SMARANDACHE RECURRENCE TYPE SEQUENCES*

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ABSTRACT

Eight particular, Smarandache Recurrence Sequences and a Smarandache General-Recurrence Sequence are defined below and exemplified (found in State Archives, Rm, Valcea, Romania).

A. 1, 2, 5, 26, 29, 677, 680, 701, 842, 845, 866, 1517, 458330, 458333, 458354, ...

(ss2(n) is the smallest number, strictly greater than the previous one, which is the squares sum of two previous distinct terms of the sequence; in our particular case the first two terms are 1 and 2.)

Recurrence definition:

(1) The numbers $a \le b$ belong to SS2;
(2) If b, c belong to SS2, then $b^2 + c^2$ belongs to SS2 too;
(3) Only numbers, obtained by rules [(1) and/or (2)] applied a finite number of times, belong to SS2.

The sequence (set) SS2 is increasingly ordered.

[Rule (1) may be changed by: the given numbers $a_1, a_2, a_3, \ldots, a_k$, where $k \ge 2$, belongs to SS2.]

B. 1, 1, 2, 4, 5, 6, 16, 17, 18, 20, 21, 22, 25, 26, 27, 29, 30, 31, 36, 37, 38, 40, 41, 42, 43, 45, 46, ...

(SS1(n) is the smallest number, strictly greater than the previous one, (for $n \ge 3$), which is the squares sum of one or more previous distinct terms of the sequence; in our particular case the first term is 1.)

Recurrence definition:

(1) The number a belongs to SS1;
(2) If $b_1, b_2, \ldots, b_k$ belong to SS1, where $k \ge 1$, then $b_1^2 + b_2^2 + \ldots + b_k^2$ belongs to SS1 too;
(3) Only numbers, obtained by rules [(1) and/or (2)] applied a finite number of times, belong to SS1.

The sequence (set) SS1 is increasingly ordered.

[Rule (1) may be changed by: the given numbers $a_1, a_2, \ldots, a_k$, where $k \ge 1$, belong to SS1.]

C. 1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 14, 15, 16, 18, 19, 21, ...

(NSS2(n) is the smallest number, strictly greater than the previous one, which is NOT the squares sum of two previous distinct terms of the sequence; in our particular case the first two terms are 1 and 2.)

Recurrence definition:

(1) The numbers $a \le b$ belong to NSS2;
(2) If b, c belong to NSS2, then $b^2 + c^2$ DOES NOT belong to NSS2; any other numbers belong to NSS2;
(3) Only numbers, obtained by rules [(1) and/or (2)] applied a finite number of times, belong to NSS2.

The sequence (set) NSS2 is increasingly ordered.

[Rule (1) may be changed by; the given numbers $a_1, a_2, ..., a_k$, where $k \geq 2$, belong to NSS2.]

D. 1, 2, 3, 6, 7, 8, 11, 12, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 38, 39, 42, 43, 44, 47, ...

(NSS1(n) is the smallest number, strictly greater than the previous one, which is NOT the squares sum of one or more of the previous distinct terms of the sequence; in our particular case the first term is 1.)

Recurrence definition:

(1) The number a belongs to NSS1;
(2) If $b_1, b_2, ..., b_k$ belong to NSS1, where $k \geq 1$, then $b_1^2 + b_2^2 + ... + b_k^2$ DOES NOT belong to NSS1; any other numbers belong to NSS1;

(3) Only numbers, obtained by rules [(1) and/or (2)] applied a finite number of times, belong to NSS1.

[Rule (1) may be changed by: the given numbers $a_1, a_2, ..., a_k$, where $k \geq 1$, belong to NSS1.]

E. 1, 2, 9, 730, 737, 389017001, 389017008,389017729, ...

(CS2(n) is the smallest number, strictly greater than the previous one, which is the cubes sum of two previous distinct terms of the sequence; in our particular case the first two terms are 1 and 2.)

Recurrence definition:

(1) The numbers $a \leq b$ belong to CS2;
(2) If c,d belong to CS2, then $c^3 + d^3$ belongs to CS2 too;
(3) Only numbers, obtained by rules [(1) and/or (2)] applied a finite number of times, belong to CS2.

The sequence (set) CS2 is increasingly ordered.

[Rule (1) may be changed by: the given numbers $a_1, a_2, ..., a_k$, where $k \geq 2$, belong to CS2.]

F. 1, 1, 2, 8, 9, 10, 512, 513, 514, 520, 521, 522, 729, 730, 731, 737, 738, 739, 1241, ...

(CS1(n) is the smallest number, strictly greater than the previous one (for $n \geq 3$), which is the cubes sum of one or more previous distinct terms of the sequence; in our particular case the first term is 1;

Recurrence definition:

(1) The number a belongs to CS1;
(2) If $b_1, b_2, ..., b_k$ belong to CS1, where $k \geq 1$, then $b_1^3 + b_2^3 + ... + b_k^3$ belongs to CS2 too;
(3) Only numbers, obtained by rules [(1) and/or (2)] applied a finite number of times, belong to CS1.

The sequence (set) CS1 is increasingly ordered.

[Rule (1) may be changed by: the given numbers $a_1, a_2, ..., a_k$, where $k \geq 2$, belong to CS1.]

G. 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 36, 37, 38, ...

(NCS2(n) is the smallest number, strictly greater than the previous one, which is NOT the cubes sum of two previous distinct terms of the sequence; in our particular case the first two terms are 1 and 2.)

Recurrence definition:

(1) The numbers $a \le b$ belong to NCS2.
(2) If c,d belong to NCS2, then $c^3 + d^3$ DOES NOT belong to NCS2; any other numbers do belong to NCS2.
(3) Only numbers, obtained by rules [(1) and/or (2)] applied a finite number of times, belong to NCS2.

The sequence (set) NCS2 is increasingly ordered.

[Rule (1) may be changed by: the given numbers $a_1, a_2, ..., a_k$, where $k \ge 2$, belong to NCS2.]

H. 1, 2, 3, 4, 5, 6, 7, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 29, 30, 31, 32, 33, 34, 37, 38, 39, ...

(NCS1(n) is the smallest number, strictly greater than the previous one, which is NOT the cubes sum of one or more previous distinct terms of the sequence; in our particular case the first term is 1.)

Recurrence definition:

(1) The number a belongs to NCS1.
(2) If $b_1, b_2, ..., b_k$ belong to NCS1, where $k \ge 1$, then $b_1^2 + b_2^2 + ... + b_k^2$ DOES NOT belong to NCS1.
(3) Only numbers, obtained by rules [(1) and/or (2)] applied a finite number of times, belong to NCS1.

The sequence (set) NCS1 is increasingly ordered.

[Rule (1) may be changed by: the given numbers $a_1, a_2, ..., a_k$, where $k \ge 2$, belong to NCS1.]

I. General recurrence type sequence:

General recurrence definition:

Let $k \ge j$ be natural numbers, and $a_1, a_2, ..., a_k$ be given elements, and R a j-relationship (relation among j elements).

Then:

(1) The elements $a_1, a_2, ..., a_k$ belong to SGR.
(2) If $m_1, m_2, ..., m_j$ belong to SGR, then $R(m_1, m_2, ..., m_j)$ belongs to SGR too.
(3) Only numbers, obtained by rules [(1) and/or (2)] applied a finite number of times, belong to SGR.

The sequence (set) SGR is increasingly ordered.

Method of construction of the general recurrence sequence:

-level 1: the given elements $a_1, a_2, ..., a_k$ belong to SGR;

-level 2: apply the relationship R for all combinations of j elements among $a_1, a_2, ..., a_k$; the results belong to SGR too;

order all elements of levels 1 and 2 together,

-level i+1:

if $b_1, b_2, ..., b_m$ are all elements of levels 1, 2, ..., i-1 and $c_1, c_2, ..., c_n$ are all elements of level i, then apply the relationship R for all combinations of j elements among $b_1, b_2, ..., b_m, c_1, c_2, ..., c_n$ such that at least an element is from the level i;

the results belong to SGR too;

order all elements of levels i and i+1 together;

and so on . . .

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