PROPOSED PROBLEMS

by

M. Bencze

(i) Solve the following equations:

1) $S^k(x) + S^k(y) = S^k(z)$, $k \in Z$, $x, y, z \in Z$

where S' is the Smarandache function and $S(-n) = -S(n)$

2) $\frac{4}{n} = \frac{1}{S(x)} + \frac{1}{S(y)} + \frac{1}{S(z)}$, $n > 4$

3) $\frac{5}{n} = \frac{1}{S(x)} + \frac{1}{S(y)} + \frac{1}{S(z)}$, $n > 5$

4) $S^{S(y)}(x) = S^{S(x)}(y)$

5) $S\left(\sum_{k=1}^{n} x_k^u\right) = S^u\left(\sum_{k=1}^{n} x_k\right)$, $u \in Z$

6) $S^y(x) - S^t(z) = S^{y-t}(x-z)$

7) $\sum_{k=1}^{n} S^m(x_k) = \sum_{k=n}^{2n} S^m(x_k)$

8) $2S(x^4) - S^2(y) = 1$

9) $S\left(\frac{x+y+z}{3}\right) + \frac{S(x)+S(y)+S(z)}{3} = \frac{2}{3}\left[S\left(\frac{x+y}{2}\right) + S\left(\frac{y+z}{2}\right) + S\left(\frac{z+x}{2}\right)\right]$

10) $S(x_1^{x_1}) \cdot S(x_2^{x_2}) \ldots S(x_n^{x_n}) = S(x_{n+1}^{x_{n+1}})$

11) $S(x_1^{x_2}) \cdot S(x_2^{x_3}) \ldots S(x_{n-1}^{x_n}) = S(x_n^{x_1})$

12) $S(x) = \mu(y)$, where μ is the Möbius function

13) $S^2(Q_n) = \sum_{Q_{n-1} | Q_n} \ldots \sum_{Q_2 | Q_3} \sum_{Q_1 | Q_2} \mu^2(Q_1)$

14) $S(x) = B_y$, where B_y is a Bernoulli number

15) $S(x+y)(S(x)-S(y)) = S(x-y)(S(x)+S(y))$

16) $S(x) = F_y$, where F_y is a Fibonacci number

17) $\sum_{k=1}^{n} S(k^p) = \sum_{k=1}^{n} S^p(k)$

18) $\sum_{k=1}^{n} S(k) = S\left(\frac{n(n+1)}{2}\right)$

19) $\sum_{k=1}^{n} S(k^2) = S\left(\frac{n(n+1)(2n+1)}{6}\right)$

20) $\sum_{k=1}^{n} S(k^3) = S\left(\frac{n^2(n+1)^2}{4}\right)$

21) $\sum_{k=1}^{n} k(S(k)!) = (S(n+1))! - 1$

22) $\sum_{k=1}^{n} \frac{1}{S(k)S(k+1)} = \frac{S(n)}{S(n+1)}$

(ii) Solve the system

$$\begin{cases} S(x) + S(y) = 2S(z) \\ S(x) \cdot S(y) = S^2(z) \end{cases}$$

(iii) Find n such that n divides the sum

$$1^{S(n-1)} + 2^{S(n-1)} + \ldots (n-1)^{S(n-1)} + 1$$

(iv) May be writen every positive integer n as

$$n = S^3(x) + 2\,S^3(y)\;3\,S^3(z) \quad ?$$

(v) Prove that

$$|S(x)+S(y)+S(z)| + |S(x)| + |S(y)| + |S(z)| \geq$$

$$\geq |S(x)+S(y)| + |S(y)+S(z)| + |S(z)+S(x)|$$

for all $x, y, z \in Z$

(vi) Find all the positive integers x, y, z for which

$$(x+y+z) + S(x) + S(y) + S(z) \geq S(x+y) + S(y+z) + S(z+x)$$

(vii) There exists an infinity of prime numbers which may be writen under the form

$$P = S^3(x) + S^3(y) + S^3(z) + S^3(t) \quad ?$$

(viii) Let $M_1, M_2, \ldots, M_n$ be finite sets and $a_{ij} = card(M_i \cap M_j)$, $b_{ij} = S(a_{ij})$. Prove that $det(a_{ij}) \geq 0$ and $det(b_{ij}) \geq 0$.

(ix) Find the sum

$$\sum_{Q_{n-1} | Q_n} \cdots \sum_{Q_2 | Q_3} \sum_{Q_1 | Q_2} \frac{1}{S^2(Q_1)}$$

(x) Prove that

$$\sum_{k=1}^{\infty} \frac{1}{S^2(k) - S(k) + 1} \quad \text{is irational}$$

(xi) Find all the positive integers x for which

$$S\left(\left[\frac{x^{n+1} - 1}{(n+1)(x-1)}\right]\right) \geq S\left(\left[x^{\frac{n}{2}}\right]\right)$$

where [x] is the integer part of x.

(xii) There exists at lest a prime between S(n)!, and S(n+1)! ?

(xiii) If $\sigma \in S_n$ is a permutation, prove that

$$\sum_{k=1}^{n} \frac{\sigma(k)}{S^{m+1}(k)} \geq \sum_{k=1}^{n} \frac{1}{k^m}$$

Current address:

RO - 2212 Sacele
Str. Harmanului, 6
Jud. Brasov, ROMANIA