S-PRIMALITY DEGREE OF A NUMBER AND S-PRIME NUMBERS

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Abstract.
In this paper we define the S-Primality Degree of a Number, the S-Prime Numbers, and make some considerations on them.

The depths involved by the Smarandache function are far from being exhausted or completely explored.
If one takes $S(1) = 1$ then

$$\sum_{1\leq n\leq x} \lfloor S(n)/n \rfloor = \begin{cases} \pi(x)+1, & \text{if } 1\leq x<4; \\ \pi(x)+2, & \text{if } x\geq 4; \end{cases}$$

where $S(n)$ is the Smarandache function, $\pi(x)$ the number of primes less than or equal to x, and $\lfloor a \rfloor$ the greatest integer less than or equal to a (integer part).

The ratio $S(n)/n$ measures the **S-Primality Degree** (S stands for Smarandache) of the number n.

Whereas n is called **S-Prime** if $S(n)/n = 1$.
Therefore, the set of S-Prime numbers is $P \cup \{1, 4\}$, with $P = \{2, 3, 5, 7, 11, 13, 17, \ldots\}$ the set of traditional prime numbers.

Traversing the natural number set $N^* = \{1, 2, 3, 4, 5, 6, \ldots\}$ we meet "the most composite" numbers (= the most "broken up"), i. e. those of the form $n = k!$ for which $S(k!)/k! = k/k! = 1/(k-1)!$
The philosophy of this clasification of the natural numbers is that number 4, for example, appears as a prime (S-Prime) and in the same time composite (broken up).
It is not surprising that in the approachment of Fermat Last THeorem's proof, $x^n+y^n=z^n$ doesn't have nonzero integer solutions for $n \geq 3$, it had had to treat besides the cases $n \in \{3, 5, 7, 11, 13, 17, \ldots\}$ the special case n=4 as well because, for example, $x^8+y^8=z^8$ is reduceable to $(x^2)^4+(y^2)^4=(z^2)^4$.
Also, it is not surprising that Einstein (intuitevily) chosed the R^4 space to treat the relativity theory.
It is not surprising that the multiplication of triplets (a,b,c)(m,n,p) does not really work when we want to sink R^2 into R^3, while the multiplication of quadruplets (a,b,c,d)(m,n,p,q) led to the quaternions theory.