ON SMARANDACHE SIMPLE FUNCTIONS

Maohua Le

Department of Mathematics, Zhanjiang Normal College
Zhanjiang, Guangdong, P.R.China.

Absatract. Let p be a prime, and let k be a positive integer. In this paper we prove that the Smarandache simple functions $S_p(k)$ satisfies $p \mid S_p(k)$ and $k(p-1) < S_p(k) \leq kp$.

For any prime p and any positive integer k, let $S_p(k)$ denote the smallest positive integer such that $p^k \mid S_p(k)!$. Then $S_p(k)$ is called the Smarandache simple function of p and k (see [1, Notion 121]). In this paper we prove the following result.

Theorem. For any p and k, we have $p \mid S_p(k)$ and

(1) $$k(p-1) < S_p(k) \leq kp.$$

Proof. Let $a = S_p(k)$. Then a is the smallest positive integer such that

(2) $$p^k \mid a!.$$

If $p \nmid a$, then from (2) we get $p^k \mid (a-1)!$, a contradiction. So we have $p \mid a$.

Since $(kp)! = 1 \ldots p \ldots (2p) \ldots (kp)$, we get $p^k \mid (kp)!$. It implies that

(3) $a \le kp$.

On the other hand, let $p^r \mid a!$, where r is a positive integer. It is a well known fact that

(4) $r = \sum_{i=1}^{\infty} [a / p^i]$

where $[a / p^i]$ is the greatest integer which does not exceed a / p^i. Since $[a / p^i] \le a / p^i$ for any i, we see from (4) that

(5) $r < \sum_{i=1}^{\infty} (a / p^i) = a / (p - 1)$

Further, since $k \le r$ by (2), we find from (5) that

(6) $a > k (p - 1)$.

The combination of (3) and (6) yields (1). The theorem is proved.

Reference

1. Editor of Problem Section, Math. Mag 61 (1988), No.3, 202.