On Certain Arithmetic Functions

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In the recent book [1] there appear certain arithmetic functions which are similar to the Smarandache function. In a recent paper [2] we have considered certain generalization or duals of the Smarandache function $S(n)$. In this note we wish to point out that the arithmetic functions introduced in [1] all are particular cases of our function F_f, defined in the following manner (see [2] or [3]).

Let $f : \mathbb{N}^* \to \mathbb{N}^*$ be an arithmetical function which satisfies the following property:

(P_1) For each $n \in \mathbb{N}^*$ there exists at least a $k \in \mathbb{N}^*$ such that $n|f(k)$.

Let $F_f : \mathbb{N}^* \to \mathbb{N}*$ defined by

$$F_f(n) = \min\{k \in \mathbb{N}^* : n|f(k)\} \tag{1}$$

In Problem 6 of [1] it is defined the "ceil function of t-th order" by $S_t(n) = \min\{k : n|k^t\}$. Clearly here one can select $f(m) = m^t$ $(m = 1, 2, \ldots)$, where $t \geq 1$ is fixed. Property (P_1) is satisfied with $k = n^t$. For $f(m) = \frac{m(m+1)}{2}$, one obtains the "Pseudo-Smarandache" function of Problem 7. The Smarandache "double-factorial" function

$$SDF(n) = \min\{k : n|k!!\}$$

where

$$k!! = \begin{cases} 1 \cdot 3 \cdot 5 \ldots k & \text{if } k \text{ is odd} \\ 2 \cdot 2 \cdot 6 \ldots k & \text{if } k \text{ is even} \end{cases}$$

of Problem 9 [1] is the particular case $f(m) = m!!$. The "power function" of Definition 24, i.e. $SP(n) = \min\{k : n|k^k\}$ is the case of $f(k) = k^k$. We note that the Definitions 39 and 40 give the particular case of S_t for $t = 2$ and $t = 3$.

In our paper we have introduced also the following "dual" of F_f. Let $g : \mathbb{N}^* \to \mathbb{N}*$ be a given arithmetical function, which satisfies the following assumption:

(P_3) For each $n \geq 1$ there exists $k \geq 1$ such that $g(k)|n$.

Let $G_g : \mathbb{N}^* \to \mathbb{N}^*$ defined by

$$G_g(n) = \max\{k \in \mathbb{N}^* : g(k)|n\}. \tag{2}$$

Since $k^t|n$, $k!!|n$, $k^k|n$, $\frac{k(k+1)}{2}|n$ all are verified for $k = 1$, property (P_3) is satisfied, so we can define the following duals of the above considered functions:

$$S_t^*(n) = \max\{k : k^t|n\};$$

$$SDF^*(n) = \max\{k : k!!|n\};$$

$$SP^*(n) = \max\{k : k^k|n\};$$

$$Z^*(n) = \max\left\{k : \frac{k(k+1)}{2}|n\right\}.$$

These functions are particular cases of (2), and they could deserve a further study, as well.

References

[1] F. Smarandache, *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, edited by M.L. Perez, Xiquan Publ. House (USA), 2000.

[2] J. Sándor, *On certain generalization of the Smarandache function*, Notes Number Theory Discrete Mathematics, **5**(1999), No.2, 41-51.

[3] J. Sándor, *On certain generalizations of the Smarandache function*. Smarandache Notions Journal, **11**(2000), No.1-2-3, 202-212.