ON THE SMARANDACHE UNIFORM SEQUENCES

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Abstract. Let t be a positive integer with $t>1$. In this paper we give a necessary and sufficient condition for t to have the Smarandache uniform sequence.

Key words. Smarandache uniform sequence, decimal notation.

Let t be a positive integer with $t>1$. If a sequence contains all multiples of t written with same digit in base 10, then it is called the Smarandache uniform sequence of t. In [2], Smith showed that such sequence may be empty for some t.

In this paper we give a necessary and sufficient ndition for t to have the Smarandache uniform sequence . Clearly, the positive integer t can be expressed as

(1) $$t=2^a5^bc,$$

where a,b are nonnegative integers, c is a positive integer satisfying gcd $(10,c)=1$. We prove the following result.

Theoren. t has the Smarandache uniform sequence if and only if

(2) $$(a,b)=(0,0),(1,0),(2,0),(3,0),(0,1).$$

Proof. Clrarly, t has the Smarandache uniform sequence if and only if there exists a multipe m of t such that

(3) $$m=dd...d, 1 \leqslant d \leqslant 9.$$

By (1) and (3), we get

$$(4) \qquad ts=2^a5^bcs=m=d\,\frac{10^r-1}{10-1},$$

where r, s are positive integers. From (4), we obtain

$$(5) \qquad 2^a5^b9cs=d(10^r-1).$$

Since $\gcd(2^a5^b,10^r-1)=1$, we see from (5) that d is a multiple of 2^a5^b. Therefore, since $1 \leqslant d \leqslant 9$, we obtain the condition (2).

On the other hand, since gcd $(10,9c)=1$,by Fermat-Euler theorem (see [1,Theorem 72]), There exists a positive integer r such that 10^r-1 is a multipe of $9c$. Thus, if (2) holds, then t has Smarandache uniform sequnce. The theorem is proved.

References

[1] G.H.Hardy and E.M.Wright, An Introdution to the Theory of Numbers, Oxford University Press, Oxford,1937.

[2] S.Smith, A set of conjectures on Smarandache sequences, Smarandache Notions J. 11(2000),86-92.

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