Unification of the two different formulae for Relativistic-Increase of Energy of Matter-particles and Photons: And the Wave-Theoretical Insight emerging from the attempt

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Abstract: While the experiments have shown similar wave-nature of matter-particles and photons, there are two different formulae for finding Relativistic-increase of energy of massive-particles and photons. This paper presents a unification of the two different formulae, in terms of Doppler-shifts of waves; leading to an insight that the 'particles-of-matter' seem to be the 'standing-waves'; and de-Broglie's 'matter-waves', and 'quantum-mechanical-waves' are 'envelop-variations' of the actual, more-fundamental waves.

Introduction

While the experiments have shown similar wave-nature of matter-particles and photons, there are two different formulae for finding Relativistic-increase of energy of massive-particles and photons; these formulae are: $m\ c^2 = m_0\ c^2\ /\ (\ 1 - v^2/c^2)^{1/2}$ for massive-particles, which is independent of direction, whether the two are approaching or receding; and $\hbar\omega = \hbar\ \omega_0\ [(c + v)/(c - v)]^{1/2}$ for the photons, for the observer and the source approaching each-other. This paper presents a reconciliation of these two different formulae, in terms of only Doppler-shifts; leading to an insight that the 'particles-of-matter' seem to be mathematically-equivalent to two waves moving in opposite directions; and de-Broglie's 'matter-waves', and 'quantum-mechanical-waves' are 'envelop-variations' of the actual, more-fundamental waves. We also get an in-sight into the wave-particle-duality; and relative-strengths of gravitational and electric-forces.

2. Mathematical-evidence for the 'standing-wave' nature of the 'fundamental-particles'

The relationship among the 'energy-momentum-four-vectors' of the Special Theory of Relativity is: $(m\ c^2)^2 - p^2\ c^2 = (m_0\ c^2)^2$. We can express this relation as a right-angle-triangle of the fig.1-a below, whose three sides are also related similarly. The three sides of the right-angle-triangle can also be viewed as vectors, as shown in the figure 1-a.

Now, we know that communications-engineers represent electric-signals like $\mathrm{Sin}\omega(t)$ and $\mathrm{Cos}\omega(t)$ as rotating 'vectors'. Similarly, we can imagine the vectors of the fig. 1-a as 'signals' shown in the fig.1-b, as was first attempted in [3-4].

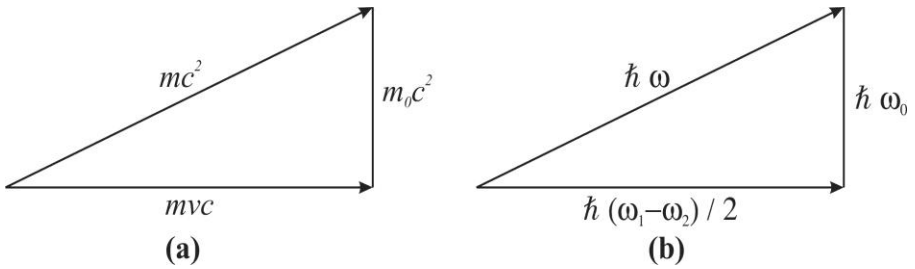

Fig.1: (a) Geometric representation of energy-momentum-four-vector of the special relativity; and (b) its wave-theoretical-translation.

In the fig. 1-a, the horizontal vector $m\,v$ represents the magnitude and direction of vector-sum of three components of momentum $m\,v_x$, $m\,v_y$ and $m\,v_z$. Since a 'particle' of 'matter' is localized in space, its energy and momentum contain a wide band of waves; and since there is no coherence between the spectral-components of a particle's energy and momentum, they get added like the addition of wideband noise, as:

$N(t) = [\,\{N_1(t)\}^2 + \{N_2(t)\}^2 + \{N_3(t)\}^2 + \ldots\ \{N_n(t)\}^2\,]^{1/2}$

Similarly: $(m\,c^2) = [(m_0\,c^2)^2 + p^2\,c^2]^{1/2}$.

This is the wave-theoretical-explanation for the Relativistic-expression for the energy-momentum four-vectors.

Now, by using Planck's relation, $E = \hbar\,w$, and Einstein's relation, $E = m\,c^2$, we get the relations: $m\,c^2 = \hbar\,w$; therefore, $m\,c = \hbar\,w/c$, $m_0\,c = \hbar\,w_0\,/c$, and :

Now, the momentum, $m\,v = m_0\,v\,/\,(1- v^2/c^2)^{1/2}$; i.e. $m\,v = m_0\,v\,c\,/\,(c^2- v^2)^{1/2}$

i.e. $m\,v = (\hbar\,w_0/2\,c\,)\,[\,2\,v\,/\,[(c - v)\,(c + v)\,]^{1/2}$

i.e. $m\,v = (\hbar\,w_0\,/\,2\,c\,)\,[\,\{(c + v)\,/\,(c - v)\,\}^{1/2} - \{\,(c - v)\,/\,(c + v)\,\}^{1/2}\,]$

i.e $m\,v = [\{\hbar\,w_0\,\{(c + v)\,/\,(c - v)\,\}^{1/2}\,\} - \hbar\,w_0\,\{\,(c - v)\,/\,(c + v)\,\}^{1/2}\,]\,/\,2c$ .....(1)

We can write w_1 for the term, $w_0\,\{(c + v)\,/\,(c - v)\,\}^{1/2}$, and we know that w_1 is a longitudinally Doppler-shifted frequency, when the source of light of frequency w_0 '*approaches'* the observer with a relative-velocity v. Similarly, we can write w_2 for the term, $w_0\,\{\,(c - v)\,/\,(c + v)\,\}^{1/2}$, and we know that w_2 is a longitudinally Doppler-shifted frequency, when the source of light of frequency w_0 '*moves away'* from the observer with a relative-velocity v. So, we can write:

$m\,v = [\,\hbar\,w_1 - \hbar\,w_2]\,/\,2c$, as shown in the figure: 1(b).....(2)

The expression-1 can be interpreted as follows: We can mathematically-consider a 'particle' of 'matter' as a 'standing-wave' formed by a combination of two waves traveling in opposite

directions with a velocity *c*. The wave traveling in the forward direction gets Doppler-shifted such that:

$w_1 = w_0 \{(c + v) / (c - v) \}^{1/2}$; and for the wave traveling in the opposite direction, we should take (-*c*) for *c*, so the Doppler-shifted-frequency $w_2 = w_0 \{ (c - v) / (c + v) \}^{1/2}$. So we can express the momentum of a particle as $m v = [\hbar w_1 - \hbar w_2] / 2c$. Of course, this mathematical-expression physically means that the wave-packet oscillates like a standing-wave; it does not mean that there are two waves coming from opposite directions forming a 'particle' of 'matter'.

Similarly, we can express the 'energy' of a moving 'particle' as:

$E = [\hbar w_1 + \hbar w_2] / 2$ . .....(3)

This discussion leads us to physical interpretation of De-Broglie's 'matter-wave' as '*envelop-variations'* of the combined wave, composed of two waves traveling in opposite directions as shown in the next section. And 'energy' of a 'moving-particle' is the '*summation of energies'* of the two constituent-waves traveling in the opposite directions and both the waves initially having half of the particle's rest-mass-energy.

Even the Relativistic Doppler-shift-formula can be re-written as geometric-mean of two Classical-Doppler-shift-formulae, as: $w_1 = w_0 \{(c + v) / (c - v) \}^{1/2}$
i.e. $w_1 = [w_0 (c + v / c) w_0 (c / (c - v)]^{1/2}$, i.e. the two classically-Doppler-shifted-frequencies when (i) the source and (ii) the observer moves with-respect-to the medium.

3. New insight into de-Broglie's 'matter-waves' and 'the q-m-waves':

The wavelength of de-Broglie's 'matter-waves' is conventionally expressed as [1-2]:

$\lambda_B = h / m v$; but based on the expression-1, $\lambda_B = 2 h c / [\hbar w_1 - \hbar w_2]$ ……..…….(4)

We can find the wavelengths: $\lambda_1 = h c / \hbar w_1$ and $\lambda_2 = h c / \hbar w_2$; where: λ_1 is wavelength of the Doppler-shifted-wave *approaching* the observer; and λ_2 is wavelength of the Doppler-shifted-wave *moving-away from* the observer. From the expression-4 we find that de-Broglie-wave is actually the 'envelop-variations', of the two Doppler-shifted constituent-waves', as shown in the fig.2 below:

When w_1 and w_2 are equal, then the next place of constructive-superimposition can be at the infinite distance. As the difference between w_1 and w_2 gets increased, the places of their next superimpositions come closer-and-closer, reducing de-Broglie's wavelength λ_B. And from the discussion of the preceding section we find that a 'particle' is a superimposition of the two Doppler-shifted-waves, traveling in opposite directions, as shown in the graphs below:

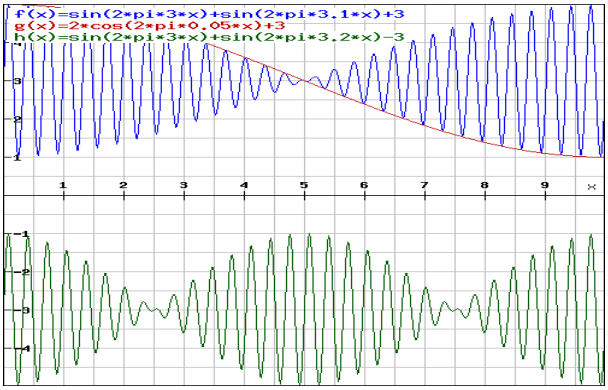

Fig.2: The waves in blue and green colors showing superimposition of two Doppler-shifted-waves; and the wave in red-color, showing envelop-variations of the superimposed-waves, which we have been knowing as the de Broglie's 'matter-wave'. As the difference between the two Doppler-shifted-waves increases, the de-Broglie-wavelength goes on reducing.

Thus, we are led to a new insight that a stationary-particle is a 'standing-wave' of frequency w_0 as was first expressed by this author in ref.[3-4].

Annihilation of the 'electron-positron-pair' leaving behind a pair of 'photons'; and the 'photon-photon-interactions' of two gamma-photons also provide a supportive-evidence for the 'standing-wave-nature' of 'particles' proposed here. We know from the experience of 'directional-antennae' and 'arrays' that when wave-amplitudes cancel in one direction, their energy gets added in the other direction; so the electron and the positron which were approaching, experience cancellation of one of their constituent waves in the forward direction and constructive addition of the other constituent waves, converting them to photons, and moving away from each other.

Finally, we know that a 'particle' of 'matter' is actually a spherical wave-packet. So, it contains a bell-shaped '*band'* of frequencies, instead of only one frequency w_0 so far considered by us. So the Doppler-shifts discussed by us are actually the shifts of the whole '*bands*' of the frequencies; and w_0 , w_1 and w_2 are just 'mean-values' of the wide bands.

4. Conclusion:

Mathematical derivation of the energy-momentum-four-vector of the special-relativity, attempted here, leads to the 'standing-wave-nature' of 'fundamental-particles' of 'matter'; and provides a new insight that de Broglie's 'matter-waves' are 'envelop-variations' of the two Doppler-shifted more-fundamental-waves. Since quantum-mechanical-waves are 'envelop-variations' of the actual more-fundamental-waves, they are able to make predictions also in an 'envelop'.

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