A conjecture about a way in which the squares of primes can be written and five other related conjectures

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Abstract. I was playing with randomly formed formulas based on two distinct primes and the difference of them, when I noticed that the formula p + q + 2*(q – p) – 1, where p, q primes, conducts often to a result which is prime, semiprime, square of prime or product of very few primes. Starting from here, I made a conjecture about a way in which any square of a prime seems that can be written. Following from there, I made a conjecture about a possible infinite set of primes, a conjecture regarding the squares of primes and Poulet numbers and yet three other related conjectures.

Conjecture 1:

The square of any prime p, p ≥ 5, can be written at least in one way as p^2 = 3*q – r – 1, where q and r are distinct primes, q ≥ 5 and r ≥ 5.

Comment:

I really have no idea yet how it could be proved the conjecture or what implications it could have if it were true, so I'll just check it for the first few squares of primes.

Verifying the conjecture:

(for the first few squares of primes)

: 5^2 = 3*11 – 7 – 1, so [p, q, r] = [5, 11, 7];
: 7^2 = 3*19 – 7 – 1, so [p, q, r] = [7, 19, 7];
: 11^2 = 3*43 – 7 – 1, so [p, q, r] = [11, 43, 7];
: 13^2 = 3*59 – 7 – 1, so [p, q, r] = [13, 59, 7];
: 17^2 = 3*101 – 13 – 1, so [p, q, r] = [17, 101, 13];
: 19^2 = 3*127 – 19 – 1, so [p, q, r] = [19, 127, 19];
: 23^2 = 3*179 – 7 – 1, so [p, q, r] = [23, 179, 7];
: 29^2 = 3*283 – 7 – 1, so [p, q, r] = [29, 283, 7];
: 31^2 = 3*331 – 31 – 1, so [p, q, r] = [31, 331, 31];
: 37^2 = 3*461 – 13 – 1, so [p, q, r] = [37, 461, 13];
: 41^2 = 3*563 – 7 – 1, so [p, q, r] = [41, 563, 7];
: 43^2 = 3*619 – 7 – 1, so [p, q, r] = [43, 619, 7];
: 47^2 = 3*739 – 7 – 1, so [p, q, r] = [47, 739, 7].

Note:

It can be seen that in few cases from the ones above we have p = r, so we make yet another conjecture:

Conjecture 2:

There exist an infinity of primes p that can be written as p = (q^2 + q + 1)/3, where q is also a prime.

Examples of such primes:

: 19 = (7^2 + 7 + 1)/3, so [p, q] = [19, 7];
: 127 = (19^2 + 19 + 1)/3, so [p, q] = [127, 19];
: 331 = (31^2 + 31 + 1)/3, so [p, q] = [331, 31].

Conjecture 3:

The square of any prime p, p ≥ 5, can be written at least in one way as p^2 = 3*q – r – 1, where q is a Poulet number and r a prime, r ≥ 5.

Verifying the conjecture:

(for the first few squares of primes)
: 5^2 = 3*341 – 997 – 1, so [p, q, r] = [5, 341, 997];
: 7^2 = 3*1387 – 4111 – 1, so [p, q, r] = [7, 1387, 4111];
: 11^2 = 3*4371 – 13921 – 1, so [p, q, r] = [11, 4371, 13921];
: 13^2 = 3*341 – 853 – 1, so [p, q, r] = [13, 341, 853].

Comment:

Considering the results from the three conjectures above, I make three other related conjectures.

Conjecture 4:

For any prime p, p ≥ 5, there exist an infinity of pairs of distinct primes [q, r] such that p = sqrt(3*q – r – 1).

Example:

(for p = 7)
: 7 = sqrt(3*19 – 7 – 1), so [q, r] = [19, 7];
: 7 = sqrt(3*23 – 19 – 1), so [q, r] = [23, 19];
: 7 = sqrt(3*29 – 37 – 1), so [q, r] = [29, 37];
: 7 = sqrt(3*31 – 43 – 1), so [q, r] = [31, 43];
: 7 = sqrt(3*37 – 61 – 1), so [q, r] = [37, 61];
: 7 = sqrt(3*41 – 73 – 1), so [q, r] = [41, 73];
(...).

Conjecture 5:

For any prime p, p ≥ 5, there exist at least a pair of distinct primes [q, r] such that p = (q^2 + r + 1)/3.

Verifying the conjecture:

(for the first few primes)
: 5 = (3^2 + 5 + 1)/3, so [p, q, r] = [5, 3, 5];
: 7 = (3^2 + 11 + 1)/3, so [p, q, r] = [7, 3, 11];
: 11 = (3^2 + 5 + 1)/3, so [p, q, r] = [5, 3, 5];
: 13 = (5^2 + 13 + 1)/3, so [p, q, r] = [13, 5, 13];
: 17 = (3^2 + 41 + 1)/3, so [p, q, r] = [17, 3, 41];
: 19 = (5^2 + 31 + 1)/3, so [p, q, r] = [19, 5, 31].

Conjecture 6:

For any prime p of the form p = 6*k + 1 there exist an infinity of pairs of distinct primes [q, r] such that p = 3*q - p^2 - 1.

Example:

(for p = 37)
:   37 = 3*29 - 7^2 - 1, so [q, r] = [29, 7];
:   37 = 3*53 - 11^2 - 1, so [q, r] = [53, 11];
:   37 = 3*109 - 17^2 - 1, so [q, r] = [109, 17];
:   37 = 3*293 - 29^2 - 1, so [q, r] = [293, 29];
:   37 = 3*1693 - 71^2 - 1, so [q, r] = [1693, 71];
:   37 = 3*1789 - 73^2 - 1, so [q, r] = [1789, 73];
(...).