On an unsolved question about the Smarandache Square-Partial-Digital Subsequence

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Abstract

In this note we report the solution of an unsolved question on Smarandache Square-Partial-Digital Subsequence. We have found it by extesive computer search.
Some new questions about palindromic numbers and prime numbers in SSPDS are posed too.

Introduction

The Smarandache Square-Partial-Digital Subsequence (SSPDS) is the sequence of square integers which admits a partition for which each segment is a square integer [1],[2],[3].
The first terms of the sequence follow:

49, 144, 169, 361, 441, 1225, 1369, 1444, 1681, 1936, 3249, 4225, 4900, 11449, 12544, 14641, 15625, 16900 ...

or

7, 12, 13, 19, 21, 35, 37, 38, 41, 44, 57, 65, 70, 107, 112, 121, 125, 130, 190, 191, 204, 205, 209, 212, 223, 253 ...

reporting the value of n^2 that can be partitioned into two or more numbers that are also squares (A048653) [5].
Differently from the sequences reported in [1], [2] and [3] the proposed ones don't contain terms that admit 0 as partition. In fact as reported in [4] we don't consider the number zero a perfect square.
So, for example, the term 256036 and the term 506 respectively, are not reported in the above sequences because the partion 256/0/36 contains the number zero.
L. Widmer explored some properties of S SPDS's and posed the following question [2]:

Is there a sequence of three or more consecutive integers whose squares are in SPDS?

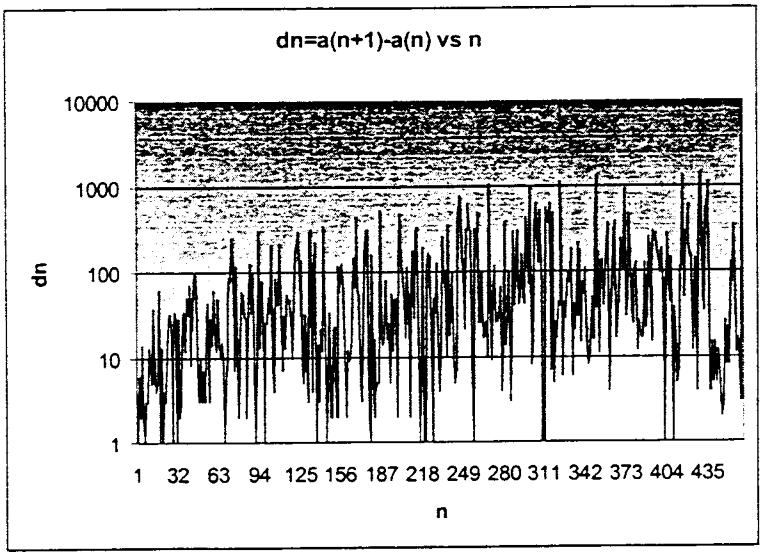
This note gives an answer to this question.

Results

A computer code has been written in Ubasic Rev. 9.
After about three week of work only a solution for three consecutive integers has been found. Those consecutive integers are: 12225, 12226,12227.

n	n^2	Partition
12225	149450625	1, 4, 9, 4, 50625
12226	149475076	1, 4, 9, 4, 75076
12227	149499529	1, 4, 9, 4, 9, 9, 529

No other three consecutive integers or more have been found for terms in SSPDS up to about 3.3E+9. Below a graph of distance dn between the terms of sequence A048653 versus n is given; in particular dn=a(n+1)-a(n) where n is the n-th term of the sequence.



According to the previous results we are enough confident to offer the following conjecture:

- ***There are no four consecutive integers whose squares are in SSPDS.***

New Questions

Starting with the sequence (A048646), reported above, the following sequence can be created [5] (A048653):

7, 13, 19, 37, 41, 107, 191, 223, 379, 487, 1093, 1201, 1301, 1907, 3019, 3371, 5081, 9041, 9721, 9907......

that we can call " Smarandache Prime-Square-Partial-Digital-Subsequence " because all the squares of these primes can be partitioned into two or more numbers that are also squares.
By looking this sequence the following questions can be posed:

1. *Are there other palindromic primes in this sequence beyond the palprime 191?*

2. *Is there at least one plandromic prime in this sequence which square is a palindromic square?*

3. *Are there in this sequence other two or more consecutive primes beyond 37 and 41?*

If we look now at the terms of the sequence A048653 we discover that two of them are very interesting:

121 and 212

Both numbers are palindromes and their squares are in SSPDS and palindromes too. In fact 121^2=14641 can be partitioned as: 1,4,64,4 and 212^2=44944 can be partitioned in five squares that are also palindromes: 4, 4, 9, 4, 4. These are the only terms found by our computer search. So the following question arises:

1. How many other SSPDS palindromes do exist ?

References

[1] Sylvester Smith, "A Set of Conjectures on Smarandache Sequences", Bulletin of Pure and Applied Sciences. (Bombay, India), Vol. 15 E (No. 1). 1996, pp. 101-107.

[2] L.Widmer, Construction of Elements of the Smarandache Square-Partial-Digital Sequence. Smarandache Notions Journal, Vol. 8, No. 1-2-3, 1997, 145-146.

[3] C. Dumitrescu and V. Seleacu, Some notions and questions in Number Theory, Erhus University Press, Glendale, Arizona, 1994

[4] E. W. Weisstein, CRC Concise Encyclopedia of Mathematics, CRC Press 1999, p. 1708

[5] N. Sloane, On-line Encyclopedia of Integer Sequences, http://www.research.att.com/~njas/sequences