Chaotic probabilities and unpredictability in physics.

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In physics, the concept of probability ρ is introduced via the Liouville equation

$$\frac{\partial \rho}{\partial t} + \nabla \bullet (\rho \mathbf{F}) = 0 \tag{1}$$

generated by the system of ODE

$$\frac{d\mathbf{v}}{dt} = \mathbf{F}[\mathbf{v}_1(t), \ldots \mathbf{v}_n(t), t] \tag{2}$$

where **v** is velocity vector. It describes the continuity of the probability density flow originated by the error distribution

$$\rho_0 = \rho(t = 0) \tag{3}$$

in the initial condition of ODE (2).

In ***Newtonian*** physics, **F** does not depend upon ρ, but in ***quantum*** mechanics it does, (see the quantum potential in the Madelung equation, [1]). Turning to the one-dimensional case of Eq. (2), let us introduce a new type of dependence F(ρ)

$$F = \frac{1}{2}\rho + \frac{1}{\rho}\frac{\partial \rho}{\partial V} + \frac{1}{\rho}\frac{\partial^3 \rho}{\partial V^3} \tag{4}$$

The system (2), (1) takes the form

$$\frac{\partial v}{\partial t} = \frac{1}{2}\rho + \frac{1}{\rho}\frac{\partial \rho}{\partial V} + \frac{1}{\rho}\frac{\partial^3 \rho}{\partial V^3} \tag{5}$$

$$\frac{\partial \rho}{\partial t} = -\rho\frac{\partial \rho}{\partial V} - \frac{\partial^2 \rho}{\partial V^2} - \frac{\partial^4 \rho}{\partial V^4} \tag{6}$$

The topology of this system is similar to that of the hydrodynamic version of the Schrödinger equation known as the Madelung equation, [1,2]; the difference is that here the gradient of quantum potential is replaced by the non-conservative force (4). As shown in [2], the hypothetical particle represented by Eqs. (5) and (6) belongs neither to Newtonian nor to quantum physics since its motion violates the second law of thermodynamics, and therefore, this particle can be associated with a physical model of livings, [3], Fig. 1.

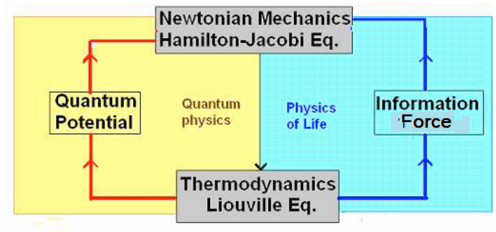

Figure 1. Classical physics, quantum physics, and physics of Life.

Now we can formulate the result presented in this note: we created a mathematical model of a particle which behavior is characterized by ***chaotic probabilities,*** and therefore, leads to a new qualitative layer of unpredictability in comparison to Newtonian or quantum chaotic systems. Indeed, turning to Eq. (6), one notice that it is the Kuramoto–Sivashinsky equation, [4], that ***dwells in probability space***. Therefore its solutions, in addition to initial and boundary conditions, must satisfy the normalization constraint

$$\int_V \rho \, dV = 1 \qquad (7)$$

As shown in [2], this constraint is satisfied if it is satisfies for the initial sensity (3). In this case, the qualitative analysis of the Kuramoto–Sivashinsky equation can be applied to Eq. (6): the first term in the right-hand part of Eq. (6) expresses the tendency to formation of shock waves; the second term represents a powerful destabilizer in the form of a negative diffusion; without the last (stabilizing) term, Eq. (6) would be non-computable because of the Hadamard instability, [2]. However, in combination with the last term, it leads to stable, but chaotic solution with a stationary statistics, [4]. This means that here we are dealing with a secondary randomness, i.e., with random-like (chaotic) behavior of the probability density which describe the random behavior of the original dynamical system.

The discovered phenomenon can be explored for deception, encryption, and data compression.

References.

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