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General Relativistic Predictions are Incompatible with Observed Solar Planetary Recessions

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ABSTRACT
It is generally assumed that Einstein's General Theory *of* Relativity (GTR) is silent on the issue of planetary recession such as has been measured recently by Standish (2005); Krasinsky and Brumberg (2004) and as-well by Williams and Boggs (2009); Williams et al. (2004). In this short note, we demonstrate that the GTR is not silent on this matter, it does make a clear predictions *albeit*, predictions that is contrary with experience and for this task, we use the same solution that was and has been used triumphantly to explain the perihelion precession of the planet Mercury. From a pure stand-point of binary logic, we expect this solution to stand-up to all its predictions for both the precession of perihelion precession and as-well the expansion of orbits. At any rate imaginable, this apparent contradiction presents an interesting state of affairs for the GTR.

Key words: astrometry – celestial mechanics – ephemerides – planets and satellites: formation.

1 INTRODUCTION

It is generally assumed that Einstein's General Theory *of* Relativity (GTR) is silent on the issue of planetary recession such as has been measured recently by Standish (2005); Krasinsky and Brumberg (2004) for the Earth-Moon system from the Sun and as-well the Moon's recession from the Earth (Williams and Boggs 2009; Williams et al. 2004). In this short note, we demonstrate that the GTR is not silent on this matter, it does make a prediction *albeit* one that runs contrary with experience. In-order to arrive at this very interesting result, we use the same solution that was and has been used triumphantly to explain the perihelion precession of the planet Mercury. Unfortunately, this solution's predictions *do not agree* with physical and natural reality.

It is interesting that that we have a theoretical result that runs contrary with experience. What this may mean is that, for the first time since the GTR was conceived, a legitimate and valid solution of the GTR disagrees with observational data. This is an interesting state of affairs. If one believes the solution leading to the the GTR's triumphant prediction of the anomalous perihelion precession of the planet Mercury, then, equally, they have to believe its solution on the secular changes in the mean Earth-Sun and Earth-Moon distance. There really is no escape from this. We might as-well have to accept this as a tiny – *albeit*, important and noticeable crack in the otherwise beautiful edifice of the GTR.

2 PRELIMINARY COMPUTATIONS

First, from the measurements of Standish (2005); Krasinsky and Brumberg (2004), we obtain a single obversational value for the recession of the Earth-Moon system the Sun. Thereafter, we obtain a formula relating the change in the eccentricity and the mean distance.

2.1 A Single Recessional Value

We need to establish a single value for the recession of the Earth-Moon system from the Sun. As is common knowledge, the mean distance from the Sun of the Earth-Moon system is referred to as the Astronomical Unit and denoted by the symbol AU. Let us represent the secular change in the Astronomical Unit by $\delta\mathrm{AU}$. At present, there are two values for this quantity, that is, the Russian astronomers Krasinsky and Brumberg (2004) find $\delta\mathrm{AU} = +150.00 \pm 3.00\,\mathrm{mm/yr}$, while the American astronomer Standish (2005) finds $\delta\mathrm{AU} = +70.00 \pm 2.00\,\mathrm{mm/yr}$. From these two values we need the best estimate. For this, we need to appeal to statistical methods to find a best estimate.

Assuming that these two measurements are governed by Gaussian statistics and that the errors in the measurements random and independent, then, the best estimate of these two measurements can be obtained by taking the weighted mean of the two values. For example if $(x_i + \delta x_i \; : \; i = 1, 2, \dots n)$ is set of n measurements of a constant quantity x, where x_i is the best value of for the n^{th} measurement and δx_i is its accompanying error margin, then, the best estimate of $x_{\rm best}$ from this set is $x_{\rm best} = \sum w_i x_i / \sum w_i$ where w_i are the weights such that $w_i = 1/(\delta x_i)^2$ and the best estimate in the error margin $\delta x_{\rm best}$ is $\delta x_{\rm best} = (\sum w_i)^{-1/2}$ (see *e.g.* Taylor 1982, p.150). Applying this prescription to the two measurements of Standish (2005); Krasinsky and Brumberg (2004), we obtain:

$$\delta {\rm AU} = +95.00 \pm 2.00 \, {\rm mm/yr}. \tag{1}$$

We shall from heron adopt this value (1) as representative of the change in the mean distance between the Sun and Earth-Moon system.

The maximum distance of the Earth from the Sun $\mathcal{R}_{\rm orb}^{\rm max} = 1.52098232 \times 10^{11}\,{\rm m}$ and minimum distance is $\mathcal{R}_{\rm orb}^{\rm min} = 1.47098290 \times 10^{11}\,{\rm m}$ (Standish and Williams 2010). In our calculation, we need one single value for the mean distance between the Sun and the Earth-Moon system. From $\mathcal{R}_{\rm orb}^{\rm min}$ and $\mathcal{R}_{\rm orb}^{\rm max}$, the best estimate would the average of these two values, that is, $\mathcal{R}_{\rm orb}^{\rm best} = (\mathcal{R}_{\rm orb}^{\rm max} + \mathcal{R}_{\rm orb}^{\rm max})/2$ and the best estimate in the error margin to this value is $\delta\mathcal{R}_{\rm orb}^{\rm best} = (\mathcal{R}_{\rm orb}^{\rm max} - \mathcal{R}_{\rm orb}^{\rm max})/2$, so that the best value for the mean distance between the Sun and the Earth-Moon system is:

$$\langle \mathcal{R}_{\oplus} \rangle = (1.50 \pm 0.03) \times 10^{11}\,{\rm m}. \tag{2}$$

For the Moon $\mathcal{R}_{\rm max} = 4.055 \times 10^{8}\,{\rm m}$ and $\mathcal{R}_{\rm min} = 3.633 \times 10^{8}\,{\rm m}$, from this $\mathcal{R}_{\rm mean} = (3.80 \pm 0.20) \times 10^{8}\,{\rm m}$

$$\langle \mathcal{R}_{\rm em} \rangle = (3.80 \pm 0.20) \times 10^{8}\,{\rm m}. \tag{3}$$

In the next section, we deduce a relationship between the secular change in the eccentricity and the mean radial distance.

2.2 Secularly Changing Eccentricity

We are going to derive the relationship between the time variation of the eccentricity and the mean distance between a planet and its parent body. This relationship we shall need in the next section. For an orbit with a *aphelion* and *perihelion* distances $\mathcal{R}_{\rm max}$ and $\mathcal{R}_{\rm min}$, respectively; the eccentricity of such an orbit is defined:

$$\epsilon = \frac{\mathcal{R}_{\rm max} - \mathcal{R}_{\rm min}}{\mathcal{R}_{\rm max} + \mathcal{R}_{\rm min}}. \tag{4}$$

Differentiating this with respect to time and then dividing the resultant equation by ϵ, one obtains:

$$\frac{\delta\epsilon}{\epsilon} = \frac{\delta\mathcal{R}_{\rm max} - \delta\mathcal{R}_{\rm min}}{\mathcal{R}_{\rm max} - \mathcal{R}_{\rm min}} - \frac{\dot{\mathcal{R}}_{\rm max} + \dot{\mathcal{R}}_{\rm min}}{\mathcal{R}_{\rm max} + \mathcal{R}_{\rm min}}. \tag{5}$$

Now, for low eccentricity orbits as those found in the Solar system, if $(\delta\mathcal{R}_{\rm max} \sim \delta\mathcal{R}_{\rm min}) := \delta\mathcal{R}_{\rm mean}$ and $(\mathcal{R}_{\rm max} \sim \mathcal{R}_{\rm min}) := \mathcal{R}_{\rm mean}$ where $\mathcal{R}_{\rm mean}$ is the mean distance of the test body from the central massive body about which it orbits, it follows that:

$$\frac{\delta\epsilon}{\epsilon} = -\frac{\delta\mathcal{R}_{\rm mean}}{\mathcal{R}_{\rm mean}}. \tag{6}$$

Thus, for expanding orbits, the eccentricity will decrease, while for contracting orbits, the eccentricity will increase.

3 EINSTEINIAN PLANETARY RECESSION

As is now common knowledge, when Einstein applied his newly discovered GTR to the problem of the precession of the perihelion of the planet mercury he obtained that the trajectory of solar planets must be described by the equation:

$$\frac{d^2u}{d\varphi^2} + u - \frac{G\mathcal{M}}{J^2} = \left(\frac{3G\mathcal{M}}{c^2}\right) u^2, \tag{7}$$

where again $u = 1/r$. To obtain a solution to this equation, we note that the left hand side is the usual Newtonian equation for the orbit of planets, *i.e.*:

$$\frac{d^2u}{d\varphi^2} + u - \frac{G\mathcal{M}}{J^2} = 0, \tag{8}$$

and the solution to this equation is: $u = (1 + \epsilon\cos\varphi)/l$ where ϵ is the *eccentricity* of the orbit as determined from Newtonian gravitational theory and $l = (1+\epsilon)\mathcal{R}_{\rm min}$ where $\mathcal{R}_{\rm min}$ is the planet's distance of closest approach to the Sun. It follows that:

$$r = \left(\frac{1+\epsilon}{1+\epsilon\cos\theta}\right) \mathcal{R}_{\rm min}. \tag{9}$$

This solution is a good approximate solution to (7) because the orbit of Mercury is nearly Newtonian. Consequently, we can rewrite the small term on the right hand side of (7) as: $3G\mathcal{M}(1+\epsilon\cos\varphi)^2/l^2c^2$; and in so doing, we make an entirely negligible error – all we do is to obtain an approximate solution to the exact solution which can only be accessed *via* a numerical solution. With this substitution in (7), we obtain:

$$\frac{d^2u}{d\varphi^2} + u - \frac{G\mathcal{M}}{J^2} = \frac{3G\mathcal{M}}{l^2c^2}\left(1 + 2\epsilon\cos\varphi + \epsilon^2\cos^2\varphi\right), \tag{10}$$

and the solution to this equation is:

$$u = \frac{1+\epsilon\cos\varphi}{l} + \frac{3G\mathcal{M}}{l^2c^2}\left[1 + \frac{\epsilon^2}{2} + \frac{\epsilon^2\cos 2\varphi}{6} + \epsilon\varphi\sin\varphi\right]. \tag{11}$$

Of the additional terms, the first *i.e.* $(1+\epsilon^2/2)$ is a constant and the second oscillates through two cycles on each orbit; both these terms are immeasurably small. However, the last term increases steadily in amplitude with φ, and hence with time, whilst oscillating through one cycle per orbit; clearly this term is responsible for the precession of the perihelion. Dropping all unimportant terms we will have:

$$u = \frac{1 + \epsilon(\cos\varphi + \eta\varphi\sin\varphi)}{l} = \frac{1+\epsilon\cos(\beta_E\varphi)}{l}, \tag{12}$$

where $\eta = 3G\mathcal{M}/lc^2$ is extremely small and $\beta_E = 1+\eta$. This is the usual way to arrive at a solution to (7). This solution is barren insofar as the secular changes in the Earth-Sun and Earth-Moon distance is concerned. What we shall do in the next section is to demonstrate that (12), does contain a legitimate and overlooked solution that predicts secular changes in say the mean Earth-Sun and Earth-Moon distance.

Unfortunately, this solution's predictions *do not agree* with experience. What this may mean is that, for the first time since the GTR was conceived, a legitimate and valid solution of the GTR disagrees with observational data. This is an interesting state of affairs. If one believes the solution (12) *vis* its predictions on the anomalous perihelion precession of the planet Mercury, then, equally, they have to believe its solution on the secular changes in the mean Earth-Sun and Earth-Moon distance.

3.1 Einsteinian Planetary Recession

As already stated, it is a generally held view that Einstein's GTR in its bare and natural form does not predict any such phenomenon as the secular increase in the mean Sun-Planet distance. We show here that the GTR does predict such phenomenon and much against experience, its predictions fall far short of explaining this phenomenon.

To arrive at this solution, we revisit the solution of (7) as given in (12). We know that if a and b are constants and φ is a variable, then, the following holds true for all a, b, and φ:

$$a\cos\varphi + b\sin\varphi \equiv \sqrt{a^2+b^2}\cos\left[\varphi + \arctan\left(\frac{b}{a}\right)\right]. \quad (13)$$

With this in mind, we note that the term $(\cos\varphi + \eta\varphi\sin\varphi)$ in (12) has the same form as above, thus, it follows that:

$$\cos\varphi + \eta\varphi\sin\varphi \equiv \sqrt{1+\eta^2\varphi^2}\cos\left[\varphi + \arctan\left(\eta\varphi\right)\right]. \quad (14)$$

Let $\Delta\varphi = \arctan(\eta\varphi)$ so that $\cos\varphi + \eta\varphi\sin\varphi \simeq \sqrt{1+\eta^2\varphi^2}\cos(\varphi+\Delta\varphi)$. From this, it follows that the solution (12) can also be written down as:

$$\frac{1}{r} = \frac{1 + \epsilon\overbrace{\sqrt{1+\eta^2\varphi^2}}^{\text{New Term}}\cos(\varphi+\Delta\varphi)}{l}. \quad (15)$$

This can be written more conveniently as:

$$\frac{1}{r} = \frac{1 + \epsilon_{\text{eff}}\cos(\varphi+\Delta\varphi)}{l}. \quad (16)$$

where $\epsilon_{\text{eff}} = \epsilon\sqrt{1+\eta^2\varphi^2}$ is the effective eccentricity. We should note at this point that the approximations made leading to (12) assume that $\eta\varphi$ is small *i.e.* $\eta\varphi \sim 0$. We will commit this to mind.

From this fact that $\epsilon_{\text{eff}} = \epsilon\sqrt{1+\eta^2\varphi^2}$, it is clear that:

$$\frac{\delta\epsilon_{\text{eff}}}{\epsilon_{\text{eff}}} = \frac{1}{2}\frac{\eta^2\delta\varphi^2}{\sqrt{1+\eta^2\varphi^2}}. \quad (17)$$

For one revolution, we will have $\delta\varphi^2 = \pm 4\pi^2/\mathcal{T}_{\text{orb}}$, where $\mathcal{T}_{\text{orb}}$ is the period of orbit of the test body about the parent body about which it orbits. The $\pm$ sign comes in as a result of the fact that depending on the direction of rotation, φ either increases of decreases. From a small $\eta\varphi$, we will have $1+\eta^2\varphi^2 \simeq 1$. Putting all this together, (17) reduces to:

$$\frac{\delta\epsilon_{\text{eff}}}{\epsilon_{\text{eff}}} = \pm 2\left(\frac{3\pi G\mathcal{M}}{c^2\mathcal{R}_{\text{orb}}}\right)^2\frac{1}{\mathcal{T}_{\text{orb}}}. \quad (18)$$

Combining this result with (6), we will have:

$$\frac{\delta\mathcal{R}_{\text{mean}}}{\mathcal{R}_{\text{mean}}} = \pm 2\left(\frac{3\pi G\mathcal{M}}{c^2\mathcal{R}_{\text{orb}}}\right)^2\frac{1}{\mathcal{T}_{\text{orb}}}. \quad (19)$$

We will not apply this result to the recession of the Earth-Moon system from the Sun and as-well the recession of the Moon from the Earth.

3.1.1 *Earth-Moon Recession*

Substituting all the relevant values for the Sun-(Earth-Moon) system in (19), one finds that this system must be drifting at a paltry rate:

$$\delta\text{AU} \;=\; 2.60 \pm 0.05\,\text{mm/yr}. \quad (20)$$

This value is about 2.5% of the measured result. Clearly, there is no agreement here with experience. However, despite being smaller than expected, it is a significant result when compared to what is actually measured. Perhaps there is need to refine these measurements and have them confirmed by a number of independent group of astronomers.

3.1.2 *Moon Recession*

Again, substituting all the relevant values for the Earth-Moon system into (19), one finds that this system must be drifting at a rate:

$$\delta\left<\mathcal{R}_{\text{em}}\right> \;=\; (1.20 \pm 0.05) \times 10^{-7}\,\text{mm/yr}. \quad (21)$$

Compared to the measured value of $38.247 \pm 0.004\,\text{mm/yr}$ (Williams and Boggs 2009; Williams et al. 2004), this value (21) is not only smaller than the measured result, it is compatible with zero. Clearly, there is no agreement here with experience.

However, we must realise that the conservation of total angular momentum (*i.e.*, orbital angular momentum plus spin angular momentum of the Earth-Moon system) requires that the very recession of the Earth-Moon system from the Sun must lead to a recession of the Moon from the Earth as demonstrated *e.g.* in Nyambuya (2014). This means that the curvature of spacetime around the Earth may be too small to cause any significant recession of the Moon from the Earth and this observed recession is in-actual fact caused by the curvature of spacetime around the Sun which affects the Earth-Moon system.

4 DISCUSSION AND CONCLUSION

Because of the impressive agreement with a plethora of observational data to which it has been submitted to (see *e.g.* Will 2009, 2006), the predictions of the GTR are usually held with great reverence, as very accurate, so much that the GTR is now such an embellished theory that it is held as a touchstone theory of gravitation. Even at a minute-level as in the present case, finding predictions of the GTR that run contrary to experience is very important as a tool for a rigorous scrutiny of the theory. Be that it may, important our result is, we do not believe that this result that we have unearthed here calls for a revision of the GTR let alone threatens its dominance.

What this result really calls for is for a closer scrutiny of both the observational result and as-well of the GTR. A scrutiny of the GTR must be conducted with in mind the idea of finding better models within or even outside of the domains of the GTR. The reason for this is simple. We here have a solution that unprecedentedly gives excellent predictions for the perihelion precession of Solar planets, especially for the planet Mercury and we have this same solution making predictions about the recession of test bodies and this time, its predictions run-afloat with physically and natural reality. Unless off-cause we have committed a fatal error in our analysis, from a purely logical standpoint, this state of affairs is at any rate, not good news.

Amongst others, it would mean that if the GTR is correct – as most physicists strongly believe; even on its predictions of the recession of test bodies, then, the currently measured phenomenon of planetary recession may not be a gravitational phenomenon, but something else other than a gravitational driven phenomenon. There are off-cause some researchers that think this maybe the case, that the recession of the Earth-Moon and the Moon may not be a gravitational phenomenon. As such, the result obtaining in present would support their assertion. It should be stated that, at present, there has not been any calculation from with the confines of the GTR that would be used to rule out the idea that this test body recessional phenomenon is not a gravitational phenomenon.

Conclusion

From what we have presented herein, we here make the following conclusion:

(i) It is pristine clear that Einstein's GTR does predict the secular drift of test bodies from the parent body about which they orbit and in the case of the Solar system, we have shown that these predictions are not in joyful tandem with physical and natural reality – *i.e.*, the measured recession of the Earth-Moon system from and Sun is only 2.5% of the observed value.

(ii) In the case of the Moon's recession from the Earth, the GTR fails to account for the observed $38.247 \pm 0.004\,\mathrm{mm/yr}$ measured recession. This does not mean the GTR is wrong, but one needs – in the light of the recession of the Earth-Moon system from the Sun; to apply the conservation of total angular momentum (orbital + spin) to the Earth-Moon system. Therefore, it is highly likely that the recession of the Moon is not caused by the same effect causing the recession of the Earth-Moon system from the Sun but is a consequence of the conservation of total angular momentum when applied to recession of the Earth-Moon system.

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