Problem in the energy and momentum conservation

Shalender Singh* and Vishnu Priya Singh Parmar

Priza Technologies Inc. R&D, 1525 McCarthy Blvd, Ste 1111,

Milpitas, California, USA 95035

Email: shalender@prizatech.com

Abstract

Lorentz transformations and special theory of relativity have existed for more than a century and mathematics related to them has been used and applied for innumerous times. Relativistic energy and relativistic momentum equations have been derived and proven to be conserved if energy/momentum transaction is seen from different frames of reference. The set of permissible inertial reference frame velocities from where the energy and momentum of a closed system of particles may be observed to be conserved forms a ball in the velocity vector space. In this paper we use the existing equations of special theory of relativity and Lorentz transformations and the mathematical structure of the observation velocity space to prove that the conservation of kinetic energy implies the conservation of momentum. We also prove that the conservation of momentum implies the conservation of kinetic energy. We further derive many more linearly independent conservation equations directly from the conservation of energy/momentum. The derivation of the conservation of kinetic energy from the conservation of momentum implies that either potential energy has a momentum thus made of inertial particles or there cannot be a net conversion of potential energy to kinetic energy. Furthermore the existence of many equations lead to extremely strict form of transfers of energy and momentum. It highly restricts the set of states particles in any closed system can assume without changing the overall energy of the system. This has a strong impact on the particle mechanics and as an example we show that the relativistic explanation of the elastic collision of particles striking each other as used by Einstein in the 1934 two blackboard derivation of mass and energy is itself inconsistent and wrong.

Keywords: Conservation of energy, Conservation of Momentum, Lorentz Transformation, Special Theory Of Relativity, Principle of Inertia, Infinite Conservation Equations.

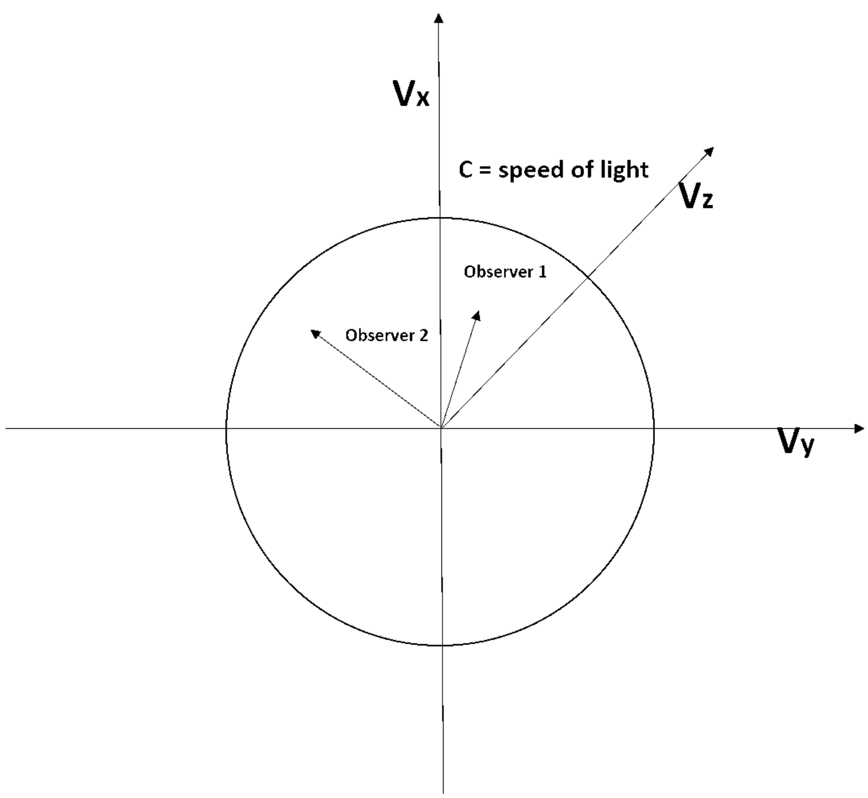
Classification: PACS 11.30.-j

1. Introduction

The conservation of energy and the conservation of momentum are considered two different principles, each of them exists independently of the other [1] [2] [3] [4] [5]. In most of the existing models of particle interactions and collisions, the conservation of momentum being a part of "principle of inertia" is considered a more sacrosanct principle than the conservation of mechanical energy as mechanical energy may be converted to some other forms of energy [1] [2] [3] [4] [6].

The first authoritative change to the long lasting formulae for kinetic energy was given by Einstein who showed equivalence of mass and energy. He and his other contemporary physicist and mathematician came up with space-time, energy and momentum transformation as per the new findings about the speed of light [7] [8] [9] [10] [11] [12] [13].

In this article we prove that the conservation of momentum can be derived from the conservation of kinetic energy. We also prove that the conservation of momentum implies conservation of kinetic energy. The derivation is based on looking at conservation of energy of particles from a continuum of inertial reference frames. As per the special theory relativity the conservation of relativistic energy and momentum remains valid if seen from any inertial frame of reference moving at a speed less that light [14]. The set of observation inertial frames of reference form a continuum of velocities. The topology of the continuum is defined only by the velocities so the position of reference frames in the space is irrelevant. A pair of inertial reference frames with very close velocities but with a separation of very large distance are very close elements in this continuum. As per the special relativity, the energy and momentum conservation is consistent across all the inertial reference frames in the continuum. Figure 1 shows the 3D continuum of velocities.



Continuum space of velocities in observation space

Figure 1: Continuum of inertial frames of references of observers as defined by velocities

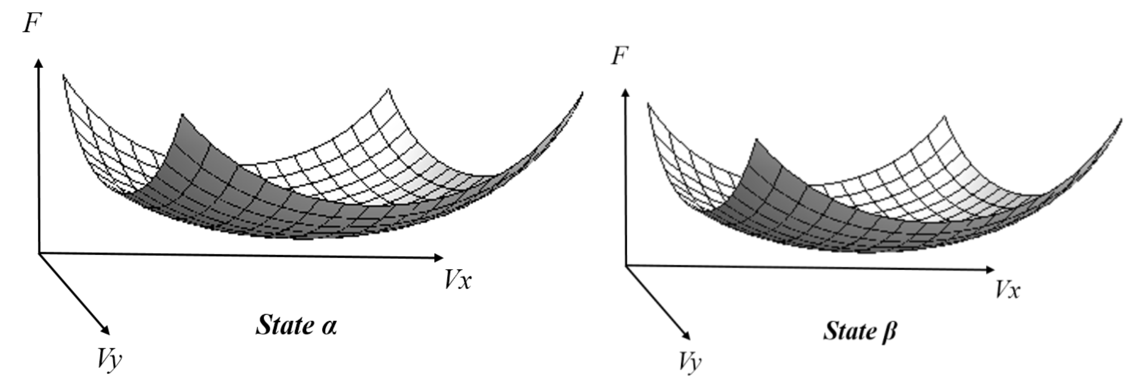


Figure 2: A conserved field in state α and β where V_x and V_y are velocity components of the continuum of allowed inertial reference frames

Assume an inertial reference frame **A** in which an observer observes a conserved property **F** of a closed system of particle. Examples of such property are total energy and momentum. Special theory of relativity allows any and all inertial frames of reference with velocity less than light relative to **A.** So any inertial frame of reference with velocity $(V_x, V_y, V_z) \in \mathbb{R}^3$ and $\|(V_x, V_y, V_z)\| < c$ is allowed and total energy and momentum of a closed system of particles is also conserved for an observer in that inertial reference frame.

For the ease of illustration assume 2D velocity space with **A** as origin $(0,0)$. With **A** as origin in the velocity space there exists allowed inertial frame of references with velocity $(V_x, V_y) \in \mathbb{R}^2$ relative to **A** and $\|(V_x, V_y)\| < c$. The energy and momentum of a closed system of particles is conserved for the observers in all of them. Figure 2 shows a 2D surface plot of a conserved field F as observed by observers in different inertial frames of reference. As we can see in the figure 2, if the field is conserved with the change of state from α to β, the topology of the field observed by observers in all allowed inertial frames of reference remains exactly the same. As the topology of the system remains exactly the same, various derivatives along the topology also remain the same, which means conserved. This means that $\left(\partial F / \partial V_x, \partial F / \partial V_y\right)$ remains same for the states α and β. Same is true for higher order derivatives. It is important to note that these derivatives are taken along the topology of the value of the conserved function as observed by observer in the allowed inertial reference frames and does not in any way mean the acceleration of the system or the reference frames.

Intuitively it is like different observers in different inertial reference frames sharing their observations with a central observer and the central observer plotting the values as a 2D surface. The central observer will notice that in the state α and β the value of the conserved property remains the same and so do the derivatives.

Using the above formulation we given a detailed mathematical derivation of our results in the section 2. In the subsection 2.4 we prove conservation of momentum from conservation of kinetic energy for an arbitrary dimensional space. In the section 2.5 we prove the conservation of kinetic energy from the conservation of momentum. In the section 2.6 we further prove that there exist infinite conservation equation, which become finite only in the case if the directional angles are quantized rational numbers. In the finite case the conservation quantities form a finite group of roots of unity. In the general case, as the number of equations become infinite, any closed system of interaction becomes over-determined.

In the section 3 we show that even elastic collision of two balls/particles as used by Einstein in two blackboard derivation in 1934 comes out to be invalid [15].

2. Generic derivation of infinite equations in M dimensions:

Let's assume there are **n** particles in a closed system and there exists a scalar conservation function **F**, which solely depends on the magnitude of the velocity of the particles (we look at states of the system where there is no inter particle potential energy).

Let the velocities of the particles be $\{\vec{V}_{a1}, \vec{V}_{a2}, ..., \vec{V}_{an}\}$ in a reference frame **A**. Let us take a continuum of inertial reference frames as a set **BC** with velocities $\vec{h}$ *w.r.t.* to **A** where $\left|\vec{h}\right| < c$. Let $\mathbf{B}\left(-\vec{h}\right) \in$ **BC** be a frame of reference in the set of inertial reference frames with velocity $-\vec{h}$ w.r.t. **A.** Let the resultant velocity of the particles in frame of reference $\mathbf{B}\left(-\vec{h}\right)$ be $\{\vec{V}_{b1}, \vec{V}_{b2}, ..., \vec{V}_{bn}\}$. Then as per the Lorentz velocity addition rule [7] [8] [16]:

$$\vec{V}_{bk} = \frac{(\vec{V}_{ak(parallel)} + \vec{h} + \sqrt{1 - \left|\vec{h}\right|^2 / c^2}\,\vec{V}_{ak(perpendicular)})}{(1 + \vec{V}_{ak} \cdot \vec{h} / c^2)} \; \forall 1 \le k \le n \qquad \dots (1)$$

2.1 Lemma 1: General conservation in any direction for m-dimensional space

Statement: Given m-dimensional space with orthonormal basis as $\{\vec{e}_1, \vec{e}_2, \vec{e}_3, ..., \vec{e}_m\}$, velocity of frame of reference $\mathbf{B}\left(-\vec{h}\right) \in$ **BC** w.r.t. **A** as $-\vec{h} = -h\vec{e}_q$ (direction in the qth dimension), velocity of kth particle in frame of reference **A** as $\vec{V}_{ak} = V_{ak}\vec{r}_k$ (where $\vec{r}_k = r_{k1}\vec{e}_1 + r_{k2}\vec{e}_2 + ... + r_{km}\vec{e}_m$ is the unit vector in the direction of $\vec{V}_{ak}$ with $r_{k1}^2 + r_{k2}^2 + ... + r_{km}^2 = 1$) and a scalar conservation function **F**, which solely depends on the magnitude of velocity V_{bk}, if $\sum_{k=1}^{n} F\left(V_{bk}\left(h, \theta_k, V_{ak}\right)\right)$ is conserved then

$$\sum_{k=1}^{n} r_{qk}\left(1 - V_{ak}^2 / c^2\right) \left.\frac{\partial F\left(V_{bk}\left(h, \theta_k, V_{ak}\right)\right)}{\partial V_{bk}}\right|_{h=0}$$ is also conserved for all $1 \le q \le m$.

Proof:

Given:

$$\vec{h} = h\vec{e}_q$$

$$\vec{V}_{ak} = V_{ak}(r_{k1}\vec{e}_1 + r_{k2}\vec{e}_2 + ... + r_{km}\vec{e}_m)$$

We have following:

$$\begin{aligned}
&\vec{V}_{\mathrm{ak}(parallel)} = V_{ak} r_{kq} \vec{e}_q \\
&\vec{V}_{\mathrm{ak}(perpendicular)} = V_{ak}(r_{k1}\vec{e}_1 + r_{k2}\vec{e}_2 + ... + r_{kq-1}\vec{e}_{q-1} + r_{kq+1}\vec{e}_{q+1} + ... + r_{km}\vec{e}_m) \qquad \dots (2) \\
&\vec{V}_{ak} \bullet \vec{h} = V_{ak} h r_{kq}
\end{aligned}$$

Substituting (2) in (1) the resultant velocity in the frame of reference **B** simplifies to be:

$$\vec{V}_{bk} = \frac{(V_{ak} r_{kq} \vec{e}_q + h\vec{e}_q + \sqrt{1-h^2/c^2} V_{ak} (r_{k1}\vec{e}_1 + r_{k2}\vec{e}_2 + ... + r_{kq-1}\vec{e}_{q-1} + r_{kq+1}\vec{e}_{q+1} + ... + r_{km}\vec{e}_m))}{(1+V_{ak} h r_{kq}/c^2)} \quad \forall 1 \le k \le n$$

So the speed (magnitude of the velocity) in the frame of reference **B** simplifies to be:

$$V_{bk} = \left\|\vec{V}_{bk}\right\| = \frac{\sqrt{\left(V_{ak} r_{kq} + h\right)^2 + \left(\sqrt{1-h^2/c^2} V_{ak}\right)^2 \left(r_{k1}^2 + r_{k2}^2 + ... + r_{kq-1}^2 + r_{kq+1}^2 + ... + r_{km}^2\right)}}{1+V_{ak} h r_{kq}/c^2}$$

But $r_{k1}^2 + r_{k2}^2 + ... + r_{km}^2 = 1$

Thus $r_{k1}^2 + r_{k2}^2 + ... + r_{kq-1}^2 + r_{kq+1}^2 + ... + r_{km}^2 = 1 - r_{kq}^2$ … (3)

$$\Rightarrow V_{bk}\left(h, \theta_k, V_{ak}\right) = \frac{\sqrt{V_{ak}^2 r_{kq}^2 + h^2 + 2hV_{ak} r_{kq} + \left(1-h^2/c^2\right) V_{ak}^2 \left(1-r_{kq}^2\right)}}{1+V_{ak} h r_{kq}/c^2}$$

$$\Rightarrow V_{bk}\left(h, \theta_k, V_{ak}\right) = \frac{\sqrt{V_{ak}^2 \left(r_{kq}^2 + 1 - r_{kq}^2\right) + h^2 + 2hV_{ak} r_{kq} - h^2 V_{ak}^2 / c^2 \left(1-r_{kq}^2\right)}}{1+V_{ak} h r_{kq}/c^2}$$

$$\Rightarrow V_{bk}\left(h, \theta_k, V_{ak}\right) = \frac{\sqrt{V_{ak}^2 + h^2 + 2hV_{ak} r_{kq} - h^2 V_{ak}^2 / c^2 \left(1-r_{kq}^2\right)}}{1+V_{ak} h r_{kq}/c^2} \qquad \text{… (4)}$$

As per the conservation of function **F** in the frame of reference **B**

$$TotalEnergy(B) = \sum_{k=1}^{n} F\left(V_{bk}\left(h, \theta_k, V_{ak}\right)\right) = const. = \mathrm{C}_b$$

Let α and β be two states of the system in which the energy is conserved then:

$$\sum_{k=1}^{n} F\left(V_{bk\alpha}\left(h, \theta_{k\alpha}, V_{ak\alpha}\right)\right) = \sum_{k=1}^{n} F\left(V_{bk\beta}\left(h, \theta_{k\beta}, V_{ak\beta}\right)\right) \qquad \text{… (5)}$$

As equation (4) is valid for any ***h* < *c*** and ***h*** is a real number thus a derivative with respect to ***h*** (along the observation continuum) should also satisfy the equality both in state α and β

$$\Rightarrow \sum_{k=1}^{n} \frac{\partial F\left(V_{bk\alpha}\left(h, \theta_{k\alpha}, V_{ak\alpha}\right)\right)}{\partial h} = \sum_{k=1}^{n} \frac{\partial F\left(V_{bk\beta}\left(h, \theta_{k\beta}, V_{ak\beta}\right)\right)}{\partial h} \qquad \text{… (6)}$$

Equation (6) implies $\sum_{k=1}^{n} \frac{\partial F\left(V_{bk}\left(h, \theta_k, V_{ak}\right)\right)}{\partial h}$ is also conserved as it remains constant with any arbitrary change of state α to β when the **F** is conserved.

Taking chain rule [17] on $\frac{\partial F\left(V_{bk}\left(h,\theta_k,V_{ak}\right)\right)}{\partial h}$

$$\frac{\partial E\left(V_{bk}\left(h,\theta_k,V_{ak}\right)\right)}{\partial h}=\frac{\partial V_{bk}\left(h,\theta_k,V_{ak}\right)}{\partial h}\frac{\partial E\left(V_{bk}\left(h,\theta_k,V_{ak}\right)\right)}{\partial V_{bk}\left(h,\theta_k,V_{ak}\right)}$$

$$\sum_{k=1}^{n}\frac{\partial F\left(V_{bk}\left(h,\theta_k,V_{ak}\right)\right)}{\partial h}=\sum_{k=1}^{n}\frac{\partial V_{bk}\left(h,\theta_k,V_{ak}\right)}{\partial h}\frac{\partial F\left(V_{bk}\left(h,\theta_k,V_{ak}\right)\right)}{\partial V_{bk}} \qquad \ldots (7)$$

Thus given any function **F:**

If $\sum_{k=1}^{n}F\left(V_{bk}\left(h,\theta_k,V_{ak}\right)\right)$ is conserved then $\sum_{k=1}^{n}\frac{\partial V_{bk}\left(h,\theta_k,V_{ak}\right)}{\partial h}\frac{\partial F\left(V_{bk}\left(h,\theta_k,V_{ak}\right)\right)}{\partial V_{bk}}$ is also conserved

Now let us simplify the term $\frac{\partial V_{bk}\left(h,\theta_k,V_{ak}\right)}{\partial h}$

$$\frac{\partial V_{bk}(h,V_{ak},\theta)}{\partial h}=\frac{\frac{1}{2}\left(2h+2V_{ak}r_{kq}-2hV_{ak}^2/c^2\left(1-r_{kq}^2\right)\right)}{\left(1+V_{ak}hr_{kq}/c^2\right)\sqrt{V_{ak}^2+h^2+2hV_{ak}r_{kq}-h^2V_{ak}^2/c^2\left(1-r_{kq}^2\right)}}$$

$$-\frac{\sqrt{V_{ak}^2+h^2+2hV_{ak}r_{qk}-h^2V_{ak}^2/c^2\left(1-r_{kq}^2\right)}\left(V_{ak}r_{kq}/c^2\right)}{\left(1+V_{ak}hr_{kq}/c^2\right)^2}$$

If we take ***h = 0***

$$\left.\frac{\partial V_{bk}(h,V_{ak},\theta)}{\partial h}\right|_{h=0}=\frac{\frac{1}{2}\left(2V_{ak}r_{qk}\right)}{(1)\sqrt{V_{ak}^2}}-\frac{\sqrt{V_{ak}^2}\left(V_{ak}r_{kq}/c^2\right)}{(1)^2}$$

$$\Rightarrow\left.\frac{\partial V_{bk}(h,V_{ak},\theta)}{\partial h}\right|_{h=0}=r_{qk}-r_{qk}V_{ak}^2/c^2$$

$$\Rightarrow\left.\frac{\partial V_{bk}(h,V_{ak},\theta)}{\partial h}\right|_{h=0}=r_{qk}\left(1-V_{ak}^2/c^2\right) \qquad \ldots (8)$$

Substituting equation (8) into equation (7) for ***h = 0***

$$\left.\sum_{k=1}^{n}r_{qk}\left(1-V_{ak}^2/c^2\right)\frac{\partial F\left(V_{bk}\left(h,\theta_k,V_{ak}\right)\right)}{\partial V_{bk}}\right|_{h=0} \text{ is also conserved.} \qquad \ldots (9)$$

As we proved for any $1\le q\le m$ thus (9) is true for any $1\le q\le m$.

Hence proved.

2.2 Lemma 2: Vector conservation for m-dimensional space

Statement: Given m-dimensional space with orthonormal basis as $\{\vec{e}_1, \vec{e}_2, \vec{e}_3, ..., \vec{e}_m\}$, velocity of frame of reference $\mathbf{B}\left(-\vec{h}\right) \in \mathbf{BC}$ w.r.t. **A** as $-\vec{h} = -h\vec{e}_q$ (direction in the q[th] dimension), velocity of k[th] particle in frame of reference **A** as $\vec{V}_{ak} = V_{ak}\vec{r}_k$ (where $\vec{r}_k = r_{k1}\vec{e}_1 + r_{k2}\vec{e}_2 + ... + r_{km}\vec{e}_m$ is the unit vector in the direction of $\vec{V}_{ak}$ with $r_{k1}^2 + r_{k2}^2 + ... + r_{km}^2 = 1$) and a scalar conservation function **F**, which solely depends on the magnitude of velocity V_{bk}, if $\sum_{k=1}^{n} F\left(V_{bk}\left(h, \theta_k, V_{ak}\right)\right)$ is conserved then

$$\sum_{k=1}^{n}\left(1 - V_{ak}^2 / c^2\right)\left.\frac{\partial F\left(V_{bk}\left(h, \theta_k, V_{ak}\right)\right)}{\partial V_{bk}}\right|_{h=0} \vec{r}_k$$ is also conserved for all $1 \le q \le m$.

Proof:

As per Lemma 1 for the above given conditions $\sum_{k=1}^{n} r_{qk}\left(1 - V_{ak}^2 / c^2\right)\left.\frac{\partial F\left(V_{bk}\left(h, \theta_k, V_{ak}\right)\right)}{\partial V_{bk}}\right|_{h=0}$ is conserved.

Putting the above equation in the vector sum with orthonormal basis $\{\vec{e}_1, \vec{e}_2, \vec{e}_3, ..., \vec{e}_m\}$ and running q from 1 to m

$$\sum_{k=1}^{n} r_{1k}\left(1 - V_{ak}^2 / c^2\right)\left.\frac{\partial F\left(V_{bk}\left(h, \theta_k, V_{ak}\right)\right)}{\partial V_{bk}}\right|_{h=0} \vec{e}_1 + ... + \sum_{k=1}^{n} r_{mk}\left(1 - V_{ak}^2 / c^2\right)\left.\frac{\partial F\left(V_{bk}\left(h, \theta_k, V_{ak}\right)\right)}{\partial V_{bk}}\right|_{h=0} \vec{e}_m$$

is also conserved in vector form as it is a linear combination of conserved quantities.

$$\Rightarrow \sum_{k=1}^{n}\left(r_{1k}\left(1 - V_{ak}^2 / c^2\right)\left.\frac{\partial F\left(V_{bk}\left(h, \theta_k, V_{ak}\right)\right)}{\partial V_{bk}}\right|_{h=0} \vec{e}_1 + ... + r_{mk}\left(1 - V_{ak}^2 / c^2\right)\left.\frac{\partial F\left(V_{bk}\left(h, \theta_k, V_{ak}\right)\right)}{\partial V_{bk}}\right|_{h=0} \vec{e}_m \right)$$

$$\Rightarrow \sum_{k=1}^{n}\left(\left(1 - V_{ak}^2 / c^2\right)\left.\frac{\partial F\left(V_{bk}\left(h, \theta_k, V_{ak}\right)\right)}{\partial V_{bk}}\right|_{h=0} \left(r_{1k}\vec{e}_1 + ... + r_{mk}\vec{e}_m\right) \right)$$

$$\Rightarrow \sum_{k=1}^{n}\left(1 - V_{ak}^2 / c^2\right)\left.\frac{\partial F\left(V_{bk}\left(h, \theta_k, V_{ak}\right)\right)}{\partial V_{bk}}\right|_{h=0} \vec{r}_k \text{ is conserved.} \qquad \text{… (10)}$$

Hence proved.

2.3 Lemma 3: With Lorentz transformation of velocity and conservation function F: $\left.\frac{\partial F\left(V_{bk}\left(h, \theta_k, V_{ak}\right)\right)}{\partial V_{bk}}\right|_{h=0}$ is equal to $\frac{\partial F\left(V_{ak}\right)}{\partial V_{ak}}$

Proof:

As the differential is taken w.r.t. $V_{bk}\left(h,\theta_k,V_{ak}\right)$ and the function is also the function of complete $V_{bk}\left(h,\theta_k,V_{ak}\right)$ and not individual h,θ_k,V_{ak}, we can take h = 0 before the differential is taken

$$\Rightarrow \left.\frac{\partial F\left(V_{bk}\left(h,\theta_k,V_{ak}\right)\right)}{\partial V_{bk}}\right|_{h=0} = \frac{\partial F\left(V_{bk}\left(h,\theta_k,V_{ak}\right)\right)\Big|_{h=0}}{\partial V_{bk}\big|_{h=0}}$$

But we derived earlier, $V_{bk}\big|_{h=0} = V_{ak}$

Thus $\left.\dfrac{\partial F\left(V_{bk}\left(h,\theta_k,V_{ak}\right)\right)}{\partial V_{bk}}\right|_{h=0} = \dfrac{\partial F\left(V_{ak}\right)}{\partial V_{ak}}$

Hence proved.

2.4 Theorem 1: Conservation of relativistic energy implies conservation of relativistic momentum.

Proof:

Take F as relativistic energy function. This means

$$F\left(V_{bk}\left(h,\theta_k,V_{ak}\right)\right) = \frac{m_k c^2}{\sqrt{1-\left(V_{bk}\left(h,\theta_k,V_{ak}\right)\right)^2/c^2}}$$

Thus

$$\frac{\partial F\left(V_{bk}\left(h,\theta_k,V_{ak}\right)\right)}{\partial V_{bk}} = m_k c^2 \frac{\partial}{\partial V_{bk}}\left(\frac{1}{\sqrt{1-\left(V_{bk}\left(h,\theta_k,V_{ak}\right)\right)^2/c^2}}\right)$$

$$\Rightarrow \frac{\partial F\left(V_{bk}\left(h,\theta_k,V_{ak}\right)\right)}{\partial V_{bk}} = -\frac{1}{2}\frac{-m_k c^2\, 2V_{bk}\left(h,\theta_k,V_{ak}\right)/c^2}{(1-\left(V_{bk}\left(h,\theta_k,V_{ak}\right)\right)^2/c^2)^{3/2}}$$

$$\Rightarrow \frac{\partial F\left(V_{bk}\left(h,\theta_k,V_{ak}\right)\right)}{\partial V_{bk}} = \frac{m_k V_{bk}\left(h,\theta_k,V_{ak}\right)}{(1-\left(V_{bk}\left(h,\theta_k,V_{ak}\right)\right)^2/c^2)^{3/2}}$$

For ***h = 0***

$$\left.\frac{\partial F\left(V_{bk}\left(h,\theta_k,V_{ak}\right)\right)}{\partial V_{bk}}\right|_{h=0} = \frac{m_k V_{bk}\left(0,\theta_k,V_{ak}\right)}{(1-\left(V_{bk}\left(0,\theta_k,V_{ak}\right)\right)^2/c^2)^{3/2}} \qquad \ldots (11)$$

Putting ***h = 0*** in the equation (1)

$$V_{bk}\left(0,\theta_k,V_{ak}\right)=\frac{\sqrt{V_{ak}{}^2}}{1}$$

$$\Rightarrow V_{bk}\left(0,\theta_k,V_{ak}\right)=V_{ak} \qquad \ldots (12)$$

Substituting (12) in (11)

$$\left.\frac{\partial F\left(V_{bk}\left(h,\theta_k,V_{ak}\right)\right)}{\partial V_{bk}}\right|_{h=0}=\frac{m_k V_{ak}}{(1-V_{ak}{}^2/c^2)^{3/2}} \qquad \ldots (13)$$

Substituting (13) in (10)

$$\Rightarrow \sum_{k=1}^{n}\left(1-V_{ak}^2/c^2\right)\frac{m_k V_{ak}}{(1-V_{ak}{}^2/c^2)^{3/2}}\vec{r}_k \quad \text{is conserved.}$$

$$\Rightarrow \sum_{k=1}^{n}\frac{m_k V_{ak}\vec{r}_k}{\sqrt{1-V_{ak}{}^2/c^2}} \text{ is conserved.}$$

But $\frac{m_k V_{ak}\vec{r}_k}{\sqrt{1-V_{ak}{}^2/c^2}}=\frac{m_k \vec{V}_{ak}}{\sqrt{1-V_{ak}{}^2/c^2}}=\vec{P}_{ak}$

$$\Rightarrow \sum_{k=1}^{n}\vec{P}_{ak} \text{ is conserved} \qquad \ldots (14)$$

Hence proved.

2.5 Theorem2: Conservation of momentum implies conservation of kinetic energy

Proof:

$\sum_{k=1}^{n}\vec{P}_{ak}=\sum_{k=1}^{n}\frac{m_k\vec{V}_{ak}}{\sqrt{1-V_{ak}{}^2/c^2}}=\sum_{k=1}^{n}\frac{m_k\vec{V}_{ak}}{\sqrt{1-V_{ak}{}^2/c^2}}$ is given to be conserved.

This implies that $\sum_{k=1}^{n}\frac{m_k V_{ak}}{\sqrt{1-V_{ak}{}^2/c^2}}r_{kq}$ is conserved for $1\le q\le m$

Using F = $\frac{m_k V_{ak} r_{kq}}{\sqrt{1-V_{ak}{}^2/c^2}}$ is the Lemma 1 and using the Lemma 3

$\sum_{k=1}^{n}r_{ks}\left(1-V_{ak}^2/c^2\right)\frac{\partial F\left(V_{ak}\right)}{\partial V_{ak}}$ is conserved for all $1\le s\le m$.

Simplifying $\left(1-V_{ak}^2/c^2\right)\frac{\partial F\left(V_{ak}\right)}{\partial V_{ak}}$

$$\left(1-V_{ak}^2/c^2\right)\frac{\partial F\left(V_{ak}\right)}{\partial V_{ak}}=\left(1-V_{ak}^2/c^2\right)\frac{\partial}{\partial V_{ak}}\left(\frac{m_k V_{ak} r_{kq}}{\sqrt{1-V_{ak}{}^2/c^2}}\right)$$

$$\Rightarrow\left(1-V_{ak}^2/c^2\right)\frac{\partial F\left(V_{ak}\right)}{\partial V_{ak}}=\left(1-V_{ak}^2/c^2\right)m_k c^2 r_{kq}\frac{1}{c^2}\left(\frac{1}{\sqrt{1-V_{ak}{}^2/c^2}}-\frac{1}{2}\frac{\left(V_{ak}\times-2V_{ak}/c^2\right)}{\left(1-V_{ak}{}^2/c^2\right)^{3/2}}\right)$$

$$\Rightarrow\left(1-V_{ak}^2/c^2\right)\frac{\partial F\left(V_{ak}\right)}{\partial V_{ak}}=\left(1-V_{ak}^2/c^2\right)m_k c^2 r_{kq}\frac{1}{c^2}\left(\frac{1-V_{ak}{}^2/c^2+V_{ak}{}^2/c^2}{\left(1-V_{ak}{}^2/c^2\right)^{3/2}}\right)$$

$$\Rightarrow\left(1-V_{ak}^2/c^2\right)\frac{\partial F\left(V_{ak}\right)}{\partial V_{ak}}=\left(1-V_{ak}^2/c^2\right)m_k c^2 r_{kq}\frac{1}{c^2}\left(\frac{1}{\left(1-V_{ak}{}^2/c^2\right)^{3/2}}\right)$$

$$\Rightarrow\left(1-V_{ak}^2/c^2\right)\frac{\partial F\left(V_{ak}\right)}{\partial V_{ak}}=\frac{1}{c^2}\left(\frac{m_k c^2 r_{kq}}{\sqrt{1-V_{ak}{}^2/c^2}}\right)$$

Thus $\sum_{k=1}^{n} r_{ks}\frac{1}{c^2}\left(\frac{m_k c^2 r_{kq}}{\sqrt{1-V_{ak}{}^2/c^2}}\right)$ is conserved for all $1\le s\le m$.

As $\frac{1}{c^2}$ is just a constant multiplication factor thus $\sum_{k=1}^{n} r_{ks}\left(\frac{m_k c^2 r_{kq}}{\sqrt{1-V_{ak}{}^2/c^2}}\right)$ is conserved for all $1\le s\le m$.

Furthermore $\sum_{k=1}^{n} r_{ks}\left(\frac{m_k c^2 r_{kq}}{\sqrt{1-V_{ak}{}^2/c^2}}\right)=\sum_{k=1}^{n} E_k r_{kq} r_{ks}$

Taking q = s

Thus $\sum_{k=1}^{n} E_k r_{ks}^2$ is conserved for all $1\le s\le m$.

As sum of conserved function is a conserved function thus $\sum_{q=1}^{m}\sum_{k=1}^{n} E_k r_{kq}^2$ is also conserved.

$\Rightarrow\sum_{k=1}^{n} E_k\sum_{q=1}^{m} r_{kq}^2$ is conserved.

But $\sum_{q=1}^{m} r_{kq}^2=1$

$\Rightarrow \sum_{k=1}^{n} E_k$ is conserved.

Here E_k is the kinetic energy of the particle k.

Hence proved.

2.6 Infinite conservation equations

2.6.1 Lemma 4: Conservation as an operator

Statement: Define an operator as $D(q,k) = r_{qk}\left(1 - V_{ak}^2 / c^2\right)\frac{\partial}{\partial V_{ak}}$. Given m-dimensional space with orthonormal basis as $\{\vec{e}_1, \vec{e}_2, \vec{e}_3, ..., \vec{e}_m\}$, velocity of frame of reference $\mathbf{B}\left(-\vec{h}\right) \in$ **BC** w.r.t. **A** as $-\vec{h} = -h\vec{e}_q$ (direction in the qth dimension), velocity of kth particle in frame of reference **A** as $\vec{V}_{ak} = V_{ak}\vec{r}_k$ (where $\vec{r}_k = r_{k1}\vec{e}_1 + r_{k2}\vec{e}_2 + ... + r_{km}\vec{e}_m$ is the unit vector in the direction of $\vec{V}_{ak}$ with $r_{k1}^2 + r_{k2}^2 + ... + r_{km}^2 = 1$) and a scalar conservation function **F**, which solely depends on the magnitude of velocity V_{bk}, if $\sum_{k=1}^{n} F\left(V_{bk}\left(h, \theta_k, V_{ak}\right)\right)$ is conserved then $\sum_{k=1}^{n} D(q,k) F\left(V_{ak}\right)$ is also conserved for all $1 \le q \le m$.

Proof:

As per Lemma 1, for the above conditions $\sum_{k=1}^{n} r_{qk}\left(1 - V_{ak}^2 / c^2\right)\left.\frac{\partial F\left(V_{bk}\left(h, \theta_k, V_{ak}\right)\right)}{\partial V_{bk}}\right|_{h=0}$ is conserved.

As per Lemma 3 $\left.\frac{\partial F\left(V_{bk}\left(h, \theta_k, V_{ak}\right)\right)}{\partial V_{bk}}\right|_{h=0} = \frac{\partial F\left(V_{ak}\right)}{\partial V_{ak}}$

It means that $\sum_{k=1}^{n} r_{qk}\left(1 - V_{ak}^2 / c^2\right)\frac{\partial F\left(V_{ak}\right)}{\partial V_{ak}}$ is conserved. … (15)

Replacing $D(q,k) = r_{qk}\left(1 - V_{ak}^2 / c^2\right)\frac{\partial}{\partial V_{ak}}$ in (15)

$\sum_{k=1}^{n} D(q,k) F\left(V_{ak}\right)$ is conserved …(16)

Hence proved.

2.6.2 Lemma 5: Infinite general conservations in any direction for m-dimensional space

Statement: Given $-\vec{h} = -h\vec{e}_q$ and $\vec{V}_{ak} = V_{ak}(r_{k1}\vec{e}_1 + r_{k2}\vec{e}_2 + ... + r_{km}\vec{e}_m)$ with $r_{k1}^2 + r_{k2}^2 + ... + r_{km}^2 = 1$ and a scalar conservation function **F**, which solely depends on the magnitude of velocity V_{bk}, if

$\sum_{k=1}^{n} F\left(V_{bk}\left(h,\theta_k,V_{ak}\right)\right)$ is conserved then $\sum_{k=1}^{n} D(q_w,k)...D(q_2,k)D(q_1,k)F\left(V_{ak}\right)$ is also conserved for arbitrary set of operators $\{D(q_w,k),...,D(q_2,k),D(q_1,k)\}$.

Proof:

Proof by induction:

A) For the $w=1$ condition the proof is Lemma 4.

B) Now assume that it is true for $w \in \mathbb{N}$ then

$\sum_{k=1}^{n} D(q_w,k)...D(q_2,k)D(q_1,k)F\left(V_{ak}\right)$ is conserved.

Take a new conservation function $G\left(V_{ak}\right) = D(q_w,k)...D(q_2,k)D(q_1,k)F\left(V_{ak}\right)$ … (17)

As per Lemma 4

$\sum_{k=1}^{n} D(q,k)\mathrm{G}\left(V_{ak}\right)$ is conserved. … (18)

Putting equation (17) in the equation (18)

$\sum_{k=1}^{n} D(q,k)D(q_w,k)...D(q_2,k)D(q_1,k)F\left(V_{ak}\right)$, is also conserved, which has $w+1$ number of operators

Thus $w \Rightarrow w+1$

Hence proved.

2.6.3 Separation of directions and derivatives in the infinite general conservations in any direction for m dimensional space

As per Lemma 5, $\sum_{k=1}^{n} D(q_w,k)...D(q_2,k)D(q_1,k)F\left(V_{ak}\right)$ is conserved for the conservation function F.

Also $D(q_w,k) = r_{q_w k}\left(1-V_{ak}^2/c^2\right)\frac{\partial}{\partial V_{ak}}$. As $r_{q_w k}$ is independent of V_{ak} it can separated and taken out of the series of operators. Let us define operator $S(k) = \left(1-V_{ak}^2/c^2\right)\frac{\partial}{\partial V_{ak}}$ then:

$$\sum_{k=1}^{n} D(q_w,k)...D(q_2,k)D(q_1,k)F\left(V_{ak}\right) = \sum_{k=1}^{n}\prod_{f=1}^{w} r_{kq_f}\left(S(k)\right)^w F\left(V_{ak}\right) \text{ is conserved} \quad \ldots (19)$$

2.6.4 Lemma 6: Repeated application of the operator $S(k) = \left(1-V_{ak}^2/c^2\right)\frac{\partial}{\partial V_{ak}}$ ***on relativistic energy***

Statement: Given the relativistic conservation function **F =** $\frac{m_k c^2}{\sqrt{1-V_{ak}{}^2/c^2}}$, the result of application of operator $S(k)$ is $\left(S(k)\right)^{2r} F = \frac{1}{c^{2r}} \frac{m_k c^2}{\sqrt{1-V_{ak}{}^2/c^2}} = \frac{1}{c^{2r}} F$ and $\left(S(k)\right)^{2r+1} F = \frac{1}{c^{2r}} \frac{m_k V_{ak}}{\sqrt{1-V_{ak}{}^2/c^2}} = \frac{1}{c^{2r}} P$ for $r \in \mathbb{N}$

Proof:

Proof by induction:

For r = 0

$$F = \frac{m_k c^2}{\sqrt{1-V_{ak}{}^2/c^2}}$$

$$\left(S(k)\right)^1 F = \left(1-V_{ak}^2/c^2\right) m_k c^2 \frac{\partial}{\partial V_{ak}} \left(\frac{1}{\sqrt{1-V_{ak}{}^2/c^2}} \right)$$

$$\Rightarrow \left(S(k)\right) F = \left(1-V_{ak}^2/c^2\right) m_k c^2 \times -\frac{1}{2} \frac{-2V_{ak}/c^2}{(1-V_{ak}^2/c^2)^{3/2}}$$

$$\Rightarrow \left(S(k)\right) F = \left(1-V_{ak}^2/c^2\right) m_k c^2 \times \frac{V_{ak}/c^2}{(1-V_{ak}^2/c^2)^{3/2}}$$

$$\Rightarrow \left(S(k)\right) F = \frac{m_k c^2 \times V_{ak}/c^2}{\sqrt{1-V_{ak}^2/c^2}} = \frac{m_k V_{ak}}{\sqrt{1-V_{ak}^2/c^2}} \qquad \ldots (20)$$

Assume it is correct for an **r**

$$\Rightarrow \left(S(k)\right)^{2r} F = \frac{1}{c^{2r}} \frac{m_k c^2}{\sqrt{1-V_{ak}{}^2/c^2}} \text{ and } \left(S(k)\right)^{2r+1} F = \frac{1}{c^{2r}} \frac{m_k V_{ak}}{\sqrt{1-V_{ak}{}^2/c^2}} \qquad \ldots (21)$$

Consider $\left(S(k)\right)^{2r+2} F$

$$\left(S(k)\right)^{2r+2} F = S(k) \left(S(k)\right)^{2r+1} F$$

From equation (21)

$$\left(S(k)\right)^{2r+2} F = S(k) \frac{1}{c^{2r}} \frac{m_k V_{ak}}{\sqrt{1-V_{ak}{}^2/c^2}}$$

$$\Rightarrow \left(S(k)\right)^{2r+2} F = \left(1 - V_{ak}^2 / c^2\right) \frac{\partial}{\partial V_{ak}} \frac{1}{c^{2r}} \frac{m_k V_{ak}}{\sqrt{1 - V_{ak}{}^2 / c^2}}$$

$$\Rightarrow \left(S(k)\right)^{2r+2} F = \left(1 - V_{ak}^2 / c^2\right) m_k c^2 \frac{1}{c^{2r+2}} \frac{\partial}{\partial V_{ak}} \left(\frac{V_{ak}}{\sqrt{1 - V_{ak}{}^2 / c^2}} \right)$$

$$\Rightarrow \left(S(k)\right)^{2r+1} F = \left(1 - V_{ak}^2 / c^2\right) m_k c^2 \frac{1}{c^{2r+2}} \left(\frac{1}{\sqrt{1 - V_{ak}{}^2 / c^2}} - \frac{1}{2} \frac{\left(V_{ak} \times -2V_{ak} / c^2\right)}{\left(1 - V_{ak}{}^2 / c^2\right)^{3/2}} \right)$$

$$\Rightarrow \left(S(k)\right)^{2r+2} F = \left(1 - V_{ak}^2 / c^2\right) m_k c^2 \frac{1}{c^{2r+2}} \left(\frac{1 - V_{ak}{}^2 / c^2 + V_{ak}{}^2 / c^2}{\left(1 - V_{ak}{}^2 / c^2\right)^{3/2}} \right)$$

$$\Rightarrow \left(S(k)\right)^{2r+2} F = \left(1 - V_{ak}^2 / c^2\right) m_k c^2 \frac{1}{c^{2r+2}} \left(\frac{1}{\left(1 - V_{ak}{}^2 / c^2\right)^{3/2}} \right)$$

$$\Rightarrow \left(S(k)\right)^{2r+2} F = \left(1 - V_{ak}^2 / c^2\right) m_k c^2 \frac{1}{c^{2r+2}} \left(\frac{1}{\left(1 - V_{ak}{}^2 / c^2\right)^{3/2}} \right)$$

$$\Rightarrow \left(S(k)\right)^{2r+2} F = \frac{1}{c^{2r+2}} \left(\frac{m_k c^2}{\sqrt{1 - V_{ak}{}^2 / c^2}} \right) \qquad \ldots (22)$$

Now consider $\left(S(k)\right)^{2r+3} F$

$$\left(S(k)\right)^{2r+3} F = S(k) \left(S(k)\right)^{2r+2} F$$

From equation (22)

$$\left(S(k)\right)^{2r+3} F = S(k) \frac{1}{c^{2r+2}} \frac{m_k c^2}{\sqrt{1 - V_{ak}{}^2 / c^2}}$$

$$\Rightarrow \left(S(k)\right)^{2r+3} F = \left(1 - V_{ak}^2 / c^2\right) \frac{\partial}{\partial V_{ak}} \frac{1}{c^{2r+2}} \frac{m_k c^2}{\sqrt{1 - V_{ak}{}^2 / c^2}}$$

$$\Rightarrow \left(S(k)\right)^{2r+3} F = \left(1 - V_{ak}^2 / c^2\right) \frac{m_k c^2}{c^{2r+2}} \frac{\partial}{\partial V_{ak}} \frac{1}{\sqrt{1 - V_{ak}{}^2 / c^2}}$$

$$\Rightarrow \left(S(k)\right)^{2r+3} F = \left(1 - V_{ak}^2 / c^2\right) m_k c^2 \frac{1}{c^{2r+2}} \frac{V_{ak} / c^2}{(1 - V_{ak}^2 / c^2)^{3/2}}$$

$$\Rightarrow \left(S(k)\right)^{2r+3} F = \frac{1}{c^{2r+2}} \frac{m_k V_{ak}}{\sqrt{1 - V_{ak}^2 / c^2}} \qquad \ldots (23)$$

Thus truth for r implies truth for r+1.

Hence proved.

2.6.5 Theorem3: Infinite independent conservation for 2D space

Statement: Given the relativistic conservation function $E_k = \frac{m_k c^2}{\sqrt{1 - V_{ak}{}^2 / c^2}}$ in the 2D space, if E_k is conserved then $\sum_{k=1}^{n} E_k e^{i 2r\theta_k} \forall r \in \mathbb{Z}$ and $\sum_{k=1}^{n} P_k e^{i(2r+1)\theta_k} \forall r \in \mathbb{Z}$ are also conserved.

Proof:

If the space is 2D, the direction vectors $\left(r_{k1}, r_{k2}\right)$ are circular angle with $\left(r_{k1}, r_{k2}\right)$ = $\left(\cos\alpha_k, \sin\alpha_k\right)$.

As per Lemma 3 conservation operators are: $D_1 = D(1,k) = \cos\alpha_k \left(1 - V_{ak}^2 / c^2\right) \frac{\partial}{\partial V_{ak}}$ and

$$D_2 = D(2,k) = \sin\alpha_k \left(1 - V_{ak}^2 / c^2\right) \frac{\partial}{\partial V_{ak}}$$

Let us take following operators:

$$D_+ = D_1 + iD_2 = e^{i\alpha_k} \left(1 - V_{ak}^2 / c^2\right) \frac{\partial}{\partial V_{ak}} = e^{i\alpha_k} S(k)$$

$$D_- = D_1 - iD_2 = e^{-i\alpha_k} \left(1 - V_{ak}^2 / c^2\right) \frac{\partial}{\partial V_{ak}} = e^{-i\alpha_k} S(k)$$

As they are linear combination of the conserved operators, they are conserved operators.

As per Lemma 4, any composition of these operators also form a conservation operator.

Thus:

$\left(D_+\right)^j \left(D_-\right)^l$ is also a conservation operator for any $j, l \in \mathbb{N}$

Consider $\left(D_+\right)^j \left(D_-\right)^l$

$$\left(D_+\right)^j \left(D_-\right)^l = \left(e^{i\alpha_k} S(k)\right)^j \left(e^{-i\alpha_k} S(k)\right)^l$$

$$\Rightarrow \left(D_+\right)^j \left(D_-\right)^l = e^{i(j-l)\alpha_k} \left(S(k)\right)^{j+l}$$

Let us 2 cases:

Case 1: $j+l=2w$

The conservation operator for $j+l=2w$ is $e^{i(2w-2l)\alpha_k}\left(S(k)\right)^{2w}$

Consider the following conserved function:

$$e^{i(2w-2l)\alpha_k}\left(S(k)\right)^{2w}\left(\frac{m_k c^2}{\sqrt{1-V_{ak}{}^2/c^2}}\right)$$

$$=e^{i(2w-2l)\alpha_k}\frac{1}{c^{2w}}\frac{m_k c^2}{\sqrt{1-V_{ak}{}^2/c^2}}$$

$$=e^{i(2w-2l)\alpha_k}\frac{1}{c^{2w}}E_{ak}$$

As $\frac{1}{c^{2w}}$ is just a constant multiplier

$=e^{i2(w-l)\alpha_k}E_{ak}$ is conserved function for arbitrary w and l. Take $r=w-l$

$\Rightarrow e^{i2r\alpha_k}E_{ak}$ is a conserved function.

Case 2: $j+l=2w+1$

The conservation operator for $j+l=2w+1$ is $e^{i(2w+1-2l)\alpha_k}\left(S(k)\right)^{2w+1}$

Consider the following conserved function:

$$e^{i(2w+1-2l)\alpha_k}\left(S(k)\right)^{2w+1}\left(\frac{m_k c^2}{\sqrt{1-V_{ak}{}^2/c^2}}\right)$$

$$=e^{i(2w+1-2l)\alpha_k}\frac{1}{c^{2w}}\frac{m_k V_{ak}}{\sqrt{1-V_{ak}{}^2/c^2}}$$

$$=e^{i(2w-2l+1)\alpha_k}\frac{1}{c^{2w}}P_{ak}$$

As $\frac{1}{c^{2w}}$ is just a constant multiplier

$=e^{i(2(w-l)+1)\alpha_k}P_{ak}$ is conserved function for arbitrary w and l. Take $r=w-l$

$\Rightarrow e^{i(2r+1)\alpha_k}P_{ak}$ is a conserved function.

Hence proved.

2.6.6 When do the conservation equations become finite?

As we proved in theorem 2, there are infinite equations of conservation not only for angle θ_k but $2r\theta_k$ for E_{ak} and $(2r+1)\theta_k$ for P_{ak}. The number of equations become finite only when all θ_k are of the form $W_k\pi / S_k$, where W_k, S_k are co-prime natural number. This makes the set $\{1, e^{i2W_k\pi/S_k}, ..., e^{i2W_k\pi(S_k-1)/S_k}\}$ a classical finite group of order S_k with element as S_k^{th} root of unity.

3. Proof of fallacy in Einstein's two blackboard derivation

The derivation by Albert Einstein was based on elastic collision of two particles, which approach each other head on and then divert at some angle. It was shown in the derivation that if this phenomena was observed from any other frame reference, the conservation still was true [15].

Here we prove using the infinite conservation equations that such an elastic collision is not at all possible thus the derivation is wrong. As we proved in the theorem 2 for any closed system of particles with energy exchange in 2D following quantities are conserved:

$$\sum_{k=1}^{n}\frac{mc^2 e^{i2r\theta_k}}{\sqrt{1-V_{ak}{}^2/c^2}}\ \forall r\in\mathbb{Z} \quad \text{and} \quad \sum_{k=1}^{n}\frac{mV_{ak}e^{i(2r+1)\theta_k}}{\sqrt{1-V_{ak}{}^2/c^2}}\ \forall r\in\mathbb{Z} \qquad \dots (24)$$

Let us take a simple case of 2 particles as in the Einstein's two blackboard derivation in 1934:

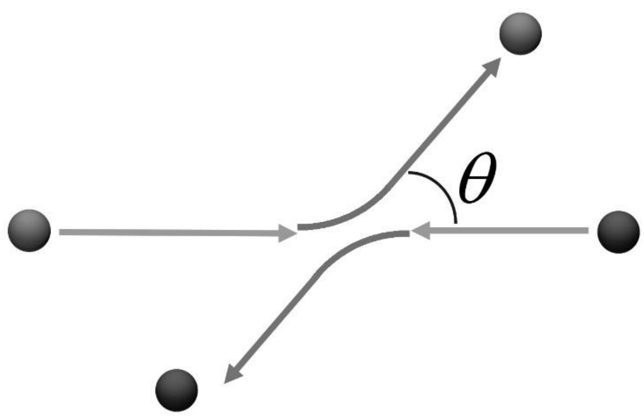


Figure 3: Elastic collision of particles as taken by Einstein for 2 blackboard $E = mc^2$ proof.

The particle have exactly same rest mass and approach each other head (initial condition) with exactly the same speed on and then move away at angles θ and $\theta+\pi$ (final condition) at exactly the same speed.

3.1 Equations for the initial condition

Energy equations

$$\sum_{k=1}^{n}\frac{mc^2 e^{i2r\theta_k}}{\sqrt{1-V_{ak}{}^2/c^2}} = mc^2\gamma(1+e^{i2r(\pi)}) = 2mc^2\gamma\ \ \forall r\in\mathbb{Z} \qquad \dots (25)$$

Momentum equations

$$\sum_{k=1}^{n}\frac{mV_{ak}e^{i(2r+1)\theta_k}}{\sqrt{1-V_{ak}{}^2/c^2}} = mV(1+e^{i(2r+1)\pi}) = 0\ \ \forall r\in\mathbb{Z} \qquad \dots (26)$$

3.2 Equations for the final condition

Energy equations

$$\sum_{k=1}^{n} \frac{mc^2 e^{i2r\theta_k}}{\sqrt{1-V_{ak}{}^2 / c^2}} = mc^2\gamma(e^{i2r\theta} + e^{i2r(\theta+\pi)})$$

$$= 2mc^2\gamma e^{i2r\theta} \ \forall r \in \mathbb{Z} \qquad \ldots (27)$$

Momentum equations

$$\sum_{k=1}^{n} \frac{mV_{ak} e^{i(2r+1)\theta_k}}{\sqrt{1-V_{ak}{}^2 / c^2}} = mVe^{i(2r+1)\theta}(1+e^{i(2r+1)\pi}) = 0 \ \forall r \in \mathbb{Z} \qquad \ldots (28)$$

For the energy equations to be conserved (25) = (27) $\forall r \in \mathbb{Z}$

But $2mc^2\gamma \neq 2mc^2\gamma e^{i2r\theta}$ for an arbitrary angle θ. The only solution for it is $\theta = 0$, which is a trivial solution with no impact.

Hence proved that the derivation is wrong.

4. Conclusion and further work

The result of derivation of conservation of kinetic energy from the conservation of momentum implies that either the potential energy has a momentum thus composed of inertial particles or there cannot be any net conversion of potential energy to the kinetic energy in a closed system.

Furthermore the existence of infinite conservation equations has a deep impact on Lagrangian formulation and path integral formulation. For example let us consider initial state of particles with inter-particle distance nearly infinity, which means that there is no inter-particle potential in the initial state. Let there be an intermediate interaction between the particles, which has some kind of inter-particle potential energy and inter particle energy exchange. Let the final state of particles be also at infinity, which means are non-interactive. In this case if there energy exchange is elastic, it must follow the infinite equations. But as we have seen in the results, that would lead to very restrictive state change and energy exchange. As the final asymptotic positions/velocity angles of particles is a result of the intermediate exchange, it would also put restrains on the how the intermediate potential energy field is setup. So it impacts every kind of potential energy and energy exchange, be it electromagnetic, gravitational, weak forces or strong forces.

As we can see in the derivations the number of equations are infinite for an arbitrary dimensional space and an arbitrary speed dependent kinetic energy function. Which means that either the definition of energy and conservation has to be re-looked into or one has to assume a stealth underlying energy compensating the equations.

References

[1] I. Newton, "Definitions," *The Mathematical Principles of Natural Philosophy,* vol. 1, p. 1, 1729.

[2] I. Newton, "Axioms or Laws of Motion," *The Mathematical Principles of Natural Philosophy,* vol. 1, p. 19, 1729.

[3] I. Newton, "Section 1," *The Mathematical Principles of Natural Philosophy,* vol. 1, p. 41, 1729.

[4] J. v. Mayer, "Remarks on the forces of inorganic nature," *Annalen der Chemie und Pharmacie,* vol. 43, p. 233, 1842.

[5] W. R. Grove, The Correlation of Physical Forces, London: Green Longmans, 1874.

[6] R. W. Hadden, On the shoulders of merchants: exchange and the mathematical conception of nature in early modern Europe, SUNY Press p 13, 1994.

[7] A. Einstein, " Zur Elektrodynamik bewegter Körper," *Annalen der Physik,* vol. 322, no. 10, pp. 891-921, 1905.

[8] A. Einstein, "Ist die Trägheit eines Körpers von seinem Energieinhalt abhängig?," *Annalen der Physik,* vol. 323, pp. 639-641, 1905.

[9] A. Einstein, "Über die vom Relativitätsprinzip geforderte Trägheit der Energie," *Annalen der Physik,* vol. 328, pp. 371-384, 1907.

[10] M. Planck, "On the Dynamics of Moving Systems," in *Sitzungsberichte der Königlich-Preussischen Akademie der Wissenschafte,* Berlin, Erster Halbband, 1907, pp. 542-570.

[11] A. Einstein, "Über das Relativitätsprinzip und die aus demselben gezogenen Folgerungen," *Jahrbuch der Radioaktivität und Elektronik,* vol. 4, pp. 411-462, 1908.

[12] H. A. Lorentz, Das Relativitätsprinzip. Drei Vorlesungen gehalten in Teylers Stiftung zu Haarlem, Leipzig and Berlin: B.G. Teubner, 1914.

[13] A. Einstein, "E = mc2: the most urgent problem of our time," *Science illustrated,* vol. 1, no. 1, pp. 16-17, 1946.

[14] A. Einstein, Relativity: The special and general theory, by Albert Einstein. Translated by Robert W. Lawson., New York: Henry Holt and Company, 1920.

[15] D. Topper, "Einstein's 1934 two-blackboard derivation of energy-mass equivalence," 2007. [Online]. Available: http://www.relativitycalculator.com/pdfs/einstein_1934_two-blackboard_derivation_of_energy-mass_equivalence.pdf.

[16] H. Poincaré, "Sur la dynamique de l'électron (On the Dynamics of the Electron)," *Comptes Rendus,* vol. 140, pp. 1504 - 1508, 1905.

[17] T. Apostle, "Theorem 5.5," in *Mathematical analysis (2nd ed. ed.),* Addison Wesley, 1974.

[18] N. D. Mermin, "It's About Time: Understanding Einstein's Relativity. Princeton University Press," Princeton University Press, 2005.

[19] S. Singh and V. P. Singh Parmar, "A method of creation of net useful energy by quantum tunneling of particles(fundamental like electrons or larger ones) from speed less than the speed of light to speed greater than the speed of light". Patent Under Process, 2013.