On Smarandache sequences and subsequences

Tiang Zhengping Xu Kanghua
(Hangzhou Teacher's College, Fuyang No.2 Middle School, Zhejiang, P. R. China)

Abstract A Smarandache sequence partial perfect additive sequence is studied completely in the first paragraph. In the second paragraph both Smarandache square-digital subsequence and square-partial-digital subsequence are studied.

Key words Smarandache partial perfect additive sequence, Smarandache square-digital subsequence , Smarandache square-partial-digital subsequence.

§1 Smarandache partial perfect additive sequence

The Smarandache partial perfect additive sequence is defined to be a sequence: 1, 1, 0, 2, −1, 1, 1, 3, −2, 0, 0, 2, 0, 2, 2, 4, −3, −1, −1, 1, −1, 1, 1, 3, −1, 1, ⋯.

This sequence has the property that:

$$\sum_{i=1}^{p} a_i = \sum_{j=p+1}^{2p} a_j, \quad \text{for all} \quad p \geq 1.$$

It is constructed in the following way:

$$a_1 = a_2 = 1,$$

$$a_{2p+1} = a_{p+1} - 1,$$

and $\quad a_{2p+2} = a_{p+1} + 1 \quad$ for all $\quad p \geq 1.$

In [1] M. Bencze raised the following two questions:

(a) Can you, readers, find a general expression of a_n (as function of n)?

Is it periodical or convergent or bounded?

(b) Please design (invent) yourselves other Smarandache perfect (or partial perfect) f-sequences.

In this paper we solved the question (a) completely.

Suppose the binary notation of $n(n \geq 2)$ as $n = (\varepsilon_k \varepsilon_{k-1} \cdots \varepsilon_1 \varepsilon_0)_2$, among which $\varepsilon_k = 1, \varepsilon_i = 0$ or 1 $(i = 0,1,\cdots,k-1)$. Define $f(n)$ are the numbers of $\varepsilon_i = 1(i = 0,1,\cdots,k)$, $g(n)$ is the minimum of i that makes $\varepsilon_i = 1$.

Thus we may prove the expression of $a(n)$ (i.e. a_n) as the following:

$$a(n) = \begin{cases} k, & \text{if} \quad \varepsilon_0 = \varepsilon_1 = \cdots = \varepsilon_{k-1} = 0, \\ -k + 2f(n) + 2g(n) - 3 & \text{otherwise} \end{cases}$$

We may use mathematical induction to prove it.

$a(2) = 1, a(3) = 0 = -1 + 2 \times 2 + 2 \times 0 - 3 = -1 + 2f(3) + 2g(3) - 3.$

So the conclusion is valid for $n = 2,3,$

Suppose that the conclusion is also valid for $2,3,\cdots,n-1(n \geq 3)$.Let's consider the cases of n.

1　When $\varepsilon_0 = \varepsilon_1 = \cdots = \varepsilon_{k-1} = 0$.

$a(n) = a((\varepsilon_k \varepsilon_{k-1} \cdots \varepsilon_1 \varepsilon_0)_2) = a((\varepsilon_k \varepsilon_{k-1} \cdots \varepsilon_1) + 1$

$= k - 1 + 1 = k.$

2　When not all the $\varepsilon_0, \varepsilon_1, \cdots, \varepsilon_{k-1}$ are zeroes, two kinds of cases should be discussed.

147

(1) If $\varepsilon_0 = 0$.

Then $f(n) = f((\varepsilon_k \varepsilon_{k-1} \cdots \varepsilon_1 \varepsilon_0)_2) = f((\varepsilon_k \varepsilon_{k-1} \cdots \varepsilon_1)_2) = f(\frac{n}{2})$,

$g(n) = g((\varepsilon_k \varepsilon_{k-1} \cdots \varepsilon_1 \varepsilon_0)_2) = g((\varepsilon_k \varepsilon_{k-1} \cdots \varepsilon_1)_2) + 1 = g(\frac{n}{2}) + 1$.

According to inductive hypothesis, we have

$$a(n) = a(\frac{n}{2}) + 1 = -(k-1) + 2f(\frac{n}{2}) + 2g(\frac{n}{2}) - 3 + 1$$

$$= -k + 2f(n) + 2(g(n) - 1) - 1$$

$$= -k + 2f(n) + 2g(n) - 3.$$

(2) If $\varepsilon_0 = 1$, three subcases exist

(i) If $\varepsilon_1 = 0$.

Then $f(n) = f((\varepsilon_k \varepsilon_{k-1} \cdots \varepsilon_1 \varepsilon_0)_2) = f((\varepsilon_k \varepsilon_{k-1} \cdots \varepsilon_2 1)_2)$

$= f(\left[\frac{n}{2}\right] + 1)$, the notation $[x]$ denotes the greatest integer not

more than x.

$$g(n) = g((\varepsilon_k \varepsilon_{k-1} \cdots \varepsilon_1 \varepsilon_0)_2) = g((\varepsilon_k \varepsilon_{k-1} \cdots \varepsilon_3 \varepsilon_2 1)_2) = g(\left[\frac{n}{2}\right] + 1) = 0.$$

so, it's easily known from inductive hypothesis

$$a(n) = a(\left[\frac{n}{2}\right] + 1) - 1 = -(k-1) + 2f(\left[\frac{n}{2}\right] + 1) + 2g(\left[\frac{n}{2}\right] + 1) - 3 - 1$$

$$= -k + 2f(n) + 2g(n) - 3.$$

(ii) If $\varepsilon_1 = \varepsilon_2 = \cdots = \varepsilon_i = 1, \varepsilon_{i+1} = 0, 1 \le i \le k - 2$.

$$n = (\varepsilon_k \varepsilon_{k-1} \cdots \varepsilon_1 \varepsilon_0)_2, \quad \left[\frac{n}{2}\right] + 1 = (\varepsilon_k \varepsilon_{k-1} \cdots \varepsilon_{i+2} \varepsilon_{i+1} \varepsilon_i \cdots \varepsilon_2 \varepsilon_1)_2 + (1)_2$$

$$= (\varepsilon_k \varepsilon_{k-1} \cdots \varepsilon_{i+2} \underbrace{100 \cdots 0}_{i \ times})_2 .$$

So, $\quad f([\frac{n}{2}]+1) = f(n) - i, \quad g([\frac{n}{2}]+1) = i = i + g(n).$

Then, According to inductive hypothesis, we have

$$a(n) = a([\frac{n}{2}]+1) - 1 = -(k-1) + 2f([\frac{n}{2}]+1) + 2g([\frac{n}{2}]+1) - 3 - 1$$

$$= -k + 2(f(n) - i) + 2(i + g(n)) - 3$$

$$= -k + 2f(n) + 2g(n) - 3 .$$

(iii) If $\quad \varepsilon_1 = \varepsilon_2 = \cdots = \varepsilon_{k-1} = \varepsilon_k = 1, \quad$ then

$$f(n) = k+1, \quad g(n) = 0. \quad [\frac{n}{2}]+1 = (\varepsilon_k \varepsilon_{k-1} \cdots \varepsilon_2 \varepsilon_1)_2 + (1)_2$$

$$= (\underbrace{100 \cdots 0}_{k \ times})_2, \quad \text{so from} \quad 1$$

$$a([\frac{n}{2}]+1) = k .$$

Then $\quad a(n) = a([\frac{n}{2}]+1) - 1 = k - 1$

$$= -k + 2(k+1) + 2 \times 0 - 3 = -k + 2f(n) + 2g(n) - 3 .$$

From the above, the conclusion is true for all the natural numbers $n(n \geq 2)$.

Having proved above fact, the remaining problem in question (a) can be solved easily. For if $n = 2^k$, we have $a(n) = k$, so sequence $\{a(n)\}$ is unbounded, therefore cannot be periodical and convergent.

149

§2 Smaranche square-digital subsequence and

Smaranche square-partial-digital subsequence

The Smaranche square-digital subsequence is defined to be a subsequence:

$$0, \ 1, \ 4, \ 9, \ 49, \ 100, \ 144, \ 400, \ 441, \ \cdots$$

i.e. from $0, \ 1, \ 4, \ 9, \ 16, \ 25, \ 36, \ \cdots, \ n^2, \ \cdots$ we choose only the terms those digits are all perfect squares (Therefore only $0, \ 1, \ 4$ and 9)

In [1] M.Bencze questioned: Disregarding the square numbers of the form $\overline{N \underbrace{0 \cdots 0}_{2k \ times}}$, where N is also a perfect square, how many other numbers belong to this sequence?

We find that $1444, \ 11449, \ 491401,$ also belong to the sequence by calculating.

In fact, we may find infinitely many numbers that belong to the sequence.

$$(2 \cdot 10^k + 1)^2 = 4 \cdot 10^{2k} + 4 \cdot 10^k + 1 \ ,$$

$$(10^k + 2)^2 = 10^{2k} + 4 \cdot 10^k + 4 \qquad \text{for all} \quad k \geq 1.$$

Smarandache square-partial-digital subsequence is defined to be a sequence:

$$49, \ 100, \ 144, \ 169, \ 400, \ 441, \ \cdots$$

i.e. the square numbers that can be partitioned into groups of digits which are also perfect squares (169 can be partitioned as $16 = 4^2$ and $9 = 3^2$, etc.).

In the same way it is questioned: Disregarding the square numbers of the form $\overline{N \underbrace{0 \cdots 0}_{2k \ times}}$, where N is also a perfect square, how many other numbers belong to this sequence?

We may find 22 numbers in the form $\overline{a0b}^2$ (here neither a or b is zero).

10404, 11025, 11449, 11664, 40401, 41616, 42025, 43681, 93025, 93636, 161604, 164025, 166464, 251001, 254016, 259081, 363609, 491401, 641601, 646416, 813604, 819025.

We may construct infinitely many numbers by adding zero in the middle of there numbers like 102^2, 105^2, 107^2, 108^2, 201^2, 204^2, 205^2, 209^2, 305^2, 306^2, 402^2, 405^2, 408^2, 501^2, 504^2, 509^2, 603^2, 701^2, 801^2, 804^2, 902^2, 905^2 as well . we may find some other numbers as the following:

3243601, 10246401, 2566404, 1036324, 4064256, 36144144, 49196196, 81324324, 64256256, 121484484, 169676676, 196784784, 484484121, 576576144, 676676169, 784784196 900900225, 1442401, 3243601, 4004001, 4844401, 10246401, 20259001, 24019801, 25010001, 49014001, 64016001 .

§ 3   Smaranche   cube-partial-digital   subsequence

1000, 8000, 10648, 27000, ⋯

i.e. the cube numbers that can be partitioned into groups of digits which are also perfect cubes (10648 can be partitioned as $1 = 1^3, 0 = 0^3, 64 = 4^3$, and $8 = 2^3$)

Same question: disregarding the cube numbers of the form: $\overline{M\underbrace{0\cdots0}_{3k\ times}}$, where M is also a perfect cube, how many other numbers belong to this sequence?

As the above said, we may find infinitely many numbers that belong to the sequence as well

$$(3 \cdot 10^{k+2} + 3)^3, (6 \cdot 10^{k+3} + 1)^3, (6 \cdot 10^{k+3} + 6)^3, (10^{k+3} + 6)^3 \qquad \text{for all} \quad k \geq 0$$

for example $\quad \underline{278\underline{18}127} = 303^3, \quad \underline{216\underline{648}648216} = 6006^3$,

$$\underline{216\underline{108}018001} = 6001^3, \underline{10\underline{18}108216} = 1006^3 .$$

REFERENCES

1. Bencze, M.(1998). "Smarandache Relationships and Subsequences", Bulletin of Pure and Applied Sciences. Vol. 17E (N0.1) 1998; P.55-62.