# ON SMARANDACHE ALGEBRAIC STRUCTURES.I :THE COMMUTATIVE MULTIPLICATIVE SEMIGROUP $A(a,n)$

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**Abstract** . In this paper , under the Smarandache algorithm , we construct a class of commutative multiplicative semigroups .

**Key words** . Smarandache algorithm , commutative multiplicative semigroup .

In this serial papers we consider some algebraic structures under the Smarandache algorithm (see [2]). Let $n$ be a positive integer with $n>1$, and let

(1) $$n=p_1^{r1}\quad p_2^{r2}\cdots p_k^{rk}$$

be the factorization of $n$, where $p_1,p_2,\ldots,p_r$ are prime with $p_1<p_2<\cdots<p_k$ and $r_1, r_2,\ldots, r_k$ are positive integers . Further , let

(2) $$n'=p_1p_2\cdots p_k .$$

Then , for any fixed nonzero integer $a$ , there exist unique integers $b,c,l,m,l',m'$ such that

(3) $$a=bc,\quad n=lm,\quad n'=l'm',$$

(4) $$l'=\gcd(l,n'),\ m'=\gcd(m,n'),$$

(5) $$l'=\gcd(a,n'),\gcd(c,n)=1,$$

and every prime divisor of $b$ divides $l'$ . Let

(6) $$e=\begin{cases} 0, & \text{if } l'=1, \\ \text{the least positive integer} & \\ \text{which make } l \mid a^e, & \text{if } l'>1. \end{cases}$$

Since $\gcd(a,m)=1$, by the Fermat – Euler theorem (see [1, Theorem 72]) , there exists a positive integer $t$ such that

(7) $$a^t \equiv 1 \pmod{m}.$$

Let $f$ be the least positive inetger $t$ satisfying (7). For any fixed $a$ and $n$, let the set

(8) $$A(a,n)=\begin{cases} \{1,a,\cdots,a^{f-1}\} \pmod{n}, & \text{if } l=1, \\ \{a,a^2,\cdots,a^{e+f-1}\} \pmod{n}, & \text{if } l>1. \end{cases}$$

In this paper we prove the following result.

**Theorem**. Under the Smarandache algorithm, $A(a,n)$ is a commutative multiplicative semigroup.

**Proof**. Since the commutativity and the associativity of $A(a,n)$ are clear, it suffices to prove that $A(a,n)$ is closed.

Let $a^i$ and $a^j$ belong to $A(a,n)$. If $i+j \leqslant e+f-1$, then from (8) we see that $a^i a^j = a^{i+j}$ belongs to $A(a,n)$. If $i+j > e+f-1$, then $i+j \geqslant e+f$. Let $u=i+f-e$. Then there exists unique integers $v, w$ such that

(9) $$u=fv+w, u \geqslant 0, \quad f>w \geqslant 0.$$

Since $a^f \equiv 1 \pmod{m}$, we get from (9) that

(10) $$a^{i+f-e}-a^w \equiv a^u-a^w \equiv a^{fv+w}-a^w \equiv a^w-a^w \equiv 0 \pmod{m}.$$

Further, since $\gcd(l,m)=1$ and $a^e \equiv 0 \pmod{l}$ by (6), we see from (10) that

(11) $$a^{i+j} \equiv a^{e+w} \pmod{m}.$$

Notice that $e \leqslant e+w \leqslant e+f-1$. We find from (11) that $a^{i+j}$ belongs to $A(a,n)$. Thus the theorem is proved.

## References


[1] G H Hardy and E M Wright, An Introduction to the Theory of Numbers, Oxford Universit Press, Oxford, 1937.

[2] R. Padilla, Smardandche algebraic structures, **Smarandache** Notions J. 9(1998), 36-38.



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