A brief account on Smarandache 2-2 subtractive relationships

Henry Ibstedt

Abstract: An analysis of the number of relations of the type S(n)-S(n+1)=S(n+2)-S(n+3) for $n<10^8$ where S(n) is the Smarandache function leads to the plausible conclusion that there are infinitely many of those.

This brief note on Smarandache 2-2 subtractive relationships should be seen in relation to the article on Smarandache k-k additive relationships in this issue of SNJ [1]. A Smarandache 2-2 subtractive relationship is defined by

S(n)-S(n+1)=S(n+2)-S(n+3)

where S(n) denotes the Smarandache function. In an article by Bencze [2] three 2-2 subtractive relationships are given

S(1)-S(2)=S(3)-S(4), 1-2=3-4
S(2)-S(3)=S(4)-S(5), 2-3=4-5
S(49)-S(50)=S(51)-S(52), 14-10=17-13

The first of these solutions must be rejected since S(1)=0 not 1. The question raised in the article is "How many quadruplets verify a Smarandache 2-2 subtractive relationship?"

As in the case of Smarandache 2-2 additive relationships a search was carried for $n \le 10^8$. In all 442 solutions were found. The first 50 of these are shown in table 1.

Table 1. The 50 first 2-2 subtractive relations.

#	n	S(n)	S(n+1)	S(n+3)	S(n+4)
1	2	2	3	4	5
2	40	5	41	7	43
3	49	14	10	17	13
4	107	107	9	109	11
5	2315	463	193	331	61
6	3913	43	103	29	89
7	4157	4157	11	4159	13
8	4170	139	97	149	107
9	11344	709	2269	61	1621
10	11604	967	211	829	73
11	11968	17	11969	19	11971
12	13244	43	883	179	1019
13	15048	19	149	43	173
14	19180	137	19181	139	19183
15	19692	547	419	229	101
16	26219	167	23	2017	1873
17	29352	1223	197	1129	103
18	29415	53	3677	1279	4903
19	43015	1229	283	1103	157
20	44358	7393	6337	1109	53
21	59498	419	601	17	199
22	140943	4271	383	4027	139

Table 1. continued.

#	n	S(n)	S(n+1)	S(n+3)	S(n+4)
23	147599	1433	41	2203	811
24	153386	283	23	1237	977
25	169533	23	79	827	883
26	181577	971	571	1697	1297
27	186056	1789	2297	2269	2777
28	201965	1303	821	1453	971
29	204189	2347	2917	139	709
30	210219	887	457	659	229
31	217591	151	461	8059	8369
32	246974	59	89	227	257
33	253672	857	167	829	139
34	257543	1801	73	2711	983
35	262905	1031	211	929	109
36	273815	2381	3803	3299	4721
37	321010	683	821	241	379
38	363653	163	227	283	347
39	407836	31	661	673	1303
40	431575	283	739	607	1063
41	451230	89	127	239	277
42	530452	202	166	419	383
43	549542	2309	2207	941	839
44	573073	2909	2837	283	211
45	589985	631	449	1291	1109
46	590569	353	809	317	773
47	608333	1907	1913	191	197
48	646333	15031	24859	271	10099
49	649702	577	1447	107	977
50	666647	666647	197	666649	199

As in the case of 2-2 additive relations there is a great number of solutions formed by pairs of prime twins.

Table 2. All 51 subtractive relations formed by pairs of prime twins for $n<10^8$.

#	n	S(n)	S(n+1)	S(n+3)	S(n+4)
1	40	5	41	7	43
2	4157	4157	11	4159	13
3	11968	17	11969	19	11971
4	19180	137	19181	139	19183
5	666647	666647	197	666649	199
6	895157	895157	137	895159	139
7	1695789	347	101	349	103
8	1995526	71	1995527	73	1995529
9	2007880	101	2007881	103	2007883
10	2272547	2272547	149	2272549	151
11	3198730	1787	3198731	1789	3198733
12	3483088	227	3483089	229	3483091
13	3546268	431	3546269	433	3546271
14	4194917	4194917	197	4194919	199

Table 2. Continued.

#	n	S(n)	S(n+1)	S(n+3)	S(n+4)
15	4503640	179	4503641	181	4503643
16	5152420	149	5152421	151	5152423
17	6634078	269	6634079	271	6634081
18	6729658	107	6729659	109	6729661
19	7455628	2729	7455629	2731	7455631
20	7831738	641	7831739	643	7831741
21	7924877	7924877	71	7924879	73
22	11001647	11001647	239	11001649	241
23	11053978	281	11053979	283	11053981
24	12466690	809	12466691	811	12466693
25	13530988	311	13530989	313	13530991
26	17293120	4157	17293121	4159	17293123
27	17424707	17424707	311	17424709	313
28	18173650	191	18173651	193	18173653
29	19222600	431	19222601	433	19222603
30	19227910	419	19227911	421	19227913
31	22208567	22208567	431	22208569	433
32	26037491	26037491	347	26037493	349
33	30468670	311	30468671	313	30468673
34	31815238	5639	31815239	5641	31815241
35	36683147	36683147	641	36683149	643
36	40881257	40881257	191	40881259	193
37	42782236	227	42782237	229	42782239
38	46238236	311	46238237	313	46238239
39	53009681	53009681	1061	53009683	1063
40	53679671	53679671	521	53679673	523
41	53906597	53906597	227	53906599	229
42	54747418	269	54747419	271	54747421
43	57935326	659	57935327	661	57935329
44	63694847	63694847	1481	63694849	1483
45	68203229	68203229	1721	68203231	1723
46	73763380	2381	73763381	2383	73763383
47	84344411	84344411	269	84344413	271
48	86250580	1667	86250581	1669	86250583
49	92596529	92596529	1019	92596531	1021
50	94788077	94788077	1031	94788079	1033
51	95489237	95489237	101	95489239	103

In the case of 2-2 additive relations only 2 solutions contained composite numbers and these were the first two. This was explained in terms of the distribution Smarandache functions values. For the same reason 2-2 subtractive relations containing composite numbers are also scarce, but there are 6 of them for $n<10^8$. These are shown in table 3.

It is interesting to note that solutions #3, #5 and #6 have in common with the solutions formed by pairs of prime twins that they are formed by pairs of numbers whose difference is 2. Finally table 4 shows a tabular comparison between the solutions to the 2-2 additive and 2-2 subtractive solutions for $n<10^8$. The great similarity between these results leads the conclusion: If the conjecture that there are infinitely many 2-2 additive relations is valid then we also have the following conjecture:

Table 3. All 2-2 subtractive relations $<10^8$ containing composite numbers.

#	n	S(n)	S(n+1)	S(n+3)	S(n+4)
1	2	2	3	4	5
2	49	14	10	17	13
3	107	107	9	109	11
4	530452	202	166	419	383
5	41839378	111	41839379	113	41839381
6	48506848	57	48506849	59	48506851

Table 4. Comparison between 2-2 additive and 2-2 subtractive relations.

	Number of 2-2 additive solutions	Number of 2-2 subtractive sol.
Total number of solutions	481	442
Number formed by pairs of prime twins	65	51
Number containing composite numbers	2	6

Conjecture: There are infinitely many Smarandache 2-2 subtractive relationships.

References

1. H. Ibstedt, Smarandache k-k additive relationships, *Smarandache Notions Journal, Vol. 12 (this issue).*
2. M. Bencze, Smarandache Relationships and Subsequences, *Smarandache Notions Journal, Vol. 11, No 1-2-3, pgs 79-85.*

7, Rue du Sergent Blandan
92130 Issy les Moulineaux, France