An Illustration of the Distribution of the Smarandache Function

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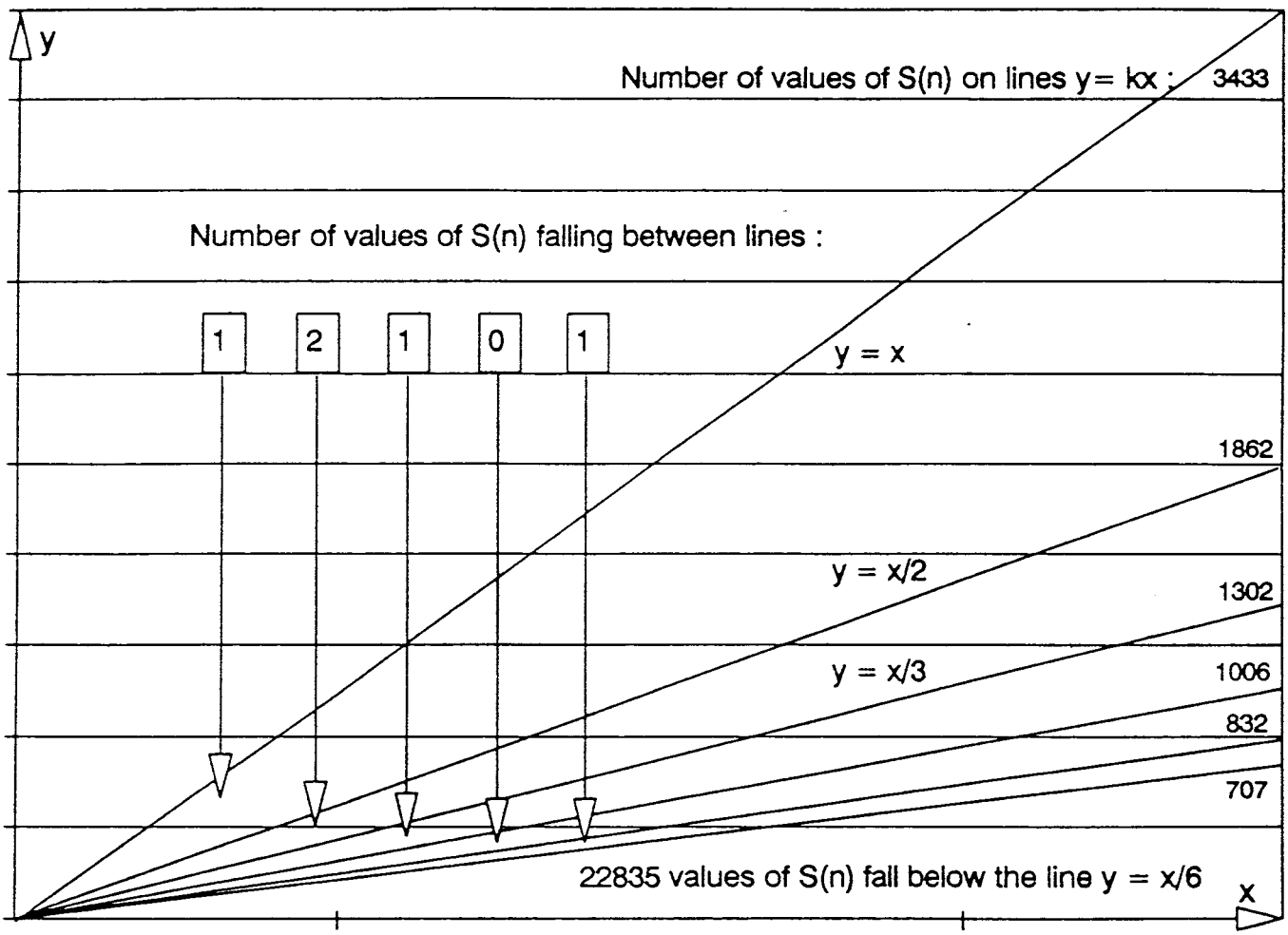
The cover illustration is a representation of the values of the Smarandache function for $n \leq 53$. The group at the back of the diagram essentially corresponds to $S(p)=p$, the middle group to $S(2p)=p$ $(p \neq 2)$ while the front group represents all the other values of $S(n)$ for $n \leq 53$.

Diagram 1. Distribution of S(n) up to n = 32000 (not to scale)



It may be interesting to take this graphical presentation a bit further. All the values of $S(n)$ for $n \leq 32000$ (conveniently chosen in order to use short integers only) have been sorted as shown in table 1. Of the 19114 points $(n,S(n))$ situated above the line $y = x/50$ only 61 points fall between lines. All of these of course correspond to cases where n is not square free. Diagram 1 illustrates this for the lines $y=x$, $y=x/2$, $y=x/3$, $y=x/4$, $y=x/5$ and $y=x/6$. The top line contains 3433 points $(n,S(n)$ although there are only 3432 primes less than 32000. This is because $(4,S(4))$ belongs to this line.

TABLE 1. On the distribution of the Smarandache Function S(n) for n≤32000.

N = number of values of S(n) on the line y=x/k, i.e. S(n)=n/k. The points (n,S(n)) are the only ones between lines y=x/k and y=x/(k+1) for k<50.

k	N	Points (n,S(n)) between lines:
1	3433	(9, 6)
2	1862	(16, 6) (25, 10)
3	1302	(49, 14)
4	1006	
5	832	(121, 22)
6	707	(169, 26)
7	616	(45, 6) (75, 10)
8	550	(125, 15) (289, 34)
9	495	(361, 38)
10	450	(147, 14)
11	417	(529, 46)
12	387	
13	359	(80, 6)
14	336	(841, 58)
15	321	(961, 62)
16	301	(250, 15) (343, 21) (363, 22)
17	283	(175, 10) (245, 14)
18	273	(1369, 74)
19	256	(507, 26)
20	250	(243, 12) (1681, 82)
21	239	(1849, 86)
22	227	(225, 10)
23	213	(2209, 94)
24	218	
25	204	(256, 10) (867, 34)
26	196	(2809,106)
27	190	(605, 22)
28	187	(1083, 38)
29	176	(3481,118)
30	179	(3721,122)
31	163	(441, 14) (625, 20)
32	164	(686, 21) (845, 26)
33	159	(500, 15) (4489,134)
34	154	(1587, 46)
35	154	(5041,142)
36	153	(5329,146)
37	139	
38	139	(539, 14) (847, 22)
39	136	(6241,158)
40	139	(486, 12) (1331, 33)
41	125	(6889,166)
42	133	(512, 12) (1445, 34)
43	119	(2523, 58)
44	125	(7921,178)
45	126	(637, 14) (1183, 26)
46	117	(2883, 62)
47	109	(1805, 38)
48	120	(729, 15) (9409,194)
49	114	(1089, 22)
50	112	

Number of elements below y = x/50: 12774 .