SMARANDACHE SEQUENCE OF HAPPY NUMBERS

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Abstract:

In this article, we present the results of investigation of Smarandache Concatenate Sequence formed from the sequence of Happy Numbers and report some primes and other results found from the sequence

Key words:

Happy numbers, Consecutive happy numbers, H-sequence, Smarandache H-sequence, Reversed Smarandache H-sequence, Prime, Happy prime, Reversed Smarandache Happy Prime, Smarandache Happy Prime

1. Introduction:

If you iterate the process of summing the squares of the decimal digits of a number and if the process terminates in 1, then the original number is called a Happy number [1].

For example:

7 -> 49 -> 97 -> 130 -> 10 -> 1, so the number 7 is a happy number.

Let us denote the sequence of Happy numbers as H-sequence. The sequence of Happy numbers [3], say H = { 1, 7, 10, 13, 19, 23, 28, 31, 32, 44, 49, 68, 70, 79, 82, 86, 91, 97 ,100 ....}.

2. Smarandache Sequence:

Let S_1 , S_2 , S_3 , . . . , S_n , . . . be an infinite integer sequence (termed as S- sequence), then the Smarandache sequence [4] or Smarandache Concatenated sequence [2] or Smarandache S-sequence is given by

$$\overline{S_1}, \quad \overline{S_1S_2}, \quad S_1S_2S_3 \quad \ldots \quad \overline{S_1S_2S_3 \ldots S_n} \qquad \ldots$$

Also Smarandache Back Concatenated sequence or Reversed Smarandache S-sequence is

$$\overline{S_1}, \quad \overline{S_2S_1}, \quad S_3S_2S_1 \quad \ldots \quad \overline{S_n \ldots S_3S_2S_1} \qquad \ldots$$

3. Smarandache H-Sequence:

Smarandache sequence of Happy numbers or Smarandache H-sequence is the sequence formed from concatenation of numbers in H-sequence (Note that H-sequence is the sequence of Happy numbers). So, Smarandache H-sequence is

1, 17, 1710, 171013, 17101319, 1710131923, 171013192328, ...

Let us denote the nth term of the Smarandache H-sequence by SH(n). So,

SH(1)=1

SH(2)=17

SH(3)=1710

SH(4)=171013 and so on.

3.1 Observations on Smarandache H-sequence:

We have investigated Smarandache H-sequence for the following two problems.

i. How many terms of Smarandache H-sequence are primes?

ii. How many terms of Smarandache H-sequence belongs to the initial H-sequence?

In search of answer to these problems, we find that

a. There are only 3 primes in the first 1000 terms of Smarandache H-sequence. These are SH(2) = 17, SH(5) = 17101319 and SH(43), which is 108 digit prime. It may be noted that SH(1000) consists of 3837 digits.

Open Problem:

Can you find more primes in Smarandache H-sequence and are there infinitely many such primes?

b. There are 1429 Happy numbers in first 10000 terms of Smarandache H-sequence and hence belongs to the initial H-sequence. The first few Happy numbers in the Smarandache H-sequence are SH(1), SH(11), SH(14), SH(30), SH(31), SH(35), SH(48), SH(52), SH(62), SH(67), SH(69), SH(71), SH(76), ..., etc.

It may be noted that SH(10000) consists of 48396 digits.

Based on the investigations we state the following:

Conjecture:

About one-seventh of numbers in the Smarandache H-sequence belong to the initial H-sequence.

In this connection, it is interesting to note that about one-seventh of all numbers are happy numbers [1].

3.2 Consecutive SH Numbers:

It is known that smallest pair of consecutive happy numbers is 31, 32. The smallest triple is 1880, 1881, 1882. The smallest example of four and 5 consecutive happy numbers are 7839, 7840, 7841, 7842 and 44488, 44489, 44490, 44491, 44492 respectively. Example of 7 consecutive happy numbers is also known [3]. The question arises as to how many consecutive terms of Smarandache H-sequence are happy numbers.

Let us define consecutive SH numbers as the consecutive terms of Smarandache H-sequence which are happy numbers. During investigation of first 10000 terms of Smarandache H-sequence, we found the following smallest values of consecutive SH numbers:

Smallest pair: SH(30), SH(31)

Smallest triple: SH(76), SH(77), SH(78)

Smallest example of four and five consecutive SH numbers are SH(153), SH(154), SH(155), SH(156) and SH(3821), SH(3822), SH(3823), SH(3824), SH(3825) respectively.

Open Problem:

Can you find the examples of six and seven consecutive SH numbers?

How many consecutive SH numbers can you have?

4.0 Reversed Smarandache H-Sequence:

It is defined as the sequence formed from the concatenation of happy numbers (H-sequence) written backward i.e. in reverse order. So, Reversed Smarandache H-sequence is

1, 71, 1071, 131071, 19131071, 2319131071, 282319131071, ...

Let us denote the nth term of the Reversed Smarandache H-sequence by RSH(n). So,

RSH(1)=1

RSH(2)=71

RSH(3)=1071

RSH(4)=131071 and so on.

4.1 Observations on Reversed Smarandache H-sequence:

Since the digits in each term of Reversed Smarandache H-sequence are same as in Smarandache H-sequence, hence the observations regarding problem (ii) including conjecture mentioned in para 3.1 above remains valid in the present case also. So, only observations regarding problem (i) mentioned in para 3.1 above are given below:

As against only 3 primes in Smarandache H-sequence, we found 8 primes in first 1000 terms of Reversed Smarandache H-sequence. These primes are:

RSH(2) = 71

RSH(4) = 131071

RSH(5) = 19131071

RSH(6) = 2319131071

RSH(10) = 443231282319131071

Other three primes are RSH(31), RSH(255) and RSH(368) which consists of 72, 857 and 1309 digits respectively.

Smarandache Curios:

It is interesting to note that there are three consecutive terms in Reversed Smarandache H-sequence, which are primes, namely RSH(4), RSH(5) and RSH(6), which is rare in any Smarandache sequence.

We also note that RSH(31) is prime as well as happy number , so, this can be termed as Reversed Smarandache Happy Prime. No other happy prime is noted in Reversed Smarandache H-sequence and Smarandache H-sequence.

Open Problem:

Can you find more primes in Reversed Smarandache H-sequence and are there infinitely many such primes?

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