Three conjectures on three possible infinite sequences of Poulet numbers

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Abstract. In this paper we present four conjectures, one of them regarding a possible infinite sequence of primes and three of them regarding three possible infinite sequences of Poulet numbers, each of them obtained starting from other possible infinite sequence of Poulet numbers.

Observation 1:

Let P be a Poulet number, $P = d_1*d_2*...*d_k$, where d_1, d_2, ..., d_k are its prime factors; then often the number $N_i = (P + d_i\text{^}2 - d_i)/d_i$, where $1 \leq i \leq k$, is a prime or a power of a prime.

: for P = 341 = 11*31:
: $N_1 = (C + d_1\text{^}2 - d_1)/d_1 = N_2 = (C + d_2\text{^}2 - d_2)/d_2 = 41$.

: for P = 561 = 3*11*17:
: $N_3 = (C + d_3\text{^}2 - d_3)/d_3 = 7\text{^}2$;

: for P = 1105 = 5*13*17:
: $N_3 = (C + d_3\text{^}2 - d_3)/d_3 = 3\text{^}4$;

: for P = 1729 = 7*13*19:
: $N_3 = (C + d_3\text{^}2 - d_3)/d_3 = 109$;

: for P = 2465 = 5*17*29:
: $N_3 = (C + d_3\text{^}2 - d_3)/d_3 = 113$;

: for P = 2821 = 7*13*31:
: $N_3 = (C + d_3\text{^}2 - d_3)/d_3 = 11\text{^}2$.

Conjecture 1:

There is an infinity of Poulet numbers P, where $P = d_1*d_2*...*d_k$, where d_1, d_2, ..., d_k are its prime factors, such that the number $N_i = P + d_i\text{^}2 - d_i$, where $1 \leq i \leq k$, is a prime or a power of a prime.

Observation 2:

Let P be a Poulet number, $P = d_1*d_2*...*d_k$, where d_1, d_2, ..., d_k are its prime factors; then sometimes there exist a prime q such that the numbers $q*N_i$ and $q*N_j$ are both Poulet numbers, where $N_i = (P + d_i\text{^}2 - d_i)/d_i$ and $N_j = (P + d_j\text{^}2 - d_j)/d_j$, where $1 \leq i \leq j \leq k$.

: for $P = 645 = 3*5*43$:

: $N_1 = (P + d_1^2 - d_1)/d_1 = 217 = 7*31$;

: $N_2 = (P + d_2^2 - d_2)/d_2 = 133 = 7*19$.

Indeed, there exist q such that $q*N_1$ and $q*N_2$ are both Poulet numbers and q = 13:

: 13*7*31 = 2821, a Poulet number;

: 13*7*19 = 1729, a Poulet number.

Conjecture 2:

There is an infinity of Poulet numbers P, where $P = d_1*d_2*...*d_k$, where d_1, d_2, ..., d_k are its prime factors, such that there exist a prime q for which the numbers $q*N_i$ and $q*N_j$ are both Poulet numbers, where $N_i = (P + d_i^2 - d_i)/d_i$ and $N_j = (P + d_j^2 - d_j)/d_j$, where $1 \le i \le j \le k$.

Observation 3:

Let P be a Poulet number, $P = d_1*d_2*...*d_k$, where d_1, d_2, ..., d_k are its prime factors; then sometimes the number $N_i = P + d_i^2 - d_i$, where $1 \le i \le k$, is also a Poulet number.

: for $P = 1387 = 19*73$:

: $N_1 = C + d_1^2 - d_1 = 1729$;

: for $P = 10585 = 5*29*73$:

: $N_3 = C + d_3^2 - d_3 = 15841$.

Indeed, the numbers 1729 and 15841 are also Poulet numbers.

Conjecture 3:

There is an infinity of Poulet numbers P, where $P = d_1*d_2*...*d_k$, where d_1, d_2, ..., d_k are its prime factors, such that the number $N_i = P + d_i^2 - d_i$, where $1 \le i \le k$, is also a Poulet number.

Observation 4:

Let P be a 2-Poulet number, $P = d_1*d_2$, where d_1 and d_2 are its prime factors; then sometimes the numbers $N_1 = P + d_1^2 - d_1$ and $N_2 = P + d_2^2 - d_2)/d_j$ are both 2-Poulet numbers also.

: for $P = 2701 = 37*73$:

: $N_1 = (P + d_1^2 - d_1)/d_1 = 4033 = 37*109$;

: $N_2 = (P + d_2^2 - d_2)/d_2 = 7957 = 73*109$.

Indeed, the numbers 4033 and 7957 are both 2-Poulet numbers also.

Conjecture 4:

There is an infinity of 2-Poulet numbers P, where $P = d_1*d_2$, where d_1 and d_2 are its prime factors, such that the numbers $N_1 = P + d_1^2 - d_1$ and $N_2 = P + d_2^2 - d_2$ are both 2-Poulet numbers also.