SMARANDACHE DETERMINANT SEQUENCES

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In this note two types of Smarandache type determinant sequences are defined and studied.

(1) Smarandache Cyclic Determinant Sequences:

(a)Smarandache Cyclic Determinant Natural Sequence:

$$\begin{vmatrix} 1 \end{vmatrix},\ \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix},\ \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{vmatrix},\ \begin{vmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \end{vmatrix},\ \ldots$$

1 , -3 , -18 , 160 , . . .

This suggests the possibility of the n th term as

$$T_n = (-1)^{[n/2]} \{(n+1)/2\} . n^{n-1} \qquad \text{——— (A)}$$

Where [ ] stands for integer part

We verify this for n = 5 , and the general case can be dealt with on similar lines.

$$T_5 = \begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 1 \\ 3 & 4 & 5 & 1 & 2 \\ 4 & 5 & 1 & 2 & 3 \\ 5 & 1 & 2 & 3 & 4 \end{vmatrix}$$

on carrying out following elementary operations

(a) R_1 = sum of all the rows, (b) taking 15 common from the first row

(c) Replacing C_k the k^{th} column by $C_k - C_1$, we get

$$T_5 = 15 \begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ 2 & 1 & 2 & 3 & -1 \\ 3 & 1 & 2 & -2 & -1 \\ 4 & 1 & -3 & -2 & -1 \\ 5 & -4 & -3 & -2 & -1 \end{vmatrix} = 15 \begin{vmatrix} 1 & 2 & 3 & -1 \\ 1 & 2 & -2 & -1 \\ 1 & -3 & -2 & -1 \\ -4 & -3 & -2 & -1 \end{vmatrix}$$

R_1-R_2 , R_3-R_2 , R_4-R_2 , gives

$$15 \begin{vmatrix} 0 & 0 & 5 & 0 \\ 1 & 2 & -2 & -1 \\ 0 & -5 & 0 & 0 \\ -5 & -5 & 0 & 0 \end{vmatrix} = 1875$$,{the proposition (A) is verified to be true}

The proof for the general case though clumsy is based on similar lines.

Generalization:

This can be further generalized by considering an arithmetic progression with the first term as **a** and the common difference as **d** and we can define **Smarandache Cyclic Arithmetic determinant sequence** as

$$\begin{vmatrix} a \end{vmatrix}, \quad \begin{vmatrix} a & a+d \\ a+d & a \end{vmatrix}, \quad \begin{vmatrix} a & a+d & a+2d \\ a+d & a+2d & a \\ a+2d & a & a+d \end{vmatrix}, \quad . \quad . \quad .$$

Conjecture-1:

$$\mathbf{T_n = (-1)^{[n/2]}\, S_n \,.d^{n-1} \,.\, n^{n-2} = (-1)^{[n/2]} \,.\{ a + (n-1)d\}.\{1/2\} \,.\{nd\}^{n-1}}$$

Where S_n is the sum of the first n terms of the AP

Open Problem: To develop a formula for the sum of n terms of the sequence.

(2) Smarandache Bisymmetric Determinant Sequences:

(a)Smarandache Bisymmetric Determinant Natural Sequence:

The determinants are symmetric along both the leading diagonals hence the name.

$$\begin{vmatrix} 1 \end{vmatrix}, \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 3 & 2 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 3 \\ 3 & 4 & 3 & 2 \\ 4 & 3 & 2 & 1 \end{vmatrix}, \ldots$$

1 , -3 , -12 , 40 , . . .

This suggests the possibility of the n^{th} term as

$$T_n = (-1)^{[n/2]} \{n(n+1)\} . 2^{n-3} \qquad \textbf{(B)}$$

We verify this for n = 5 , and the general case can be dealt with on similar lines.

$$T_5 = \begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 4 \\ 3 & 4 & 5 & 4 & 3 \\ 4 & 5 & 4 & 3 & 2 \\ 5 & 4 & 3 & 2 & 1 \end{vmatrix}$$

on carrying out following elementary operations
(b) R_1 = sum of all the rows, (b) taking 15 common from the first row, we get

$$15 \begin{vmatrix} 1 & 2 & 3 & 2 \\ 1 & 2 & 1 & 0 \\ 1 & 0 & -1 & -2 \\ -1 & -2 & -3 & -4 \end{vmatrix}$$

$R_1 = R_1 + R_4$ gives

$$15 \begin{vmatrix} 0 & 0 & 0 & -2 \\ 1 & 2 & 1 & 0 \\ 1 & 0 & -1 & -2 \\ -1 & -2 & -3 & -4 \end{vmatrix} = 120$$, which confirms with (B)

The proof of the general case can be based on similar lines.

Generalization: We can generalize this also in the same fashion by considering an arithmetic progression as follows:

$$\begin{vmatrix} a \end{vmatrix}, \begin{vmatrix} a & a+d \\ a+d & a \end{vmatrix}, \begin{vmatrix} a & a+d & a+2d \\ a+d & a+2d & a+d \\ a+2d & a+d & a \end{vmatrix}, \ldots$$

Conjecture-2: The general term of the above sequence is given by

$$T_n = (-1)^{[n/2]} .\{ a + (n-1)d \} .2^{n-3}. d^{n-1}$$