (\ln f)_{,r}], \tag{57}$$

$$\Gamma^2{}_{22} = -\sin 2\theta, \tag{58}$$

$$\Gamma^2{}_{33} = -\frac{\Im}{rf}\cos\theta, \tag{59}$$

$$\Gamma^3{}_{13} = \frac{p}{\Im}\sin\theta, \tag{60}$$

$$\Gamma^3{}_{23} = \frac{rf}{\Im}\cos\theta, \tag{61}$$

$$\Gamma^3{}_{30} = (\ln\Im)_{,t}, \tag{62}$$

$$\Gamma^{3}{}_{31} = (\ln\Im)_{,r}. \tag{63}$$

Hence, the corresponding non-vanishing components of the of the torsion tensor are given as

$$T^{0}{}_{01} = -T^{0}{}_{10} = -2(\ln p)_{,r} + (\ln\aleph)_{,r} - \frac{p^3 h_{,r} + \Im\aleph p_{,t} + p\aleph\aleph_{,t}}{hp^3}, \tag{64}$$

$$T^{1}{}_{01} = -T^{1}{}_{10} = (\ln p)_{,t}, \tag{65}$$

$$T^{1}{}_{12} = -T^{1}{}_{21} = -\frac{rf}{p}, \tag{66}$$

$$T^{2}{}_{01} = -T^{2}{}_{10} = \frac{p_{,r}}{rf}\sin 2\theta, \tag{67}$$

$$T^{2}{}_{12} = -T^{2}{}_{21} = \frac{f - p + rf_{,r}}{rf}, \tag{68}$$

$$T^{2}{}_{02} = -T^{2}{}_{20} = (\ln f)_{,t}\cos 2\theta, \tag{69}$$

$$T^{3}{}_{13} = -T^{3}{}_{31} = \frac{\Im_{,r} - p\sin\theta}{\Im}, \tag{70}$$

$$T^{3}{}_{23} = -T^{3}{}_{32} = \frac{rf}{\Im}\cos\theta, \tag{71}$$

$$T^{3}{}_{03} = -T^{3}{}_{30} = (\ln\Im)_{,t}. \tag{72}$$

The corresponding non-vanishing components of the vector torsion turn out to be

$$V_0(t,r,\theta) = -\frac{f_{,t}}{f}\cos 2\theta - \frac{p_{,t}}{p} - \frac{\Im_{,t}}{\Im}, \tag{73}$$

$$V_1(t,r,\theta) = -\frac{f - p + rf_{,t}}{rf}\cos 2\theta - \frac{2p_{,r}}{p} + \frac{p\sin\theta - \Im_{,r}}{\Im} + \frac{\aleph_{,r}}{\aleph} - \frac{h_{,r}}{h} - \frac{\Im\aleph p_{,t}}{hp^3} - \frac{\aleph\aleph_{,t}}{hp^2}, \tag{74}$$

$$V_2(t,r) = rf\left(\frac{\cos\theta}{\Im} - \frac{1}{p}\right), \tag{75}$$

and, the non-vanishing component of the axial-vector torsion is

$$A^{(3)}(t,r,\theta) = \frac{1}{3rf\sqrt{w}}\left(h\csc\theta - 2\cos\theta\frac{p_{,t}}{p}\right). \tag{76}$$

In space-like vector form, the axial vector becomes

$$\overrightarrow{A} = \sqrt{-g_{33}}A^{(3)}\hat{e}_\phi = \frac{\sin\theta}{3\sqrt{w}}\left(h\csc\theta - 2\cos\theta\frac{p_{,t}}{p}\right)\hat{e}_\phi. \tag{77}$$

Hence, the spin precision of a Dirac particle in torsion gravity turns out to be

$$\frac{d\overrightarrow{S}}{dt} = \frac{\sin\theta}{2\sqrt{w}}\left(2\cos\theta\frac{p_{,t}}{p} - h\csc\theta\right)\hat{e}_\phi \times \overrightarrow{S}, \tag{78}$$

and the corresponding hamiltonian will be

$$\delta H = \frac{\sin\theta}{2\sqrt{w}}\left(2\cos\theta\frac{p_{,t}}{p} - h\csc\theta\right)\hat{e}_\phi.\overrightarrow{\sigma}. \tag{79}$$

4. FINAL REMARKS

Long distance phenomena is described in Einstein's theory of general relativity very successfully, but on microscopic distances the theory encounters serious difficulties. It is known that a covariant conserved energy-momentum tensor for the gravitational field can not be constructed in the framework of general relativity, thus the investigation of alternative gravity theories is justified from the physical as well as from the mathematical point of view [34].

This paper is devoted to discuss torsion gravity version of monotonically increasingly boosted coordinates. For this purpose, a tetrad having four unknown functions is applied to the field equation of the torsion gravity.

If we take $\Delta(r) = 0$ in the MIB system's metric, we obtain a line-element describing spherical symmetric flat spacetime written in standard spherical polar coordinates. Therefore, the axial-vector vanishes, i.e. $\overrightarrow{A} = 0$. This shows that, under $\Delta(r) \to 0$ limit, the spin vector of the Dirac particle will be constant and the corresponding Hamiltonian term induced by the axial-vector spin coupling will be equal to zero. Since the torsion plays the role of the gravitational force in torsion gravity, a spinless particle will obey the force equation [28, 33] in the gravitational field

$$\frac{du_\lambda}{ds} - \Gamma_{\mu\lambda\nu}u^\mu u^\nu = T_{\mu\lambda\nu}u^\mu u^\nu. \tag{80}$$

The left part of this relation is the Weitzenböck covariant derivative of u_λ along the world line of the particle. The presence of the torsion tensor given in the right part of the relation means essentially that torsion plays the role of an external force in torsion gravity.

Finally, it is worth to mention here that the tetrad formalism itself has some important advantages come mainly from its independence from the equivalence principle and consequent suitability to the discussion of quantum issues. We know that it is always enriching to investigate known issues from another point of view, so that the endeavor is in itself commendable.

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REFERENCES

1. B. Mashoon, Class. Quant. Grav. 17 (2000) 2399.
2. B. Mashoon, Phys. Rev. Lett. 61 (1988) 2639.
3. F.W. Hehl and W.T. Ni, Phys. Rev. D 42 (1990) 2045.
4. C.M. Zhang, and A. Beesham, Mod. Phys. Lett. A16 (2001) 2319.
5. J. Huang, Ann. Physik. 3 (1994) 53.
6. B. Mashhoon, R. Neutze, M. Hannam, and G.E. Sted- man, Phys. Lett. A 249 (1998) 161.
7. M.P. Silverman, Phys. Lett. A 152 (1991) 133.
8. G.E. Stedman, Rep. Prog. Phys. 60 (1997) 615.
9. G. Papini and G. Lambiase, Phys. Lett. A 294 (2002) 175.
10. C.M. Zhang, Gen. Rel. Grav. 35 (2003) 1465.
11. J.W. Maluf, J.F. da Rocha-Neto, T.N.L. Toribio and K.H. Castello-Branco, Phys. Rev. D65 (2002) 124001.
12. F.W. Hehl, J.D. McCrea, E.W. Mielke and Y. Neemann: Phys. Rep. 258 (1995) 1.
13. K. Hayashi and T. Shirafuji, Phys. Rev. D 19 (1979) 3524.
14. R. Weitzenbck, *Invariantten Theorie* (Gronningen: Noordhoff, 1923).
15. F. Gronwald and F.W. Hehl, Proceedings of the School of Cosmology and Gravitation on Quantum Gravity, Italy, ed. P.G. Bergmann et al. (World Scientific, 1995).
16. M. Blagojecvic, Gravitation and Gauge Symmetries (IOP publishing, 2002).
17. R.T. Hammond, Gen. Rel. Grav. 26 (1994) 247.
18. F. Gronwald and F.W. Hehl, On the Gauge Aspects of Gravity, Proceedings of the 14th School of Cosmology and Gravitation, Italy, ed. P.G. Bergmann et al. (World Scientific, 1996).
19. P.A.M. Dirac, Proc. Roy. Soc. London 117A, 610 (1928); Proc. Roy. Soc. London 118A (1928) 351.
20. K. Hayashi and T. Nakano, Prog. Theor. Phys. 38 (1967) 491.
21. C.Y. Cardall and G.M. Fuller, Phys. Rev. D 55 (1996) 7960.
22. J. Nitsch and F.W. Hehl, Phys. Lett. B90 (1980) 98.
23. R.T. Hammond, Rep. Prog. Phys.65 (2002) 599.
24. P.B. Yasskin and W.R. Stoeger, Phys. Rev. D21 (1980) 2081.
25. J. Audretsch, Phys. Rev. D24 (1980) 1470.
26. G.L. Gamal Nashed, Phys. Rev. D 66 (2002) 064015; Gen. Rel. Grav. 34 (2002) 1047 ; IL Nuovo Cimento B 117 (2002) 521.
27. J. G. Pereira, T. Vargas and C. M. Zhang, Class. Quant. Grav. 18 (2001) 833.
28. R. Aldrovandi and J.G. Pereira, *An Introduction to Geometrical Physics*, (World Scientific, Singapore, 1995).
29. A. Vilenkin, Phys. Rep. 121 (1985) 263.
30. E.P. Honda and M.W. Choptuik, Phys. Rev. D 65 (2002) 084037.
31. R. Arnowitt, S. Deser and C.W. Misner, in Gravitation: An introduction to current research, L. Witten Ed., (Wiley, New York, 1962).
32. H.P. Robertson, Ann. of Math. (Princeton) 33 (1932) 496.
33. J.G. Pereira, T. Vargas and C.M. Zhang, Class. Quant. Grav. 18 (2001) 833.
34. M. Sharif and M.J. Amir, Gen. Rel. Grav. 38 (2006) 1735.