SMARANDACHE CONCATENATED POWER DECIMALS AND THEIR IRRATIONALITY

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Abstract

In this paper we prove that all Smarandache concatenated k-power decimals are irrational numbers.

For any positive integer k, we define the Smarandache concatenated k-power decimal α_k as follows:

$$\alpha_1 = 0.1234567891011..., \ \alpha_2 = 0.149162536496481100121...$$
(1)
$$\alpha_3 = 0.18276412521634351272910001331..., \ ..., \ etc.$$

In this peper we discuss the irrationally of α_k . We prove the following result:

Theorem. For any positive integer k, α_k is an irrational number.

Proof. We nou suppose that α_k is a rational number.

Then, by [1, Theorem 135], α_k is an infinite periodical decimal such that

(2)
$$\alpha_k = 0.a_1...a_r a_{r-1}...a_{r-t}$$

were r, t are fixed integers, with $r \ge 0$ and t > 0, $a_1, ..., a_r, a_{r+1}, ..., a_{r-t}$ are integers satisfying $0 \le a_i \le 9$ (i = 1, 2, ..., r+t).

However, we see from (1) that there exist arbitrary many

continuous zeros in the expansion of α_k . Therefore, we find

from (2) that $a_{r-1} = ... = a_{r-r} = 0$. It implies that α_k is a finite decimal; a contradiction.

Thus, α_k must be an irrational number. The theorem is proved.

Finally, we pose a further question as follows:

Question. Is α_k a transcedental number for any positive integer k?

By an old result of Mahler [2], the answer of our question is positive for k=1.

References:

- G.H.Hardy and E.M.Wright, "An Introduction to the Theory of Numbers", Oxford University Press, Oxford, 1938.
- 2. K.Mahler, "Aritmetische Eigenschaften einer Klasse von Dezimalbruchen", Nederl. Akad. Wetesch. Proc., Ser.A, 40 (1937), 421-428.