Introducing the SMARANDACHE-KUREPA and SMARANDACHE-WAGSTAFF Functions

by

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Definition A.

The left-factorial function is defined by D.Kurepa thus:

$$!n = 0! + 1! + 2! + 3! + \ldots + (n-1)!$$

whilst S.S.Wagstaff prefers:

$$B_n = !(n+1) - 1 = 1! + 2! + 3! + \ldots + n!$$

The following properties should be observed:

(i) $!n$ is only divisible by n when n = 2.
(ii) 3 is a factor of B_n if n is greater than 1.
(iii) 9 is a factor of B_n if n is greater than 4.
(iv) 99 is a factor of B_n if n is greater than 9.

There are no other such cases of divisibility ob B_n for n less than a thousand.

The tabulated values of these two functions together with their prime factors begin:

TABLE I.

n	!n	B_n
1	1	1
2	2	3
3	4=2·2	9=3·3
4	10=2·5	33=3·11
5	34=2·17	153=3·3·17
6	154=2·7·11	873=3·3·97
7	8742·19·23	5913=3·3·3·3·73
8	5914=2·2957	46233=3·3·11·467
9	46234=2·23117	409113=3·3·131·347
10	409114=2·204557	

"Intuitive Thought": There appear to be a disproportionate (unexpectedly high) number of large primes in this table?

Definition B.

For prime p not equal to 3 define the SMARANDACHE-KUREPA Function, SK(p), as the smallest integer such that !SK(p) is divisible by p.
For prime p not equal to 2 or 5 define the SMARANDACHE-WAGSTAFF Function, SW(p), as the smallest integer such that $B_{SW(p)}$ is divisible by p.

The tabulation of these two functions begins:

TABLE II.

p	2	3	5	7	11	13	17	19	23	131
SK(p)	2	*	4	6	6	?	5	7	7	?
SW(p)	*	2	*	?	4	?	5	?	?	9

Where the entry * denotes that the value is not defined and the entry ? denotes not avaible from TABLE I above.

Some unanswered questions:

1. Are there other (*) - entries i.e. undefined values in the above table.
2. What is the distribution function of integers in both SK(p), SW(p) and their union ?
3. When, in general, is SK(p) = SW(p) ?

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