

On a Concatenation Problem

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Abstract: This article has been inspired by questions asked by Charles Ashbacher in the *Journal of Recreational Mathematics*, vol. 29.2. It concerns the Smarandache Deconstructive Sequence. This sequence is a special case of a more general concatenation and sequencing procedure which is the subject of this study. Answers are given to the above questions. The properties of this kind of sequences are studied with particular emphasis on the divisibility of their terms by primes.

1. Introduction

In this article the concatenation of a and b is expressed by a_b or simply ab when there can be no misunderstanding. Multiple concatenations like $abcabcabc$ will be expressed by $3(abc)$.

We consider n different elements (or n objects) arranged (concatenated) one after the other in the following way to form:

$$A = a_1 a_2 \dots a_n.$$

Infinitely many objects A , which will be referred to as cycles, are concatenated to form the chain:

$$B = a_1 a_2 \dots a_n a_1 a_2 \dots a_n a_1 a_2 \dots a_n \dots$$

B contains identical elements which are at equidistant positions in the chain. Let's write B as

$$B = b_1 b_2 b_3 \dots b_k \dots \text{ where } b_k = b_j \text{ when } j \equiv k \pmod{n}, 1 \leq j \leq n.$$

An infinite sequence $C_1, C_2, C_3, \dots, C_k, \dots$ is formed by sequentially selecting $1, 2, 3, \dots, k, \dots$ elements from the chain B :

$$C_1 = b_1 = a_1$$

$$C_2 = b_2 b_3 = a_2 a_3$$

$$C_3 = b_4 b_5 b_6 = a_4 a_5 a_6 \text{ (if } n \leq 6, \text{ if } n=5 \text{ we would have } C_3 = a_4 a_5 a_1)$$

The number of elements from the chain B used to form the first $k-1$ terms of the sequence C is $1+2+3+\dots+k-1 = (k-1)k/2$. Hence

$$C_k = b_{\frac{(k-1)k}{2}+1} b_{\frac{(k-1)k}{2}+2} \dots b_{\frac{(k-1)k}{2}+k}$$

However, what is interesting to see is how C_k is expressed in terms of a_1, \dots, a_n . For sufficiently large values of k C_k will be composed of three parts:

The first part: $F(k) = a_u a_{u+1} \dots a_n$

The middle part: $M(k) = AA \dots A$. The number of concatenated A 's depends on k .

The last part: $L(k) = a_1 a_2 \dots a_w$

$$\text{Hence } C_k = F(k)M(k)L(k) \quad (1)$$

The number of elements used to form C_1, C_2, \dots, C_k is $(k-1)k/2$. Since the number of elements in A is finite there will be infinitely many terms C_k which have the same first element a_u . u can be determined from $\frac{(k-1)k}{2} + 1 \equiv u \pmod{n}$. There can be at most n^2 different combinations to form $F(k)$ and $L(k)$. Let C_j and C_i be two different terms for

which $F(i)=F(j)$ and $L(i)=L(j)$. They will then be separated by a number m of complete cycles of length n , i.e.

$$\frac{(j-1)j}{2} - \frac{(i-1)i}{2} = mn$$

Let's write $j=i+p$ and see if p exists so that there is a solution for p which is independent of i .

$$(i+p-1)(i+p)-(i-1)i=2mn$$

$$i^2+2ip+p^2-i-p-i^2+i=2mn$$

$$2ip+p^2-p=2mn$$

$$p^2+p(2i-1)=2mn$$

If n is odd we will put $p=n$ to obtain $n+2i-1$, or $m=(n+2i-1)/2$. If n is even we put $p=2n$ to obtain $m=2n+2i-1$. From this we see that the terms C_k have a peculiar periodic behaviour. The periodicity is $p=n$ for odd n and $p=2n$ for even n . Let's illustrate this for $n=4$ and $n=5$ for which the periodicity will be $p=8$ and $p=5$ respectively.

Table 1. $n=4$. $A=abcd$. $B=abcd\ abcd\ abcd\ abcd\dots$

i	C_i	Period #	$F(i)$	$M(i)$	$L(i)$
1	a		a		
2	bc		bc		
3	dab	1	d		ab
4	cdab	1	cd		ab
5	cdabc	1	cd		abc
6	dabcda	1	d	abcd	a
7	bcdabcd	1	bcd	abcd	
8	abcdabcd	1		2(abcd)	
9	abcdabcda	1		2(abcd)	a
10	bcdabcdabc	1	bcd	abcd	abc
11	dabcdabcdab	2	d	2(abcd)	ab
12	cdabcdabcdab	2	cd	2(abcd)	ab
13	cdabcdabcdabc	2	cd	2(abcd)	abc
14	dabcdabcdabcda	2	d	3(abcd)	a
15	bcdabcdabcdabcd	2	bcd	3(abcd)	
16	abcdabcdabcdabcd	2		4(abcd)	
17	abcdabcdabcdabcda	2		4(abcd)	a
18	bcdabcdabcdabcdabc	2	bcd	3(abcd)	abc
19	dabcdabcdabcdabcdab	3	d	4(abcd)	ab
20	cdabcdabcdabcdabcdab	3	cdcd	4(abcd)	ab

Note that the periodicity starts for $i=3$.

Numerals are chosen as elements to illustrate the case $n=5$. Let's write $i=s+k+pj$, where s is the index of the term preceding the first periodical term, $k=1,2,\dots$, p is the index of members of the period and j is the number of the period (for convenience the first period is numbered 0). The first part of C_i is denoted $B(k)$ and the last part $E(k)$. C_i is now given by the expression below where q is the number of cycles concatenated between the first part $B(k)$ and the last part $E(k)$.

$$C_i=B(k)_qA_E(k), \text{ where } k \text{ is determined from } i-s \equiv k \pmod{p} \quad (2)$$

Table 2. $n=5$. $A=12345$. $B=123451234512345\dots$

i	C_i	k	q	$F(i) \leftrightarrow B(k)$	$M(i)$	$L(i) \leftrightarrow E(k)$
1	1			1		
$s=2$	23			2		
$j=0$						
3	451	1	0	45		1
4	2345	2	0	2345		
5	12345	3	1		12345	
6	123451	4	1		12345	1
7	2345123	5	0	2345		123
$j=1$						
$3+5j$	45123451	1	j	45	12345	1
$4+5j$	234512345	2	j	2345	12345	
$5+5j$	1234512345	3	$j+1$		2(12345)	
$6+5j$	12345123451	4	$j+1$		2(12345)	1
$7+5j$	234512345123	5	j	2345	12345	123
$j=2$						
$3+5j$	4512345123451	1	j	45	2(12345)	1
$4+5j$	23451234512345	2	j	2345	2(12345)	
....						

2. The Smarandache Deconstructive Sequence

The Smarandache Deconstructive Sequence of integers [1] is constructed by sequentially repeating the digits 1 to 9 in the following way

1,23,456,7891,23456,789123,4567891,23456789,123456789,1234567891,...

The sequence was studied in a booklet by Kashihara [2] and a number of questions on this sequence were posed by Ashbacher [3]. In thinking about these questions two observations lead to this study.

1. Why did Smarandache exclude 0 from the integers used to create the sequence?
After all 0 is indispensable in all arithmetics most of which can be done using 0 and 1 only.
2. The process used to create the Deconstructive Sequence is a process which applies to any set of objects as has been shown in the introduction.

The periodicity and the general expression for terms in the "generalized deconstructive sequence" shown in the introduction may be the most important results of this study. These results will now be used to examine the questions raised by Ashbacher. It is worth noting that these divisibility questions are dealt with in base 10 although only nine digits 1,2,3,4,5,6,7,8,9 are used to express terms in the sequence. In the last part of this article questions on divisibility will be posed for a deconstructive sequence generated from $A="0123456789"$.

For $i > 5$ ($s=5$) any term C_i in the sequence is composed by concatenating a first part $B(k)$, a number q of cycles $A="123456789"$ and a last part $E(k)$, where $i=5+k+9j$, $k=1,2, \dots, 9$, $j \geq 0$, as expressed in (2) and $q=j$ or $j+1$ as shown in table 3.

Members of the Smarandache Deconstructive Sequence are now interpreted as decimal integers. The factorization of B(k) and E(k) is shown in table 3. The last two columns of this table will be useful later in this article.

Table 3. Factorization of Smarandache Deconstructive Sequence
 $i=5+k+9j$

i	k	B(k)	q	E(k)	Digit sum	$3 \mid C_i$?
6+9j	1	789=3·263	j	123=3·41	30+j·45	3
7+9j	2	456789=3·43·3541	j	1	40+j·45	No
8+9j	3	23456789	j		44+j·45	No
9+9j	4		j+1		(j+1)·45	$9 \cdot 3^z$ *
10+9j	5		j+1	1	1+(j+1)·45	No
11+9j	6	23456789	j	123=3·41	50+j·45	No
12+9j	7	456789=3·43·3541	j	123456=2 ⁶ ·3·643	60+j·45	3
13+9j	8	789=3·263	j+1	1	25+(j+1)·45	No
14+9j	9	23456789	j	123456=2 ⁶ ·3·643	65+j·45	No

*) where z depends on j.

Together with the factorization of the cycle $A=1223456789=3^2 \cdot 3607 \cdot 3803$ it is now possible to study some divisibility properties of the sequence. We will first find a general expression for C_i in terms of j and k. For this purpose we introduce:

$$\begin{aligned} q(k) &= 0 \text{ for } k=1,2,3,6,7,9 \text{ and } q(k)=1 \text{ for } k=4,5,8 \\ u(k) &= 1 + [\log_{10}(E(k))] \text{ if } E(k) \text{ exists otherwise } u(k)=0, \text{ i.e. } u(3)=u(4)=0 \\ \delta(j,k) &= 0 \text{ if } j=0 \text{ and } q(k)=0 \text{ otherwise } \delta(j,k)=1 \end{aligned}$$

With the help of these functions we can now use table 3 to formulate the general expression

$$C_{5+k+9j} = E(k) + \delta(j,k) \cdot A \cdot 10^{u(k)} \cdot \sum_{r=0}^{j-1+q(k)} 10^{9r} + B(k) \cdot 10^{9(j+q(k))+u(k)} \quad (3)$$

Before dealing with the questions posed by Ashbacher we recall the familiar rules: An even number is divisible by 2; a number whose last two digits form a number which is divisible by 4 is divisible by 4. In general we have the following:

Theorem. Let N be an n-digit integer such that $N > 2^\alpha$ then N is divisible by 2^α if and only if the number formed by the α last digits of N is divisible by 2^α .

Proof. To begin with we note that

- If x divides a and x divides b then x divides (a+b).
- If x divides one but not the other of a and b the x does not divide (a+b).
- If neither a nor b is divisible by x then x may or may not divide (a+b).

Let's write the n-digit number in the form $a \cdot 10^\alpha + b$. We then see from the following that $a \cdot 10^\alpha$ is divisible by 2^α .

$$\begin{aligned} 10 &\equiv 0 \pmod{2} \\ 100 &\equiv 0 \pmod{4} \\ 1000 &= 2^3 \cdot 5^3 \equiv 0 \pmod{2^3} \end{aligned}$$

$$\dots$$

$$10^\alpha \equiv 0 \pmod{2^\alpha}$$

and then

$$a \cdot 10^\alpha \equiv 0 \pmod{2^\alpha} \text{ independent of } a.$$

Now let b be the number formed by the α last digits of N , we then see from the introductory remark that N is divisible by 2^α if and only if the number formed by the α last digits is divisible by 2^α .

Question 1. Does every even element of the Smarandache Deconstructive Sequence contain at least three instances of the prime 2 as a factor?

Question 2. If we form a sequence from the elements of the Smarandache Deconstructive Sequence that end in a 6, do the powers of 2 that divide them form a monotonically increasing sequence?

These two questions are related and are dealt with together. From the previous analysis we know that all even elements of the Smarandache Deconstructive end in a 6. For $i \leq 5$ they are:

$$C_3 = 456 = 57 \cdot 2^3$$

$$C_5 = 23456 = 733 \cdot 2^5$$

For $i > 5$ they are of the forms:

$$C_{12+9j} \text{ and } C_{14+9j} \text{ which both end in } \dots 789123456.$$

Examining the numbers formed by the 6, 7 and 8 last digits for divisibility by 2^6 , 2^7 and 2^8 respectively we have:

$$123456 = 2^6 \cdot 3 \cdot 643$$

$$9123456 = 2^7 \cdot 149 \cdot 4673$$

$$89123456 \text{ is not divisible by } 2^8$$

From this we conclude that all even Smarandache Deconstructive Sequence elements for $i \geq 12$ are divisible by 2^7 and that no elements in the sequence are divisible by higher powers of 2 than 7.

Answer to Qn 1. Yes

Answer to Qn 2. The sequence is monotonically increasing for $i \leq 12$. For $i \geq 12$ the powers of 2 that divide even elements remain constant = 2^7 .

Question 3. Let x be the largest integer such that $3^x | i$ and y the largest integer such that $3^y | C_i$. It is true that x is always equal to y ?

From table 3 we see that the only elements C_i of the Smarandache Deconstructive Sequence which are divisible by powers of 3 correspond to $i = 6 + 9j$, $9 + 9j$ or $12 + 9j$. Furthermore, we see that $i = 6 + 9j$ and C_{6+9j} are divisible by 3, no more no less. The same is true for $i = 12 + 9j$ and C_{12+9j} . So the statement holds in these cases. From the congruences

$$9 + 9j \equiv 0 \pmod{3^x} \text{ for the index of the element}$$

and

$$45(1+j) \equiv 0 \pmod{3^y} \text{ for the corresponding element}$$

we conclude that $x = y$.

Answer: The statement is true. It is interesting to note that, for example, the 729 digit number C_{729} is divisible by 729.

Question 4. Are there other patterns of divisibility in this sequence?

A search for patterns would continue by examining divisibility by the next lower primes 5, 7, 11, ... It is obvious from table 3 and the periodicity of the sequence that there are no elements divisible by 5. Algorithm (3) will prove useful. For each value of k the value of C_i depends on j only. The divisibility by a prime p is therefore determined by finding out for which values of j and k the congruence $C_i \equiv 0 \pmod{p}$

holds. We evaluate $\sum_{r=0}^{j-1+q(k)} 10^{9r} = \frac{10^{9(j+q(k))} - 1}{10^9 - 1}$ and introduce $G=10^9-1$. We note that $G=3^4 \cdot 37 \cdot 333667$. From (3) we now obtain:

$$G \cdot C_i = G \cdot E(k) + (\delta(j,k) \cdot A \cdot + G \cdot B(k)) 10^{9(j+q(k))+u(k)} - \delta(j,k) \cdot A \cdot 10^{9k} \quad (3')$$

The divisibility of C_i by a prime p other than 3, 37 and 333667 is therefore determined by solutions for j to the congruences $G \cdot C_i \equiv 0 \pmod{p}$ which are of the form

$$a \cdot (10^9)^j + b \equiv 0 \pmod{p} \quad (4)$$

Table 4 shows the results from computer implementation of the congruences $G \cdot C_i \equiv 0 \pmod{p}$ for $k=1,2,\dots,9$ and $p < 100$. The appearance of elements divisible by a prime p is periodic, the periodicity is given by $j=j_1+m \cdot d$, $m=1,2,3, \dots$. The first element divisible by p appears for i_1 corresponding to j_1 . In general the terms C_i divisible by p are $C_{j+k+9(j_1+md)}$ where d is specific to the prime p and $m=1,2,3, \dots$. We note from table 4 that d is either equal to $p-1$ or a divisor of $p-1$ except for the case $p=37$ which as we have noted is a factor of A . Indeed this periodicity follows from Euler's extension of Fermat's little theorem because we can write \pmod{p} :

$$a \cdot (10^9)^j + b = a \cdot (10^9)^{j+md} + b = a \cdot (10^9)^j + b \text{ for } d=p-1 \text{ or a divisor of } p-1.$$

Finally we note that the periodicity for $p=37$ is $d=37$, which is found by examining (3') modulus 37^2 .

Table 4. Smarandache Deconstructive elements divisible by p.

p	k	i _i	j _i	d	p	k	i _i	j _i	d
7	4	18	1	2	47	1	150	16	46
11	4	18	1	2	47	2	250	27	46
13	4	18	1	2	47	3	368	40	46
13	8	22	1	2	47	4	414	45	46
13	9	14	0	2	47	5	46	4	46
17	1	6	0	16	47	6	164	17	46
17	2	43	4	16	47	7	264	28	46
17	3	44	4	16	47	8	400	43	46
17	4	144	15	16	47	9	14	0	46
17	5	100	10	16	53	1	24	2	13
17	6	101	10	16	53	4	117	12	13
17	7	138	14	16	53	7	93	9	13
17	8	49	4	16	59	1	267	29	58
17	9	95	9	16	59	2	511	56	58
19	1	15	1	2	59	3	413	45	58
19	4	18	1	2	59	4	522	57	58
19	7	21	1	2	59	5	109	11	58
23	1	186	20	22	59	6	11	0	58
23	2	196	21	22	59	7	255	27	58
23	3	80	8	22	59	8	256	27	58
23	4	198	21	22	59	9	266	28	58
23	5	118	12	22	61	2	79	8	20
23	6	200	21	22	61	4	180	19	20
23	7	12	0	22	61	6	101	10	20
23	8	184	19	22	67	4	99	10	11
23	9	14	0	22	67	8	67	6	11
29	1	24	2	28	67	9	32	2	11
29	2	115	12	28	71	1	114	12	35
29	3	197	21	28	71	3	53	5	35
29	4	252	27	28	71	4	315	34	35
29	5	55	5	28	71	5	262	28	35
29	6	137	14	28	71	7	201	21	35
29	7	228	24	28	73	4	72	7	8
29	8	139	14	28	79	4	117	12	13
29	9	113	11	28	83	1	348	38	41
31	3	26	2	5	83	2	133	14	41
31	4	45	4	5	83	4	369	40	41
31	5	19	1	5	83	6	236	25	41
37	1	222	24	37	83	7	21	1	41
37	2	124	13	37	83	8	112	11	41
37	3	98	10	37	83	9	257	27	41
37	4	333	36	37	89	2	97	10	44
37	5	235	25	37	89	4	396	43	44
37	6	209	22	37	89	6	299	32	44
37	7	111	11	37	97	1	87	9	32
37	8	13	0	37	97	2	115	12	32
37	9	320	34	37	97	3	107	11	32
41	4	45	4	5	97	4	288	31	32
43	1	33	3	7	97	5	181	19	32
43	4	63	6	7	97	6	173	18	32
43	7	30	2	7	97	7	201	21	32
					97	8	202	21	32
					97	9	86	8	32

Question: Table 4 indicates some interesting patterns. For instance, the primes 19, 43 and 53 only divides elements corresponding to $k=1, 4$ and 7 for $j < 250$ which was set as an upper limit for this study. Similarly, the primes 7, 11, 41, 73 and 79 only divides elements corresponding to $k=4$. Is 5 the only prime that cannot divide an element of the Smarandache Deconstructive Sequence?

3. A Deconstructive Sequence generated by the cycle A=0123456789

Instead of sequentially repeating the digits 1-9 as in the case of the Smarandache Deconstructive Sequence we will use the digits 0-9 to form the corresponding sequence:

0,12,345,6789,01234,567890,1234567,89012345,678901234,5678901234,56789012345,678901234567, ...

In this case the cycle has $n=10$ elements. As we have seen in the introduction the sequence then has a period $2n=20$. The periodicity starts for $i=8$. Table 5 shows how, for $i > 7$, any term C_i in the sequence is composed by concatenating a first part $B(k)$, a number q of cycles $A="0123456789"$ and a last part $E(k)$, where $i=7+k+20j$, $k=1,2, \dots, 20$, $j \geq 0$, as expressed in (2) and $q=2j$, $2j+1$ or $2j+2$. In the analysis of the sequence it is important to distinguish between the cases where $E(k)=0$, $k=6,11,14,19$ and cases where $E(k)$ does not exist, i.e. $k=8,12,13,14$. In order to cope with this problem we introduce a function $u(k)$ which will at the same time replace the functions $\delta(j,k)$ and $u=1+[\log_{10}E(k)]$ used previously. $u(k)$ is defined as shown in table 5. It is now possible to express C_i in a single formula.

$$C_i = C_{7+k+20j} = E(k) + A \cdot \sum_{r=0}^{q(k)+2j-1} (10^{10})^r + B(k) \cdot (10^{10})^{q(k)+2j} 10^{u(k)} \quad (5)$$

The formula for C_i was implemented modulus prime numbers less than 100. The result is shown in table 6. Again we note that the divisibility by a prime p is periodic with a period d which is equal to $p-1$ or a divisor of $p-1$, except for $p=11$ and $p=41$ which are factors of $10^{10}-1$. The cases $p=3$ and 5 have very simple answers and are not included in table 6.

Table 5. $n=10$, $A=0123456789$

i	k	B(k)	q	E(k)	u(k)
8+20j	1	89	2j	012345=3·5·823	6
9+20j	2	6789=3·31·73	2j	01234=2·617	5
10+20j	3	56789=109·521	2j	01234=2·617	5
11+20j	4	56789=109·521	2j	012345=3·5·823	6
12+20j	5	6789=3·31·73	2j	01234567=127·9721	8
13+20j	6	89	2j+1	0	1
14+20j	7	123456789=3 ² ·3607·3803	2j	01234=2·617	5
15+20j	8	56789=109·521	2j+1		0
16+20j	9		2j+1	012345=3·5·823	6
17+20j	10	6789=3·31·73	2j+1	012=2 ² ·3	3
18+20j	11	3456789=3·7·97·1697	2j+1	0	1
19+20j	12	123456789=3 ² ·3607·3803	2j+1		0
20+20j	13		2j+2		0
21+20j	14		2j+2	0	1
22+20j	15	123456789=3 ² ·3607·3803	2j+1	012=2 ² ·3	3
23+20j	16	3456789=3·7·97·1697	2j+1	012345=3·5·823	6
24+20j	17	6789=3·31·73	2j+2		0
25+20j	18		2j+2	01234=2·617	5
26+20j	19	56789=109·521	2j+2	0	1
27+20j	20	123456789=3 ² ·3607·3803	2j+1	01234567=127·9721	8

Table 6. Divisibility of the 10-cycle deconstructive sequence by primes $p \leq 97$

p	k	i ₁	j ₁	d	p	k	i ₁	j ₁	d
7	3	30	1	3	11	11	18	0	11
7	6	13	0	3	11	12	219	10	11
7	7	14	0	3	11	13	220	10	11
7	8	15	0	3	11	14	221	10	11
7	11	38	1	3	11	15	202	9	11
7	12	59	2	3	11	16	83	3	11
7	13	60	2	3	11	17	44	1	11
7	14	61	2	3	11	18	185	8	11
7	15	22	0	3	11	19	146	6	11
7	18	45	1	3	11	20	87	3	11
7	19	46	1	3	13	2	49	2	3
7	20	47	1	3	13	3	30	1	3
11	1	88	4	11	13	4	11	0	3
11	2	9	0	11	13	12	59	2	3
11	3	110	5	11	13	13	60	2	3
11	4	211	10	11	13	14	61	2	3
11	5	132	6	11	17	1	48	2	4
11	6	133	6	11	17	5	32	1	4
11	7	74	3	11	17	10	37	1	4
11	8	35	1	11	17	12	79	3	4
11	9	176	8	11	17	13	80	3	4
11	10	137	6	11	17	14	81	3	4

Table 6, cont. Divisibility of the 10-cycle deconstructive sequence by primes $p \leq 97$

p	k	i_1	j_1	d	p	k	i_1	j_1	d
17	16	43	1	4	41	11	678	33	41
19	1	128	6	9	41	12	819	40	41
19	2	149	7	9	41	13	820	40	41
19	3	90	4	9	41	14	821	40	41
19	4	31	1	9	41	15	142	6	41
19	5	52	2	9	41	16	703	34	41
19	10	117	5	9	41	17	384	18	41
19	12	179	8	9	41	18	205	9	41
19	13	180	8	9	41	19	206	9	41
19	14	181	8	9	41	20	467	22	41
19	16	63	2	9	43	2	109	5	21
23	1	168	8	11	43	3	210	10	21
23	2	149	7	11	43	4	311	15	21
23	3	110	5	11	43	6	173	8	21
23	4	71	3	11	43	10	217	10	21
23	5	52	2	11	43	12	419	20	21
23	10	217	10	11	43	13	420	20	21
23	12	219	10	11	43	14	421	20	21
23	13	220	10	11	43	16	203	9	21
23	14	221	10	11	43	20	247	11	21
23	16	223	10	11	47	1	28	1	23
29	2	129	6	7	47	2	69	3	23
29	4	11	0	7	47	3	230	11	23
29	10	97	4	7	47	4	391	19	23
29	12	139	6	7	47	5	432	21	23
29	13	140	6	7	47	6	113	5	23
29	14	141	6	7	47	7	214	10	23
29	16	43	1	7	47	8	15	0	23
31	3	30	1	3	47	9	376	18	23
31	9	56	2	3	47	12	459	22	23
31	12	59	2	3	47	13	460	22	23
31	13	60	2	3	47	14	461	22	23
31	14	61	2	3	47	17	84	3	23
31	17	64	2	3	47	18	445	21	23
37	2	9	0	3	47	19	246	11	23
37	3	30	1	3	47	20	347	16	23
37	4	51	2	3	53	3	130	6	13
37	12	59	2	3	53	12	259	12	13
37	13	60	2	3	53	13	260	12	13
37	14	61	2	3	53	14	261	12	13
41	1	788	39	41	59	2	269	13	29
41	2	589	29	41	59	3	290	14	29
41	3	410	20	41	59	4	311	15	29
41	4	231	11	41	59	7	474	23	29
41	5	32	1	41	59	8	395	19	29
41	6	353	17	41	59	9	496	24	29
41	7	614	30	41	59	10	297	14	29
41	8	615	30	41	59	11	78	3	29
41	9	436	21	41	59	12	579	28	29
41	10	117	5	41	59	13	580	28	29

Table 6, cont. Divisibility of the 10-cycle deconstructive sequence by primes $p \leq 97$

p	k	i_1	j_1	d	p	k	i_1	j_1	d
59	14	581	28	29	71	8	95	4	7
59	15	502	24	29	71	12	139	6	7
59	16	283	13	29	71	13	140	6	7
59	17	84	3	29	71	14	141	6	7
59	18	185	8	29	71	18	45	1	7
59	19	106	4	29	71	19	26	0	7
61	12	59	2	3	73	7	14	0	2
61	13	60	2	3	73	9	36	1	2
61	14	61	2	3	73	12	39	1	2
67	1	328	16	33	73	13	40	1	2
67	2	509	25	33	73	14	41	1	2
67	3	330	16	33	73	17	44	1	2
67	4	151	7	33	73	19	26	0	2
67	5	332	16	33	79	1	228	11	13
67	6	273	13	33	79	3	130	6	13
67	7	234	11	33	79	5	32	1	13
67	8	95	4	33	79	12	259	12	13
67	9	56	2	33	79	13	260	12	13
67	10	557	27	33	79	14	261	12	13
67	11	378	18	33	83	3	410	20	41
67	12	659	32	33	83	9	476	23	41
67	13	660	32	33	83	12	819	40	41
67	14	661	32	33	83	13	820	40	41
67	15	282	13	33	83	14	821	40	41
67	16	103	4	33	83	17	344	16	41
67	17	604	29	33	89	12	219	10	11
67	18	565	27	33	89	13	220	10	11
67	19	426	20	33	89	14	221	10	11
67	20	387	18	33	97	8	455	22	24
71	1	8	0	7	97	12	479	23	24
71	3	70	3	7	97	13	480	23	24
71	5	132	6	7	97	14	481	23	24
71	7	114	5	7	97	18	25	0	24

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