SMARANDACHE CONTINUED FRACTIONS

by Jose Castillo, Navajo Community College,
Tsaile, Arizona, USA

Abstract:
Open problems are studied using Smarandache type sequences in the composition of simple and general continued fractions.

Key Words:
Simple and General Continued Fractions, Smarandache Simple and Continued Fractions

1) A Smarandache Simple Continued Fraction is a fractionn of the form:

$$a(1) + \cfrac{1}{a(2) + \cfrac{1}{a(3) + \cfrac{1}{a(4) + \cfrac{1}{a(5) + \dots}}}}$$

where a(n), for n >= 1, is a Smarandache type Sequence or Sub-Sequence.

2) And a Smarandache General Continued Fraction is a fraction of the form:

$$a(1) + \cfrac{b(1)}{a(2) + \cfrac{b(2)}{a(3) + \cfrac{b(3)}{a(4) + \cfrac{b(4)}{a(5) + \dots}}}}$$

where a(n) and b(n), for n >= 1, are both Smarandache type Sequences or Sub-Sequences.

(Over 200 such sequences are listed in Sloane's database of Encyclopedia of Integer sequences -- online).

For example:
a) if we consider the smarandache consecutive sequence:
1, 12, 123, 1234, 12345, ..., 123456789101112, ...
we form a smarandache simple continued fraction:

$$1 + \cfrac{1}{12 + \cfrac{1}{123 + \cfrac{1}{1234 + \cfrac{}{12345 + \ldots}}}}$$

b) if we include the smarandache reverse sequence:
1, 21, 321, 4321, 54321, ..., 121110987654321, ...
to the previous one we get a smarandache general continued fraction:

$$1 + \cfrac{1}{12 + \cfrac{21}{123 + \cfrac{321}{1234 + \cfrac{4321}{12345 + \ldots}}}}$$

With a mathematics software it is possible to calculate such continued fractions to see which ones of them converge, and eventually to make conjectures, or to algebraically prove those converging towards certain constants.

Open Problem: Are the previous two examples of continued fractions convergent?

References:
[1] Smarandoiu, Stefan, "Convergence of Smarandache Continued Fractions", <Astracts of Papers Presented to the American Mathematical Society>, Vol. 17, No. 4, Issue 106, 1996, p. 680.
[2] Zhong, Chung, "On Smarandache Continued Fractions", American Research Press, to appear.