ON SMARANDACHE ALGEBRAIC STRUCTURES. III : THE COMMUTATIVE RING $B(a,n)$

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Abstract In this paper we construat a class of commutaive rings under the Smarandache algorithm.

Key words. Smarandache algorithm, commutative ring.

Let a,n be integers such that $a \neq 0$ and $n>1$. Under the definitions and notitions in [1], let

$$(1) \qquad B(a,n)=\begin{cases} \{0,1,a,\cdots,a^{f-1}\} \pmod n, \text{ if } l'=1, \\ \{0,a,a^2,\cdots a^{e+f-1}\} \pmod n, \text{ if } l'>1. \end{cases}$$

In this paper we prove the following result.

Theorem. If m is a prime and a is a primitive root modulo m, then $B(a,n)$ is a commutative ring under the Smarandache additive and multiplicative.

Proof. Since $B(a,n)=A(a,n) \cup \{0\}$ by (1), $B(a,n)$ is a commutative multiplicative semigroup under the Smarandache algorithm (see [2]).

Notice that m is a prime and a is a primitive root modulo m. Then we have $f=m-1$. If $l'=1$, then $B(a,n)=\{0,1,2,\cdots m-1\} \pmod m$. Therefore, $B(a,n)$ is a commutative additive group. It implies that $B(a,n)$ is a commutative ring under additive and multiplicative. If $l'>1$, since $l \mid a^e$, then from (1) we see that $B(a,n)=\{0,l,2l,\cdots,(m-1)l\} \pmod n$. Therefore, $B(a,n)$ is also a commutative ring. The theorem is proved.

References

[1] M.-H. Le, On Smarandache algebraic structures I: The commutative multiplicative semigroup *A(a,n)*, Smarandache Notions J., **12(2001).**

[2] R.Padilla, Smarandache algebraic structures, Smarandache Notions J. 9(1998),36-38.

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