Concept of the Effective Mass Tensor in GR Field Equations

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Abstract: *In the previous papers [1, 2] we presented the concept of the effective mass tensor* $m_{\mu\nu}$ *the in General Relativity (GR). According to this concept under the influence of the gravitational field the bare mass tensor* $m_{\mu\nu}^{bare}$ *becomes* $m_{\mu\nu}$*. Assuming that there is a relationship between metric tensor* $g_{\mu\nu}$ *and* $m_{\mu\nu}$*, we obtained modified Einstein's field equations, which gives new meaning to GR.*

Introduction

In the previous papers [1, 2] we presented the concept of *the effective mass tensor* (EMT) $m_{\mu\nu}$ in the General Relativity (GR). This concept was based on the following assumptions:

1. The mass of the body is not a scalar but the tensor.
2. In the absence of the fields (gravitational, *etc.*) the mass of the body is described by *the bare mass tensor* (BMT) $m_{\mu\nu}^{bare}$, where: $m_{\mu\nu}^{bare} = m \cdot \eta_{\mu\nu} = \mathrm{diag}(-m,+m,+m,+m)$, $\eta_{\mu\nu}$ is the Minkowski tensor, *m* is the bare mass of the body.
3. Under the influence of the fields (gravitational, *etc.*) $m_{\mu\nu}^{bare}$ becomes $m_{\mu\nu}$.
4. Effective mass of the body is a result of the mutual interactions between the body and all others bodies (*Mach's Principle*).
5. Tensor $m_{\mu\nu}$ includes full information about all fields surrounding the body without their exact analysis.
6. We assume that there exists relation between *the metric tensor* $g_{\mu\nu}$ and $m_{\mu\nu}$ in the simple form $g_{\mu\nu} = \frac{m_{\mu\nu}}{m}$, where the space-time components: $\mu, \nu = 0, 1, 2, 3$.
7. The metric $ds^2(g_{\mu\nu}) = ds^2(m_{\mu\nu})$, where: $ds^2(g_{\mu\nu}) = g_{\mu\nu}dx^\mu dx^\nu$, $ds^2(m_{\mu\nu}) = \frac{m_{\mu\nu}}{m}dx^\mu dx^\nu$.
8. In the weak gravitational field we can decompose $m_{\mu\nu}$ to the simple form: $m_{\mu\nu} = m_{\mu\nu}^{bare} + m_{\mu\nu}^{*}$, where: $m_{\mu\nu}^{*} = m \cdot |h_{\mu\nu}| << 1$ is *the small perturbation*.
9. In the absence of the all fields $m_{\mu\nu} \to m_{\mu\nu}^{bare}$.
10. All mathematical calculations are realized in the GR framework.

As a consequence of the above assumptions we obtained the new equation of the motion [2] and in this paper the new fields equation, which satisfy the classical tests of GR – for example: *the perihelion shift*, *the deflection of light by the Sun* and *the gravitational redshift*.

Metric in the EMT world

In GR the reference frames are massless, clocks and rods also. This is a consequence of apply metrics $ds^2(g_{\mu\nu}) = g_{\mu\nu}dx^\mu dx^\nu$ to describe the gravity. In the gravitational field all clocks and rods have the effective mass m^* [2, 3]. When there is no gravitational field all clocks and rods have the bare masses m. All clocks with the effective mass runs slower than the clocks with the bare mass while a rods with the effective mass will show a length greater than the rods with the bare mass. This is a consequence of apply metrics $ds^2(m_{\mu\nu}) = \frac{m_{\mu\nu}}{m}dx^\mu dx^\nu$ to describe the gravity. In the EMT world the reference frames have a mass, clocks and rods also.

Reference frame in EMT world

The equation of motion in EMT world have the form [2]

$$\frac{d^2x^\beta}{d\tau^2} + \Gamma^{*\beta}_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = 0 \tag{1}$$

where: the term $\Gamma^{*\beta}_{\mu\nu}$ we will call *the modified Christoffel symbols of the second kind* and

$$\Gamma^{*\beta}_{\mu\nu} = \frac{1}{2}m^{\beta\alpha}\left(\frac{\partial m_{\alpha\mu}}{\partial x^\nu} + \frac{\partial m_{\alpha\nu}}{\partial x^\mu} - \frac{\partial m_{\mu\nu}}{\partial x^\alpha}\right) \tag{2}$$

The term $\Gamma^{*\beta}_{\mu\nu}$ describes the anisotropy of EMT in the space-time. For $m_{\mu\nu} = m^{bare}_{\mu\nu}$ the modified Christoffel symbols $\Gamma^{*\beta}_{\mu\nu} = 0$. The reference frame, where the condition $\Gamma^{*\beta}_{\mu\nu} = 0$ is satisfied, we will call the *inertial reference frame* in EMT world.

The geodesic deviation equation in EMT world

In GR *the geodesic deviation* describes the tendency of objects to approach or recede from one another while moving under the influence of a gravitational field, which varies in the space [4, 5].

If two objects are moving along two initially parallel trajectories, the presence of *the tidal gravitational force* will cause the trajectories to bend towards or away from each other, producing a relative acceleration between the objects.

Let us consider two particles, which are moving along two initially parallel trajectories. For the point $x^\mu(\tau)$ and the point $x^\mu(\tau) + \delta x^\mu(\tau)$, where $\delta x^\mu(\tau)$ is a very small distance, we have two equations of motion respectively

$$\begin{gathered}\frac{d^2x^\mu}{d\tau^2} + \Gamma^{*\mu}_{\alpha\beta}(x)\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} = 0 \\ \frac{d^2(x^\mu + \delta x^\mu)}{d\tau^2} + \Gamma^{*\mu}_{\alpha\beta}(x + \delta x)\frac{d(x^\alpha + \delta x^\alpha)}{d\tau}\frac{d(x^\beta + \delta x^\beta)}{d\tau} = 0\end{gathered} \tag{3}$$

Calculation gives *the equation of geodesic deviation* in the form

$$\frac{D^2 \delta x^\mu}{D\tau^2} + R^{*\mu}_{\nu\rho\sigma} \frac{dx^\nu}{d\tau} \delta x^\rho \frac{dx^\sigma}{d\tau} = 0 \tag{4}$$

where:

$$R^{*\mu}_{\nu\rho\sigma} = \partial_\rho \Gamma^{*\mu}_{\sigma\nu} - \partial_\sigma \Gamma^{*\mu}_{\rho\nu} + \Gamma^{*\mu}_{\rho\lambda} \Gamma^{*\lambda}_{\sigma\nu} - \Gamma^{*\mu}_{\sigma\lambda} \Gamma^{*\lambda}_{\rho\nu} \tag{5}$$

Equation (4) describes the evolution of the separation δx^μ between two small particles to *the modified Riemann curvature tensor* $R^{*\mu}_{\nu\rho\sigma}$. Dimension of the $R^{*\mu}_{\nu\rho\sigma}$ is [1/m^2]. The modified Riemann curvature tensor is expressed by the second-order derivatives of EMT

$$R^{*\alpha}_{\beta\mu\nu} = \frac{1}{2} m^{\alpha\sigma} \left(\partial_\beta \partial_\mu m_{\sigma\nu} - \partial_\beta \partial_\nu m_{\sigma\mu} + \partial_\sigma \partial_\nu m_{\beta\mu} - \partial_\sigma \partial_\mu m_{\beta\nu} \right) \tag{6}$$

and describes distribution of the effective mass in the space-time, *i.e.* $m_{\mu\nu}$. If $R^{*\mu}_{\nu\rho\sigma} = 0$ then space-time is empty or distribution of the effective masses in the space-time is isotropic.

The modified Ricci curvature tensor (*the modified Ricci tensor*) we will define as the contraction of the modified Riemann tensor.

$$R^*_{\alpha\beta} \equiv R^{*\mu}_{\alpha\mu\beta} \tag{7}$$

Finally, we have *the modified scalar curvature* R^* (*the modified Ricci curvature*), defined by

$$R^* = R^{*\mu}_{\mu} \tag{8}$$

In the GR the tidal forces is described by *the Riemann curvature tensor*. The *geodesic deviation equation* describes relationship between the Riemann curvature tensor and the relative acceleration of two neighboring geodesics lines. In the GR the Riemann curvature tensor $R^{\mu}_{\nu\rho\sigma} = 0$ describes the empty space only.

In EMT world the tidal force is described by *the modified Riemann curvature tensor*. The geodesic deviation equation describes relationship between the modified Riemann curvature tensor and the relative acceleration of two neighboring particles.

In the weak gravitational field the modified Christoffel symbols (with accuracy to first order) have the form [2]

$$\Gamma^{*i}_{00} = \frac{1}{2m} \delta^{ij} \partial_j m^*_{00} \tag{9}$$

(components *i* and *j* are the Roman indices to denote spatial components: *i*, *j* = 1, 2, 3). Now the equation of the motion (1) have the form

$$\frac{d^2 x^i}{dt^2} = -\delta^{ij} \partial_j \left(\frac{c^2 m^*_{00}}{2m} \right) \tag{10}$$

Equation (10) is the Newtonian counterpart of the equation of motion in EMT world. We will call them *the modified Newton's equations*.

Let us consider two nearby particles with coordinates: $x^i(t)$ and $x^i(t)+\delta x^i(t)$, where $\delta x^i(t)$ is *the separation vector*. Their evolution in time we can described by the two modified Newton's equations:

$$\frac{d^2}{dt^2}\left(x^i\right)=-\delta^{ij}\partial_j\left(\frac{c^2 m_{00}^*(x)}{2m}\right)$$
$$\frac{d^2}{dt^2}\left(x^i+\delta x^i\right)=-\delta^{ij}\partial_j\left(\frac{c^2 m_{00}^*(x+\delta x)}{2m}\right) \tag{11}$$

Combining and expanding to first order in δx^i yields a dynamical equation for the separation vector

$$\frac{d^2}{dt^2}\delta x^i=-\frac{c^2}{2m}\delta^{ik}\left(\partial_j\partial_k m_{00}^*(x)\right)\delta x^j \tag{12}$$

We see that the separation vector δx^i is proportional to the $\partial_j\partial_k m_{00}^*(x)$, which varies in the space. Equation (12) represents geodesic deviation equation in weak gravitational field in EMT world and expressed the fact the distance between the two freely falling particles will vary if they move in a non-uniform gravitational field. Comparing the equation

$$\frac{D^2\delta x^\mu}{D\tau^2}+K_\nu^\mu\delta x^\nu=0 \tag{13}$$

where: $K_\nu^\mu\equiv-R_{\sigma\rho\nu}^{*\mu}\frac{dx^\sigma}{d\tau}\frac{dx^\rho}{d\tau}$ with the equation

$$\frac{d^2\delta x^i}{dt^2}+K_j^i\delta x^j=0 \tag{14}$$

where: $K_j^i\equiv\frac{c^2}{2m}\delta^{ik}\left(\partial_j\partial_k m_{00}^*(x)\right)$ we see their similarity. Dimension of the K_ρ^μ and K_j^i are the same and is [1/s^2]. In the weak gravitational field the term K_j^i is the Newtonian counterpart of the term K_ν^μ. Equations (13) and (14) show *the harmonic oscillator equation* [4, 5].

In search of the field equations in EMT world

In the GR the metric tensor $g_{\mu\nu}$ include two kinds of information:

1. The relatively unimportant information concerning the specific coordinate system used (*e.g.* spherical coordinates, Cartesian coordinates, *etc.*).
2. Important information regarding the existence of any gravitational potentials.

In GR the Einstein's field equations has the form:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{15}$$

where: $R_{\mu\nu}$ is the Ricci curvature tensor, R is the Ricci scalar, $g_{\mu\nu}$ is the metric tensor, G is Newton's gravitational constant, c is the speed of light in the vacuum, and $T_{\mu\nu}$ is the stress-energy tensor. Physical sense of the equation (15) is following: *the surrounding matter curves the space-time*. In the Newtonian limits, in the Cartesian coordinate system, the g_{00} is the Newtonian potential V.

We assumed that the metric tensor is proportional to EMT, *i.e.* $g_{\mu\nu} \sim m_{\mu\nu}$ then we will expect that Einstein's field equations will take the form of *the modified Einstein's field equations* in EMT world in the form:

$$\tau^*_{\mu\nu} - \frac{1}{2} \frac{m_{\mu\nu}}{m} \tau^* = 8\pi T^*_{\mu\nu} \tag{16}$$

where: $\tau^*_{\mu\nu} = \frac{c^4}{G} R^*_{\mu\nu}$ we will call *the effective energy tensor*, which should include full information regarding the existence of any surrounding fields, the term $\frac{c^4}{G}$ is *the Planck force*, $\tau^* = \frac{c^4}{G} R^*$ we will call *the bare energy*, tensor $T^*_{\mu\nu} = \left(2 \frac{\partial L_m}{\partial \left(\frac{m^{\mu\nu}}{m} \right)} - \frac{m_{\mu\nu}}{m} L_m \right)$ is *the energy - momentum tensor* in EMT world and L_m is the non-gravitational part of the Lagrangian density of the action and describes any matter fields. The stress–energy tensor $T^*_{\mu\nu}$ is a source of the effective energy.

For the fluid in the thermodynamic equilibrium the stress–energy tensor $T^*_{\mu\nu}$ in EMT world takes a form

$$T^*_{\mu\nu} \equiv \left(\rho + \frac{p}{c^2} \right) u_\mu u_\nu - \frac{m_{\mu\nu}}{m} p \tag{17}$$

where: ρ is *the mass-energy density*, p is *the hydrostatic pressure*, u_μ is the fluid's *four velocity*. In the weak gravitational field, for the small velocity and for the condition $\frac{p}{c^2} << \rho$ equation (16) takes a form

$$\tau^*_{00} = 8\pi T^*_{00} \tag{18}$$

where: $\tau^*_{00} = \frac{c^4}{G} \nabla^2 \left(\frac{m^*_{00}}{m} \right)$, $T^*_{00} = \rho c^2$.

The field equation (18) we can express in a slightly different form and we get *the modified Poisson equation* for the gravitational field

$$\nabla^2\left(\frac{m^*_{00}}{m}\right)=\frac{8\pi G}{c^2}\rho \tag{19}$$

We assume that there exists relation

$$\frac{m^*_{00}}{m}=\frac{2V}{c^2} \tag{20}$$

and we will obtain well-known *the Poisson equation* for the gravitational field

$$\nabla^2 V=4\pi G\rho \tag{21}$$

Conclusion

In this paper we considered the concept of the EMT in GR. According to this concept under the influence of the gravitational field BMT $m^{bare}_{\mu\nu}$ becomes the EMT $m_{\mu\nu}$. Our assumptions and calculation give the new field equations, the modified Einstein's field equation (16), which gives a new physical sense of GR, what we illustrated in the table below.

General Relativity World	**The EMT World**
Geometry = Matter	*Effective Energy = Matter*

So, *Geometry* and *Effective Energy* are equivalent.

We hope that the concept of the effective mass tensor will help better understand a fascinating gravitational phenomena and will solve *the problem of the missing mass in the Universe*.

References

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