Approach for multi-period PD calibration for LDP portfolios.

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In the paper we propose PD calibration framework for LDP that allows producing smooth non-zero PD estimates for any given time horizon within the length of economic cycle. The advantages of the approach is that produced PDs are consistent with two main anchors – PIT and TTC PD estimates and are subject to smooth, monotonic transition between those two anchors. In practise, proposed framework could be applied to risk-based pricing of mid-term deals, whose duration is too long compared with PIT PD horizon and significantly shorter that the length of the whole economic cycle.

Currently, there are two main approaches to probability of default (PD) calibration: so-called TTC (through-the cycle) and PIT (point-in-time), see details, for example, in [1], [5].

TTC PDs should be stable, e.g. it is expected that TTC PDs will not significantly vary over the whole economic cycle. TTC PDs are used for capital calculations according to the recommendations of Basel Committee on Banking Supervision (BCBS) [2], mainly because their usage helps to decrease pro-cyclicality of capital requirements. Usually, the starting point for TTC PD calibration for a given rating class “R” (especially for portfolio with sufficient default statistics) is to calculate average long run defaults frequency estimate over the time frame T that covers the whole economic cycle:

$$TTC\ PD_R = \frac{1}{T}\sum_{t=1}^{T} PD_R(t)(1)$$

On another hand, TTC PD usage for pricing purposes always leads to “out of market prices” due to overestimation of PDs for expansion periods and underestimation for stress periods.
Therefore, for pricing purposes PIT PDs are commonly used. PIT PDs tends to fluctuate in accordance with economic cycle, e.g. to be higher for stress periods and lower for expansion time. The simplest approach used by banks is to take actual default frequency in the rating class “R” for the most recent period as a proxy for a PIT PD_R for the next period (more advanced banks makes forward-looking adjustment using a variety of different approaches).

Proposed in the paper approach to PD calibration for pricing purposes fulfills the following requirements:

1. It is applicable for portfolios with scarce defaults statistics, including low default portfolios (LDP). Default frequencies LDP portfolios could be very volatile or even zero for high rating classes, therefore it is a challenge to produce monotonic PIT PD estimates for rating classes.

PIT-TTC approach could produce a risk-based price for a loan only for next period or for a whole cycle. It’s unclear how should we price loans whose maturity extends beyond the time horizon that you can reasonably predict but still does not cover a full economic cycle

2. (for example, 2 or 3 years maturity). PD calibration framework for risk-based pricing should allow to produce PD estimates for any given maturity within the credit cycle..

3. It is doubtful assumption that the default rate in the next period should be the same as it was recently. This assumption means that we think, that the default rate is stable between current and the next periods. That is obviously does not hold, especially for emerging economies.
4. The common problem for banks, especially in emerging economies, is a lack of statistical data for a whole economic cycle. In that case the approach should allow us to make consistent PIT and TTC PD estimations.

From our point of view, further proposed PD calibration framework will allow the banks, even in case of LDP portfolios with short observable historical time frame, to build a consistent, transparent PD calibration system with non-zero PDs for any given rating class and maturity.

First of all, we should switch from separate PD estimation for each rating class to a portfolio wide default frequencies. Even for portfolios with sufficient historical data, it is hard to get consistent default frequencies data by rating classes for the whole economic cycle (due to changes in the models with the time). Therefore, the most consistent and robust figure which could be estimated almost for any credit portfolio is an average default frequency for the portfolio: $PD_p(t)$, where t=1..T is a time index. Hereinafter, TTC PD_p means through the cycle portfolio average default frequency (e.g. central tendency PD), PIT PD_p means forecast of the expected default frequency in the portfolio in the next period.

In order to find PIT PD_p, taking into account issue (3), we have to make some forecast of the default frequency in the next period. PD forecast could be made using variety of different technics and models and is a separate deep topic. The main requirement of the model (and a common economic sense) is a consistency between PIT and TTC estimates. One of the ways to achieve consistency is to calibrate dependence between PD and external indicators:
$PD_p = F(M_i) + \varepsilon\ (2), i = 1..N$, where M_i - i-th factor, ε - random error.

Given the calibrated functional dependence (2) we can consistently estimate TTC PD_p and PIT PD_p, even in case of issue (4) (lack of statistical data for the whole cycle). PIT PD_p would be estimated using forecasted values of macro-variables for the next period $\breve{M}_i\ i = 1..N$.

Using historical dynamics of macro-variables we could get estimate of portfolio PD for any point in time, therefore we could extend our factual statistics by «recovered» default frequencies. Given such partly «recovered» statistics for the whole economic cycle, we could estimate using TTC PD_p using formula (1). This approach to portfolio central tendency estimation is quite close to the quasi-TTC idea, described in [3].

Now we have PIT PD_p that could be using for pricing short-term loans (maturity is equal or less than forecast of macro-variables) and TTC PD_p that should be used for long-term deals with maturity close to the length of economic cycle (because pool of long term loans will pass through all the points of economic cycle, therefore averaging default rates through the time is a correct estimation).

Next step is to solve the (2) issue regarding mid-term loans pricing. By mid-term maturity loans we mean loans maturity extends beyond the time horizon that you can reasonably predict but still does not cover a full economic cycle.

The general idea of our approach is the following. The first assumption in our pricing framework is that the probability of the occurrence of stress event within the time frame equal to economic cycle

is close to 1, the probability of two stress events within the same time frame is close to 0. This is equal to the “from crisis to crisis” definition of the of the economic cycle.
Asymptotically you would expect a loan with maturity equal to economic cycle to see (on average across his lifetime) all stages of an economic cycle (i.e. good vs. bad periods in the same ratio as would correspond to the good-bad-ratio of an economic cycle). For mid-term loans, obviously, the longer the maturity of the loan the lower the reliability of the forecast of states of economy that the loan passes through across its lifetime. In case we are not anticipating stress event in our forecasting horizon, it is reasonable to assume that the probability to catch a stress in increasing with the time until it become 1 at the economic cycle maturity (and therefore such loan will be prices at TTC PD). On the other hand, in case we anticipate stress, it is reasonable to assume that after it marginal PIT PDs should decrease (according to assumptions we could have only 1 stress event in economic cycle) after our forecasting horizon. Thus it is natural to interpolate between current/ PIT (and better predictable) default rates towards TTC default rates when setting price levels comparing short-term vs. mid/long-term loan.

For example, let’s assume that we have a forecast of 1 year PIT PD_p and it is less than TTC PD_p estimate. That means that according to our macro-forecast we are not anticipating stress event in the forecasting period (in case of stress anticipation, PIT PD should be greater than TTC). Since crisis is always unpredictable, the longer the maturity of the deal, the lesser is our confidence in the absence of stress event in that time frame. Longer maturity leads the higher probability to catch stress event in case we are not anticipating it in our forecasting horizon. Therefore, for example, the chances for a 4-years loans pool to pass through the stress event is much higher than for 2-years loans pool Since the stress event is unpredictable, using above described logic, prices should converge from the PIT PD to TTC PD in accordance with their maturity.
As the boundary we use TTC PD because for a pool with maturity equal to economic cycle the chances to catch one stress event and spend the rest of the time under normal economic conditions is equal to 1 under our assumptions, therefore we should use TTC PD_p to price it.
In case of PIT PD_p estimate is less than TTC PD_p estimate, using the same logic, marginal PDs for a longer maturity should decrease (because there could be only one crisis during economic cycle and probability of it’s occurrence next period is high according to the macro-forecast), therefore PIT PD once again will converge to TTC PD with the time but from “another side”. Mathematically, it could be formulated as:

$$\widetilde{PD}_p(t) = PIT\ PD_p + \left(TTC\ PD_p - PIT\ PD_p\right) \cdot \text{Convergence}\ factor(t) (3)$$

where $\widetilde{PD}_p(t)$ – is a term structure of annual average default rates (spot PDs) for the portfolio.
By definition, convergence factor should be close to 0 for t = 1 and close to 1 for t equal to the maturity of the economic cycle (T). One of the simplest implementation of the Convergence factor is the following:

$$\text{Convergence}\ factor(t) = 1 - e^{-\lambda(t-1)}\ (4)$$

In case of formula (3) Convergence factor, as required, is equal to 0 for the first period (t=1) and asymptotically convergence to 1 for $t \to +\infty$. The speed of convergence λ could be calibrated in two ways: based on the assumption of the duration of the economic cycle and market-based approach.

In case we are fixing economic cycle duration T, we should require the convergence factor to be smaller than some reasonable threshold since we approach duration equal to T, therefore we can find a low bound estimate $\tilde{\lambda}$ for the λ:

$$1 - e^{-\tilde{\lambda}(T-1)} \leq 1 - Ths\ (5)$$

One of the reasonable ways to calibrate the threshold Ths, is to require the precision of convergence to be equal or less to the precision of the pricing system. In case, precision of our pricing system is 1 basis point and we use standard rounding rules, the convergence should be achieved at least on the 0.4 b.p. level. Because the Convergence factor influences only second summand in formula (3), the Ths could be found as:

$$\text{Ths} \leq \frac{Required\ precision}{TTC\ PD_p - PIT\ PD_p} \quad (6)$$

Therefore, using (5) and (6) convergence speed is equal to:

$$\tilde{\lambda} = -ln \frac{\frac{Required\ precision}{TTC\ PD_p - PIT\ PD_p}}{T-1}$$

According to above described methodology, Table 1 and Picture 2 show results for the following assumptions:
Cycle duration equal to 10 years;
TTC PD = 4%
PIT PD (expansion) = 2.5%, PID PD (stress) = 8%;
Precision of pricing system is equal to 0.4 b.p.

Table 1. PD term structure duration based example.

Time (years)	1	2	3	4	5	6	7	8	9	10
Expansion	2.50%	3.224%	3.598%	3.792%	3.892%	3.944%	3.971%	3.985%	3.992%	3.996%
Stress	8.00%	5.857%	4.862%	4.400%	4.186%	4.086%	4.040%	4.019%	4.009%	4.004%

One can see that all requirements are fulfilled in both expansion and stress cases: PIT PDs meets exactly, TTC PD convergence error at the cycle duration point is less than the rounding error for pricing system (1 b.p. precision).

The second, market derived approach, is based on the fitting of some market quoted PD term structure by the function (4). As an example, we could use some liquid (country) CDS spreads term structure (or an averages term structure of CDS spreads because we need to capture only dynamics). In that case, short term (1 year) CDS is a proxy for a PIT PD, while the long term CDS spread (10 years) approximates average default rate over the whole economic cycle. High volatility of market indicators could be a problem under these approach, possible mitigations could be:

- Averaging of the CDS quotes for a significant time horizon;
- Usage of the most liquid instruments in the market (as a last resort, convergence of the most liquid instruments like LIBOR rates could be taken as a proxy).

Therefore, after simple fitting procedure, we get market based speed of convergence λ.

The calibration procedure is quite simple. On the first step we normalize CDS quotes (using 1-year CDS= CDS(1) as PIT estimate, 10 years CDS = CDS(10) as TTC) :

$$\widetilde{\text{CDS}}(\text{t}) = 1 - \frac{CDS(t) - CDS(1)}{CDS(10) - CDS(1)}$$

After normalization we could find market convergence speed λ using routine fitting procedure:

$$\widetilde{\text{CDS}}(\text{t}) \sim e^{-\lambda t}$$

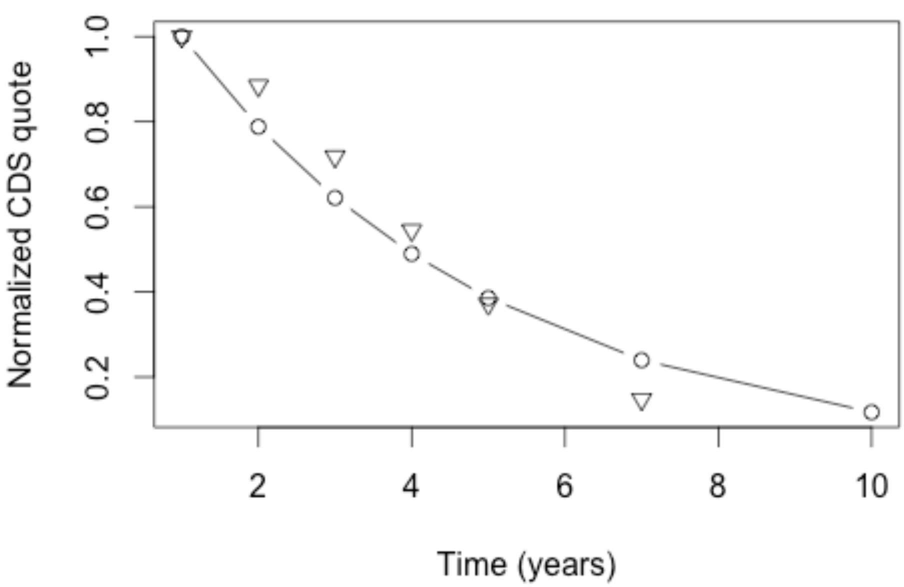
Given the real CDS quotes on Russia Federation (provided in the Table 2) we have λ = 0.2382 (RSS = 0.0442). The Picture 1 describes goodness of the fit. This results shows, that our duration based estimate is quite conservative (low bounds for λ are: expansion = 0.66, stress = 0.77). Graphical comparison of the results you can see at the Picture 2. Market based prices, based on the same PIT and TTC PD values, as in duration approach, you can see in Table 2.

As the most prudential approach we propose to use maximum of the speeds of convergence produced by market prices and our assumption regarding economic cycle duration.

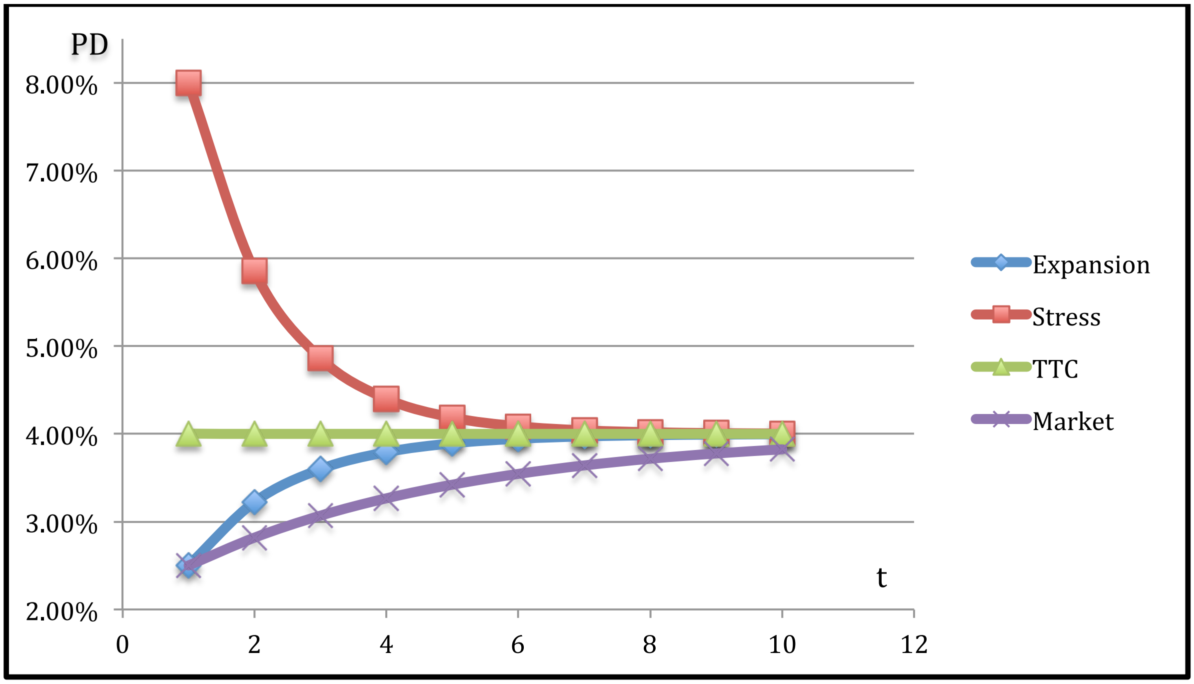
Table 2. PD term structure market based example.

Time (years)	1	2	3	4	5	6	7	8	9	10
CDS quote	0.44%	0.62%	0.88%	1.15%	1.42%	-	1.77%	-	-	2.00%
Market driven PD	2.50%	2.818%	3.068%	3.266%	3.422%	3.544%	3.641%	3.717%	3.777%	3.824%

Picture 1. Fit of normalized CDS quotes



Picture 2. PD term structure example.



The final step of the pricing framework is to decompose the $\widetilde{PD}_p(t)$ for each duration to rating structure. That could be done, for example, using central tendency calibration approach described in [4]. The only difference is that we replace TTC PD by $\widetilde{PD}_p(t)$ in each duration bucket (year). The idea of the approach is the following. Fit the CAP function (parameter k) using concave function:

$$y(x) = \frac{1-e^{-kx}}{1-e^{-k}} \quad (7)$$

Further, for each duration t, $\widetilde{PD}_p(t)$ is decomposed for rating grades using derivative of the function (7):

$$\widetilde{PD}_p(R,t) = \widetilde{PD}_p(t)\frac{dy}{dx} = \frac{\widetilde{PD}_p(t)}{1-e^{-k}}e^{-kx_R}$$

where x_R represents the cumulative percentage of counterparties in rating class R.
In case of such decomposition, we should make two assumptions:

- changes in rating structure of the portfolio.
- changes in AR through the time;

We can deal with the first one relatively easy. The first option is too calculate historical average rating structure. The second approach could be based on the application of the rating migration matrixes to current portfolio paired with the business plan of portfolio growth by rating classes. Regarding AR value, we can again can average historic AR values. Another, more prudent approach is to calculate AR standard deviation value (see [1]), scale it using square root of time coefficient for each duration and subtract it from mean of AR. The second approach, in our opinion, have more economic sense, because it produces less convex PD estimates for long term deals than for short term. This effect is reasonable, because the longer horizon we have, the worse is the predictive power of our model and the more higher is the probability of significant rating migration.
In case of sufficient default statistics, Gini decay could be measured using actual data – by changing time distance between assigned ratings and default events.

As we result, we’ve constructed PD calibration framework that meets all above mentioned requirements (1-4).

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[2] Basel Committee on Banking Supervision, International Convergence of Capital Measurement and Capital Standards, June 2006.
[3] Financial Service Authority, A regulatory response to the global banking crisis, March 2009.
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[5] B. Ozdemir, P. Miu, Basel II Implementation: A Guide to Developing and Validating a Compliant, Internal Risk Rating System, McGraw-Hill Sep 2008