

On some Smarandache conjectures and unsolved problems

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In this paper some Smarandache conjectures and open questions will be analysed. The first three conjectures are related to prime numbers and formulated by F. Smarandache in [1].

1) First Smarandache conjecture on primes

The equation:

$$B_n(x) = p_{n+1}^x - p_n^x = 1 ,$$

where p_n is the n-th prime, has a unique solution between 0.5 and 1;

- the maximum solution occurs for $n = 1$, i.e.

$$3^x - 2^x = 1 \quad \text{when } x = 1;$$

- the minimum solution occurs for $n = 31$, i.e.

$$127^x - 113^x = 1 \quad \text{when } x = 0.567148K = a_0$$

First of all observe that the function $B_n(x)$ which graph is reported in the fig. 5.1 for some values of n , is an increasing function for $x > 0$ and then it admits a unique solution for $0.5 \leq x \leq 1$.

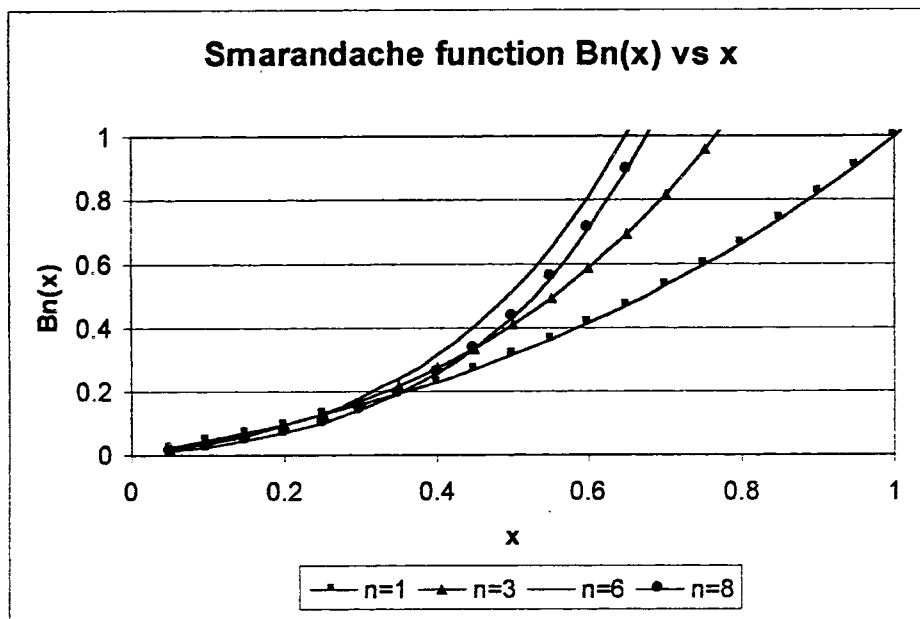


Fig. 5.1

In fact the derivate of $B_n(x)$ function is given by:

$$\frac{d}{dx} B_n(x) = p_{n+1}^x \cdot \ln(p_{n+1}) - p_n^x \cdot \ln(p_n)$$

and then since $p_{n+1} > p_n$ we have:

$$\ln(p_{n+1}) > \ln(p_n) \quad \text{and} \quad p_{n+1}^x > p_n^x \quad \text{for } x > 0$$

This implies that $\frac{d}{dx} B_n(x) > 0$ for $x > 0$ and $n > 0$.

Being the $B_n(x)$ an increasing function, the Smarandache conjecture is equivalent to:

$$B_n^0 = p_{n+1}^{a_0} - p_n^{a_0} \leq 1$$

that is, the intersection of $B_n(x)$ function with $x = a_0$ line is always lower or equal to 1. Then an Ubasic program has been written to test the new version of Smarandache conjecture for all primes lower than 2^{27} . In this range the conjecture is true. Moreover we have created an histogram for the intersection values of $B_n(x)$ with $x = a_0$:

| Counts | Interval |
|---------|------------|
| 7600437 | [0, 0.1] |
| 2640 |]0.1, 0.2] |
| 318 |]0.2, 0.3] |
| 96 |]0.3, 0.4] |
| 36 |]0.4, 0.5] |
| 9 |]0.5, 0.6] |
| 10 |]0.6, 0.7] |
| 2 |]0.7, 0.8] |
| 3 |]0.8, 0.9] |
| 1 |]0.9, 1] |

This means for example that the function $B_n(x)$ intersects the axis $x = a_0$, 318 times in the interval $]0.2, 0.3]$ for all n such that $p_n < 2^{27}$.

In the fig. 5.2 the graph of normalized histogram is reported (black dots).

According to the experimental data an interpolating function has been estimated (continuous curve):

$$B_n^0 = 8 \cdot 10^{-8} \cdot \frac{1}{n^{6.2419}}$$

with a good R^2 value (97%).

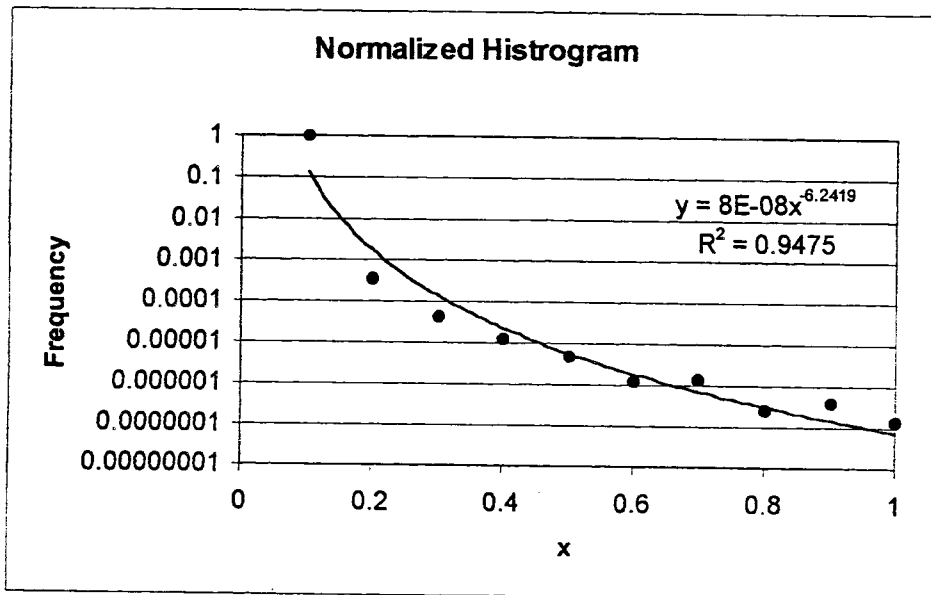


Fig. 5.2

Assuming this function as empirical probability density function we can evaluate the probability that $B_n^0 > 1$ and then that the Smarandache conjecture is false. By definition of probability we have:

$$P(B_n^0 > 1) = \frac{\int_c^\infty B_n^0 dn}{\int_c^\infty B_n^0 dn} \approx 6.99 \cdot 10^{-19}$$

where $c=3.44E-4$ is the lower limit of B_n^0 found with our computer search. Based on those experimental data there is a strong evidence that the Smarandache conjecture on primes is true.

2) Second Smarandache conjecture on primes.

$$B_n(x) = p_{n+1}^x - p_n^x < 1$$

where $x < a_0$. Here p_n is the n -th prime number.

This conjecture is a direct consequence of conjecture number 1 analysed before. In fact being $B_n(x)$ an increasing function if:

$$B_n^0 = p_{n+1}^{a_0} - p_n^{a_0} \leq 1$$

is verified then for $x < a_0$ we have no intersections of the $B_n(x)$ function with the line $B_n(x) = 1$, and then $B_n(x)$ is always lower than 1.

3) Third Smarandache conjecture on primes.

$$C_n(k) = p_{n+1}^{\frac{1}{k}} - p_n^{\frac{1}{k}} < \frac{2}{k} \quad \text{for } k \geq 2 \text{ and } p_n \text{ the } n\text{-th prime number}$$

This conjecture has been verified for prime numbers up to 2^{25} and $2 \leq k \leq 10$ by the author [2]. Moreover a heuristic that highlight the validity of conjecture out of range analysed was given too.

At the end of the paper the author reformulated the Smarandache conjecture in the following one:

Smarandache-Russo conjecture

$$C_n(k) \leq \frac{2}{k^{2 \cdot a_0}} \quad \text{for } k \geq 2$$

where a_0 is the Smarandache constant $a_0 = 0.567148...$ (see [1]).

So in this case for example the Andrica conjecture (namely the Smarandache conjecture for $k=2$) becomes:

$$\sqrt{p_{n+1}} - \sqrt{p_n} < 0.911111\dots$$

Thanks to a program written with Ubasic software the conjecture has been verified to be true for all primes $p_n < 2^{25}$ and $2 \leq k \leq 15$.

In the following table the results of the computer search are reported.

| k | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|--------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| Max_C(n,k) | 0.6708 | 0.3110 | 0.1945 | 0.1396 | 0.1082 | 0.0885 | 0.0756 | 0.0659 | 0.0584 | 0.0525 | 0.0476 | 0.0436 | 0.0401 | 0.0372 |
| $2/k^{2a_0}$ | 0.4150 | 0.1654 | 0.0861 | 0.0519 | 0.0343 | 0.0242 | 0.0178 | 0.0136 | 0.0107 | 0.0086 | 0.0071 | 0.0060 | 0.0050 | 0.0043 |
| delta | 0.2402 | 0.2641 | 0.2204 | 0.1826 | 0.1538 | 0.1314 | 0.1134 | 0.0994 | 0.0883 | 0.0792 | 0.0717 | 0.0654 | 0.0600 | 0.0554 |

Max_C(n,k) is the largest value of the Smarandache function $C_n(k)$ for $2 \leq k \leq 15$

and $p_n < 2^{25}$ and delta is the difference between $\frac{2}{k^{2a_0}}$ and Max_C(n,k).

Let's now analyse the behaviour of the delta function versus the k parameter. As highlighted in the following graph (fig. 5.3),

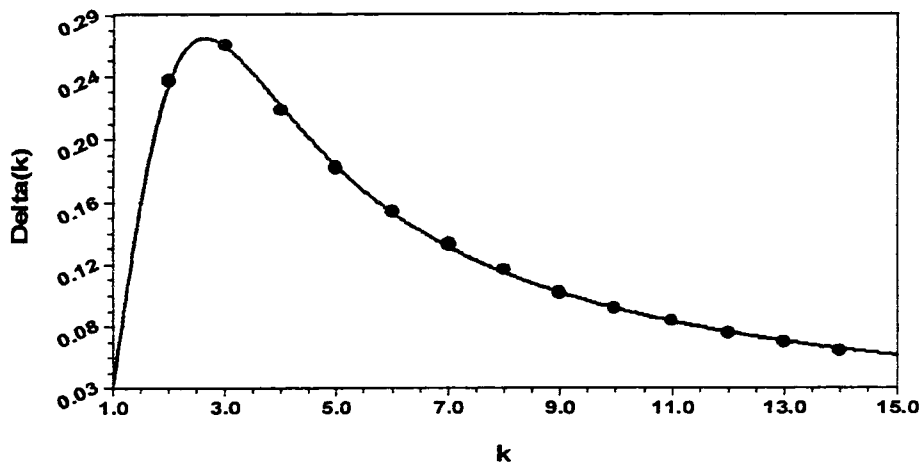


Fig. 5.3

an interpolating function with good R^2 (0.999) has been estimated:

$$\Delta(k) = \frac{a + b \cdot k}{1 + c \cdot k + d \cdot k^2}$$

where: $a = 0.1525\dots$, $b = 0.17771\dots$, $c = -0.5344\dots$, $d = 0.2271\dots$

Since the Smarandache function decrease asymptotically as n increases it is likely that the estimated maximum is valid also for $p_n > 2^{25}$. If this is the case then the interpolating function found reinforce the Smarandache-Russo conjecture being:

$$\Delta(k) \rightarrow 0 \text{ for } k \rightarrow \infty$$

Let's now analyse some Smarandache conjectures that are a generalization of Goldbach conjecture.

4) Smarandache generalization of Goldbach conjectures

C. Goldbach (1690-1764) was a German mathematician who became professor of mathematics in 1725 in St. Petersburg, Russia. In a letter to Euler on June 7, 1742, He speculated that every even number is the sum of three primes.

Goldbach in his letter was assuming that 1 was a prime number. Since we now exclude it as a prime, the modern statements of Goldbach's conjectures are [5]:

Every even number equal or greater than 4 can be expressed as the sum of two primes, and every odd number equal or greater than 9 can be expressed as the sum of three primes.

The first part of this claim is called the Strong Goldbach Conjecture, and the second part is the Weak Goldbach Conjecture.

After all these years, the strong Goldbach conjecture is still not proven, even though virtually all mathematicians believe it is true.

Goldbach's weak conjecture has been proven, almost!

In 1937, I.M. Vonogradov proved that there exist some number N such that all odd numbers that are larger than N can be written as the sum of three primes. This reduce the problem to finding this number N , and then testing all odd numbers up to N to verify that they, too, can be written as the sum of three primes.

How big is N? One of the first estimates of its size was approximately [6]:

$$10^{6846168}$$

But this is a rather large number; to test all odd numbers up to this limit would take more time and computer power than we have. Recent work has improved the estimate of N. In 1989 J.R. Chen and T. Wang computed N to be approximately [7]:

$$10^{43000}$$

This new value for N is much smaller than the previous one, and suggests that some day soon we will be able to test all odd numbers up to this limit to see if they can be written as the sum of three primes.

Anyway assuming the truth of the generalized Riemann hypothesis [5], the number N has been reduced to 10^{20} by Zinoviev [9], Saouter [10] and Deshouillers. Effinger, te Riele and Zinoviev[11] have now successfully reduced N to 5.

Therefore the weak Goldbach conjecture is true, subject to the truth of the generalized Riemann hypothesis.

Let's now analyse the generalizations of Goldbach conjectures reported in [3] and [4]; six different conjectures for odd numbers and four conjectures for even numbers have been formulated. We will consider only the conjectures 1, 4 and 5 for the odd numbers and the conjectures 1, 2 and 3 for the even ones.

4.1 First Smarandache Goldbach conjecture on even numbers.

Every even integer n can be written as the difference of two odd primes, that is $n = p - q$ with p and q two primes.

This conjecture is equivalent to:

For each even integer n , we can find a prime q such that the sum of n and q is itself a prime p .

A program in Ubasic language to check this conjecture has been written.

The result of this check has been that the first Smarandache Goldbach conjecture is true for all even integers equal or smaller than 2^{29} .
The list of Ubasic program follows.

```

1 ' *****
2 '           Smarandache Goldbach conjecture
3 '           on even numbers: n=p-q with p and q two primes
4 '           by Felice Russo Oct. 1999
5 ' *****
10 cls
20 for N=2 to 2^28 step 2
22 W=3
25 locate 10,10:print N
30 for Q=W to 10^9
40 gosub *Pspr(Q)
50 if Pass=0 then goto 70
60 cancel for:goto 80
70 next
75 print N,"The Smarandache conjecture is not true up to 10^9 for q=";Q
80 Sum=N+Q
90 gosub *Pspr(Sum)
100 if Pass=1 then goto 120
110 W=Q+1:goto 30
120 next
130 print "The Smarandache conjecture has been verified up to:";N-2
140 end
1000 ' *****
1010 '           Strong Pseudoprime Test Subroutine
1020 '           by Felice Russo 25/5/99
1030 ' *****
1040 '
1050 ' The sub return the value of variable PASS.
1060 ' If pass is equal to 1 then N is a prime.
1070 '
1080 '
1090 *Pspr(N)
1100 local I,J,W,T,A,Test

```

```

1110 W=3:if N=2 then Pass=1:goto 1290
1120 if even(N)=1 or N=1 then Pass=0:goto 1290
1130 if prmdiv(N)=N then Pass=1:goto 1290
1140 if prmdiv(N)>0 and prmdiv(N)<N then Pass=0:goto 1290
1150 I=W
1160 if gcd(I,N)=1 then goto 1180
1170 W=I+1:goto 1150
1180 T=N-1:A=A+1
1190 while even(T)=1
1200 T=T\2:A=A+1
1210 wend
1220 Test=modpow(I,T,N)
1230 if Test=1 or Test=N-1 then Pass=1:goto 1290
1240 for J=1 to A-1
1250 Test=(Test*Test)@N
1260 if Test=N-1 then Pass=1:cancel for:goto 1290
1270 next
1280 Pass=0
1290 return

```

For each even integer n the program check if it is possible to find a prime q , generated by a subroutine (rows from 1000 to 1290) that tests the primality of an integer, such that the sum of n and q , $\text{sum}=n+q$ (see rows 80 and 90) is again a prime.

If yes the program jumps to the next even integer. Of course we have checked only a little quantity of integers out of infinite number of them.

Anyway we can get some further information from experimental data about the validity of this conjecture.

In fact we can calculate the ratio q/n for the first 3000 values, for example, and then graphs this ratio versus n (see fig. 5.4).

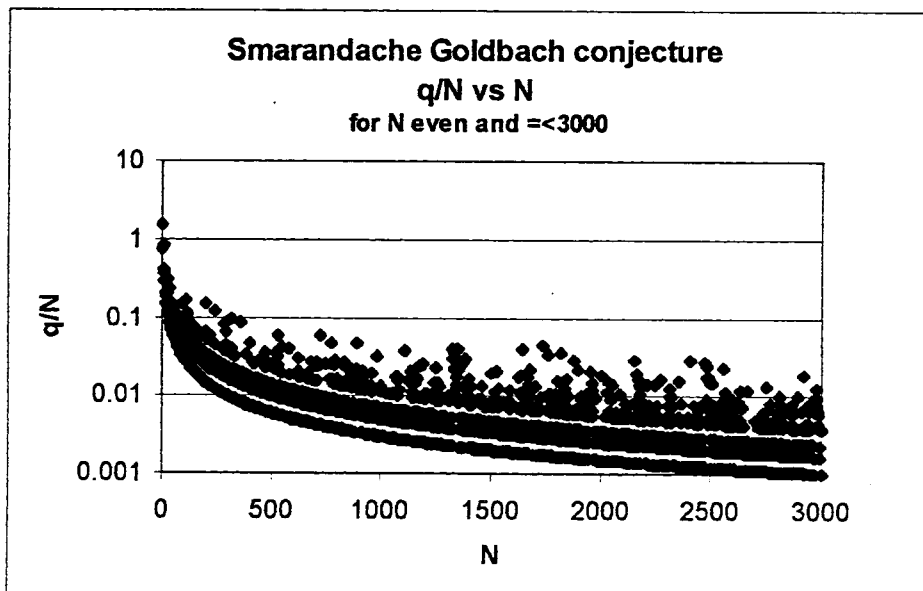


Fig. 5.4

As we can see this ratio is a decreasing function of n ; this means that for each n is very easy to find a prime q such that $n+q$ is a prime. This heuristic well support the Smarandache-Goldbach conjecture.

4.2 Second Smarandache-Goldbach conjecture on even numbers.

Every even integer n can be expressed as a combination of four primes as follows:

$$n=p+q+r-t \text{ where } p, q, r, t \text{ are primes.}$$

For example: $2=3+3+3-7$, $4=3+3+5-7$, $6=3+5+5-7$, $8=11+5+5-13$

Regarding this conjecture we can notice that since n is even and t is an odd prime their sum is an odd integer.

So the conjecture is equivalent to the weak Goldbach conjecture because we can always choose a prime t such that $n+t \geq 9$.

4.3 Third Smarandache-Goldbach conjecture on even numbers.

Every even integer n can be expressed as a combination of four primes as follows:

$$n=p+q-r-t \text{ where } p, q, r, t \text{ are primes.}$$

For example: $2=11+11-3-17$, $4=11+13-3-17$, $6=13+13-3-17$, $8=11+17-7-13$

As before this conjecture is equivalent to the strong Goldbach conjecture because the sum of an even integer plus two odd primes is an even integer. But according to the Goldbach conjecture every even integer ≥ 4 can be expressed as the sum of two primes.

4.4 First Smarandache Goldbach conjecture on odd numbers.

Every odd integer n , can be written as the sum of two primes minus another prime:

$$n=p+q-r \text{ where } p, q, r \text{ are prime numbers.}$$

For example: $1=3+5-7$, $3=5+5-7$, $5=3+13-11$, $7=11+13-17$ $9=5+7-3$

Since the sum of an odd integer plus an odd prime is an even integer this conjecture is equivalent to the strong Goldbach conjecture that states that every even integer ≥ 4 can be written as the sum of two prime numbers.

A little variant of this conjecture can be formulated requiring that all the three primes must be different.

For this purpose an Ubasic program has been written. The conjecture has been verified to be true for odd integers up to 2^{29} .

The algorithm is very simple. In fact for each odd integer n , we put $r=3$, $p=3$ and q equal to the largest primes smaller than $n+r$.

Then we check the sum of p and q . If it is greater than $n+r$ then we decrease the variable q to the largest prime smaller than the previous one. On the contrary if the sum is smaller than $n+r$ we increase the variable p to the next prime. This loop continues until p is lower than q . If this is not the case then we increase the variable r to the next prime and we restart again the check on p and q . If the sum of n and r

coincide with that of p and q the last check is on the three primes r, p and q that must be of course different. If this is not the case then we reject this solution and start again the check.

```

1 ' *****
2 '      First Smarandache-Goldbach conjecture
3 '              on odd integers
4 '              by Felice Russo Oct. 99
5 ' *****
10 cls:Lim=2^29
20 for N=1 to Lim step 2
30 S=3:W=3
40 locate 10,10:print N
50 r=S
60 gosub *Pspr(r)
70 if Pass=0 then goto 260
80 Sum1=N+r:L=0:H=Sum1-1
90 p=W
100 gosub *Pspr(p)
110 if Pass=1 and L=0 then goto 140
120 if Pass=1 and L=1 then goto 190
130 W=p+1:goto 90
140 q=H
150 gosub *Pspr(q)
160 if Pass=1 then goto 190
170 H=q-1:goto 140
190 Sum2=p+q
200 if p>=q then goto 260
210 if Sum2>Sum1 then H=q-1:goto 140
220 if Sum2<Sum1 then W=p+1:L=1:goto 90
230 if r=p or r=q and p<q then W=p+1:goto 90
240 if r=p or r=q and p>=q then goto 260
250 goto 270
260 S=r+1:if r>2^25 goto 290 else goto 50
270 next
280 cls:print "Conjecture verified up to";Lim:goto 300
290 cls:print "Conjecture not verified up to 2^25 for";N

```

```

300 end
310 ' *****
320 '      Strong Pseudoprime Test Subroutine
330 '      by Felice Russo 25/5/99
340 ' *****
350 '
360 ' The sub return the value of variable PASS.
370 ' If pass is equal to 1 then N is a prime.
380 '
390 '
400 *Pspr(N)
410 local I,J,W,T,A,Test
420 W=3:if N=2 then Pass=1:goto 600
430 if even(N)=1 or N=1 then Pass=0:goto 600
440 if prmdiv(N)=N then Pass=1:goto 600
450 if prmdiv(N)>0 and prmdiv(N)<N then Pass=0:goto 600
460 I=W
470 if gcd(I,N)=1 then goto 490
480 W=I+1:goto 460
490 T=N-1:A=A+1
500 while even(T)=1
510 T=T\2:A=A+1
520 wend
530 Test=modpow(I,T,N)
540 if Test=1 or Test=N-1 then Pass=1:goto 600
550 for J=1 to A-1
560 Test=(Test*Test)@N
570 if Test=N-1 then Pass=1:cancel for:goto 600
580 next
590 Pass=0
600 return

```

4.5 Fourth Smarandache Goldbach conjecture on odd numbers.

Every odd integer n can be expressed as a combination of five primes as follows:

$$n=p+q+r-t-u \text{ where } p, q, r, t, u \text{ are all prime numbers.}$$

For example: $1=3+7+17-13-13$, $3=5+7+17-13-13$, $5=7+7+17-13-13$,
 $7=5+11+17-13-13$

Also in this case the conjecture is equivalent to the weak Goldbach conjecture. In fact the sum of two odd primes plus an odd integer is always an odd integer and according to the weak Goldbach conjecture it can be expressed as the sum of three primes.

Now we will analyse a conjecture about the wrong numbers introduced in Number Theory by F. Smarandache and reported for instance in [8] and then we will analyse a problem proposed by Castillo in [12].

5) Smarandache Wrong numbers

A number $n = \overline{a_1 a_2 a_3 \dots a_k}$ of at least two digits, is said a Smarandache Wrong number if the sequence:

$$a_1, a_2, a_3, K K, a_k, b_{k+1}, b_{k+2}, K K$$

(where b_{k+i} is the product of the previous k terms, for any $i \geq 0$) contains n as its term [8].

Smarandache conjectured that there are no Smarandache Wrong numbers.

In order to check the validity of this conjecture up to some value N_0 , an Ubasic program has been written.

N_0 has been choose equal to 2^{28} . For all integers $n \leq N_0$ the conjecture has been proven to be true. Moreover utilizing the experimental data obtained with the computer program a heuristic that reinforce the validity of conjecture can be given. First of all let's define what we will call the Smarandache Wrongness of an integer n with at least two digits. For any integer n , by definition of Smarandache Wrong number we must create the sequence:

$$a_1, a_2, a_3, K K, a_k, b_{k+1}, b_{k+2}, K K$$

as reported above. Of course this sequence is stopped once a term b_{k+i} equal or greater than n is obtained.

Then for each integer n we can define two distance:

$$d_1 = |b_{k+i} - n| \quad \text{and} \quad d_2 = |b_{k+i-1} - n|$$

The Smarandache Wrongness of n is defined as $\min\{d_1, d_2\}$ that is the minimum value between d_1 and d_2 and indicate with $W(n)$. Based on definition of $W(n)$, if the Smarandache conjecture is false then for some n we should have $W(n)=0$.

Of course by definition of wrong number, $W(n)=n$ if n contains any digit equal to zero and $W(n)=n-1$ if n is repunit (that is all the digits are 1). In the following analysis we will exclude this two species of integers. With the Ubasic program utilized to test the smarandache conjecture we have calculated the $W(n)$ function for $12 \leq n \leq 3000$. The graph of $W(n)$ versus n follows.

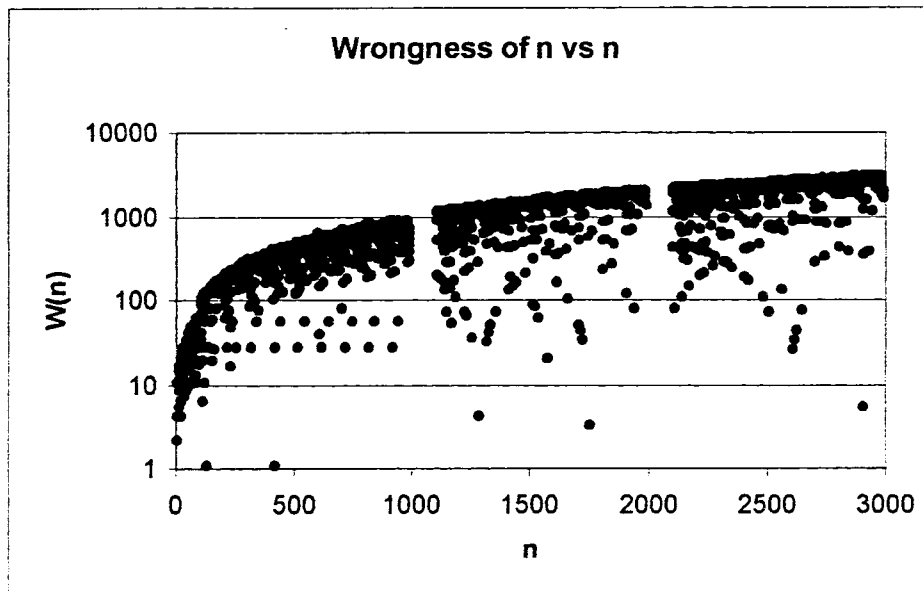


Fig. 5.5

as reported above. Of course this sequence is stopped once a term b_{k+i} equal or greater than n is obtained.

Then for each integer n we can define two distance:

$$d_1 = |b_{k+i} - n| \quad \text{and} \quad d_2 = |b_{k+i-1} - n|$$

The Smarandache Wrongness of n is defined as $\min\{d_1, d_2\}$ that is the minimum value between d_1 and d_2 and indicate with $W(n)$. Based on definition of $W(n)$, if the Smarandache conjecture is false then for some n we should have $W(n)=0$.

Of course by definition of wrong number, $W(n)=n$ if n contains any digit equal to zero and $W(n)=n-1$ if n is repunit (that is all the digits are 1). In the following analysis we will exclude this two species of integers. With the Ubasic program utilized to test the smarandache conjecture we have calculated the $W(n)$ function for $12 \leq n \leq 3000$. The graph of $W(n)$ versus n follows.

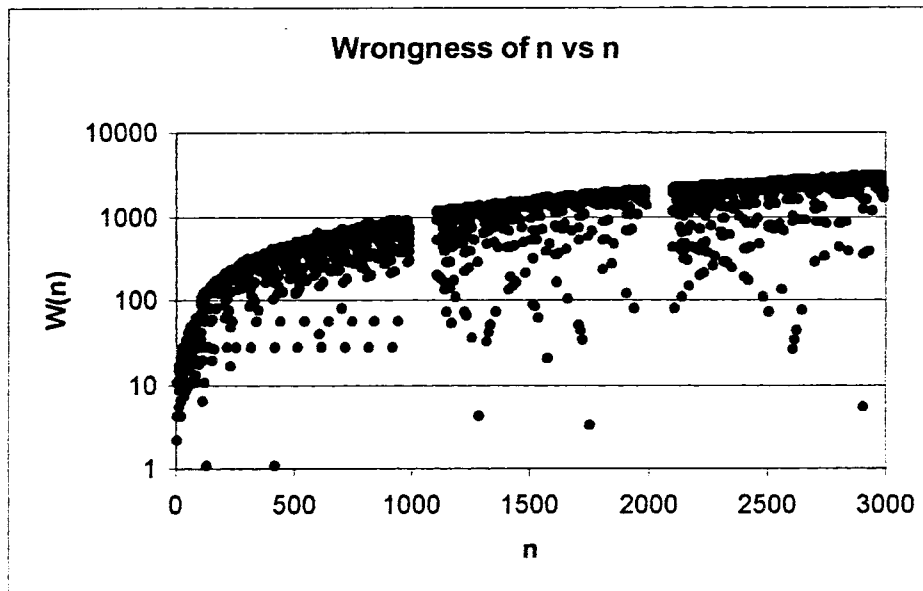


Fig. 5.5

As we can see $W(n)$ in average increases linearly with n even though at a more close view (see fig. 5.6) a nice triangular pattern emerges with points scattered in the region between the x-axis and the triangles.

Anyway the average behaviour of $W(n)$ function seems to support the validity of Smarandache conjecture.

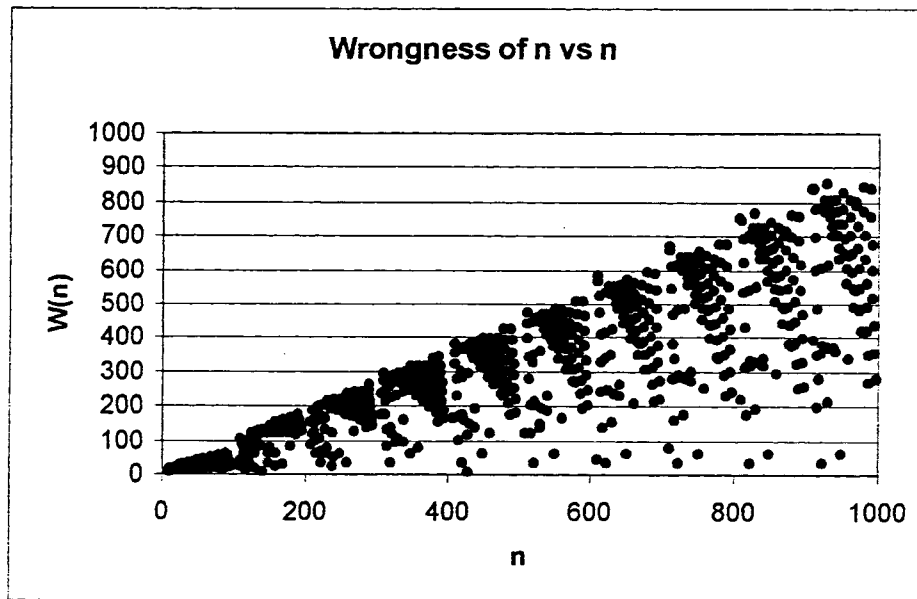


Fig. 5.6

Let's now divide the integers n into two families: those for which the $W(n)$ function is smaller than 5 and those for which it is greater than 5.

The integers with $W(n)$ smaller than 5 will be called the Smarandache Weak Wrong numbers.

Up to 2^{28} the sequence of weak wrong numbers is given by the following integers n :

| n | W(n) | interv. | C_Ww(n) |
|---------|------|----------|---------|
| 12 | 4 | 10^2 | 5 |
| 13 | 4 | 10^3 | 2 |
| 14 | 2 | 10^4 | 4 |
| 23 | 5 | 10^5 | 2 |
| 31 | 4 | 10^6 | 1 |
| 143 | 1 | 10^7 | 1 |
| 431 | 1 | 10^8 | 0 |
| 1292 | 4 | 2^{28} | 0 |
| 1761 | 3 | 2^{29} | 0 |
| 2911 | 5 | | |
| 6148 | 4 | | |
| 11663 | 1 | | |
| 23326 | 2 | | |
| 314933 | 5 | | |
| 5242881 | 1 | | |

Here $W(n)$ is the Wrongness of n and $C_Ww(n)$ is the number of the weak wrong numbers between 10 and 10^2 , 10^2 and 10^3 and so on.

Once again the experimental data well support the Smarandache conjecture because the density of the weak wrong numbers seems goes rapidly to zero.

6) About a problem on continued fraction of Smarandache consecutive and reverse sequences.

In [12] J. Castillo introduced the notion of Smarandache simple continued fraction and Smarandache general continued fraction. As example he considered the application of this new concept to the two well-know Smarandache sequences:

Smarandache consecutive sequence

1, 12, 123, 1234, 12345, 123456, 1234567

Smarandache reverse sequence

1, 21, 321, 4321, 54321, 654321, 7654321

At the end of its article the following problem has been formulated:

Is the simple continued fraction of consecutive sequence convergent? If yes calculate the limit.

$$1 + \frac{1}{12 + \frac{1}{123 + \frac{1}{1234 + \frac{1}{12345 + \Lambda}}}}$$

Is the general continued fraction of consecutive and reverse sequences convergent? If yes calculate the limit.

$$1 + \frac{1}{12 + \frac{21}{123 + \frac{321}{1234 + \frac{4321}{12345 + \Lambda}}}}$$

Using the Ubasic software a program to calculate numerically the above continued fractions has been written. Here below the result of computation.

$$1 + \frac{1}{12 + \frac{1}{123 + \frac{1}{1234 + \frac{1}{12345 + \Lambda}}}} \approx 1.0833\dots$$

References

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