On the Reconstruction of Graphs

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Abstract

Reconstruction conjecture asks whether it is possible to reconstruct a unique (up to isomorphism) graph from set of its one vertex deleted subgraphs. We show here the validity of reconstruction conjecture for every connected graph which is uniquely reconstructible from the set of all its spanning trees. We make use of a well known result, namely, the reconstruction of a tree from the deck of its pendant point deleted subtrees.

1. Introduction: All graphs in this paper are connected and further are reconstructible from the set of their spanning trees. We give a crisp proof of the famous Reconstruction Conjecture due to Kelly and Ulam [1], [2] for such graphs.

We have based our proof upon two things: An assumption and a well known theorem given below:

Assumption: Graphs considered here are assumed to be uniquely reconstructible from their set of all spanning trees.

Theorem: A tree can be uniquely reconstructed from its pendant point deleted deck of subtrees [3].

2. The main result: We settle Reconstruction Conjecture for graphs satisfying assumption given above.

Theorem 2.1 (Reconstruction Conjecture): Let G and H be two simple graphs with p points, p>2, which satisfy assumption given above and let there exist a (1-1), onto map,

$$\theta: V(G) \rightarrow V(H),$$

$$v_i \in V(G) \rightarrow u_i \in V(H)$$

such that $G_i \equiv G - v_i \cong H_i \equiv H - u_i, \forall i, i = 1,2,\cdots,p$, then $G \cong H$. (The symbol $\cong$ stands for isomorphism.)

Remark 2.1: The existence of such a map is called hypomorphism. The reconstruction conjecture states that hypomorphism implies isomorphism.

Proof: Let $\{T_1^G, T_2^G, \cdots\}$ and $\{T_1^H, T_2^H, \cdots\}$ be the spanning trees of *G* and *H* respectively.
Consider all spanning trees of $\{G_i \cong G - v_i\}$ and $\{H_i \cong H - u_i\}$.
Now, since $G_i \equiv G - v_i \cong H_i \equiv H - u_i, \forall i, i = 1,2,\cdots, p$, therefore for every spanning tree of $\{G_i \cong G - v_i\}$ there will exists spanning tree of $\{H_i \cong H - u_i\}$ isomorphic to it. So, let $\{T_1^{v_i}, T_2^{v_i}, \cdots\}$ be the spanning trees of $\{G_i \cong G - v_i\}$ and $\{T_1^{u_i}, T_2^{u_i}, \cdots\}$ be the spanning trees of $\{H_i \cong H - u_i\}$ such that $T_j^{v_i} \cong T_j^{u_i} \forall j$.
Now, consider first tree T_1^G in the above mentioned set of spanning trees of *G*, namely, $\{T_1^G, T_2^G, \cdots\}$. Let $\{v_{j_1}, v_{j_1}, \cdots v_{j_k}\}$ be the pendant points of T_1^G. In the spanning trees of $\{G_{j_l} \cong G - v_{j_l}, 1 \le l \le k\}$ there will exist suitable spanning trees which will give rise to tree T_1^G by the well known **Theorem** given in [3] as mentioned above. Now, the isomorphic copies of these suitable spanning trees will exists in $\{H_{j_l} \cong H - u_{j_l}, 1 \le l \le k\}$ and they will give rise, again due to **Theorem** above, to a spanning tree, without any loss of generality say T_1^H, such that T_1^G will be isomorphic to T_1^H. In this way we can build all spanning trees of *G* such that for every spanning tree of *G* there will be exactly one isomorphic spanning tree of *H*. Now, as per **Assumption** for all graphs made above and so for *G* and *H* all spanning trees of *G* will reconstruct unique *G* and all spanning trees of *H* will reconstruct unique *H*, therefore $G \cong H$ as desired!

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References

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2. Ulam S. M., A Collection of Mathematical Problems, Wiley (Interscience), New York, 1960.
3. Harary Frank, Palmer E. M., The reconstruction of a tree from its maximal proper subtrees. Canadian Journal of Mathematics, 18, pages 803-810, 1966.