D - Form of SMARANDACHE GROUPOID

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Abstract :

The set of p different equivalence classes is $\mathbf{Z}p = \{ [0], [1], [2], ------ [k] ----- [p-1] \}$. For convenience, we have omitted the brackets and written k in place of [k]. Thus

$$\mathbf{Z}p = \{ 0, 1, 2, ------ k ------ p-1 \}$$

The elements of $\mathbf{Z}p$ can be written uniquely as m - adic numbers. If $r = (a_{n-1}a_{n-2} ----- a_1a_0)_m$ and $s = (b_{n-1}b_{n-2} ------ b_1b_0)_m$ be any two elements of $\mathbf{Z}p$, then $r\Delta s$ is defined as $(|a_{n-1} - b_{n-1}|\, |a_{n-2} - b_{n-2}| ------ |a_1 - b_1|\, |a_0 - b_0|)_m$ then $(\mathbf{Z}p, \Delta)$ is a groupoid known as SMARANDACHE GROUPOID. If we define a binary relation $r \equiv s \Leftrightarrow r \Delta C(r) = s \Delta C(s)$, where C(r) and C(s) are the complements of r and s respectively, then we see that this relation is equivalence relation and partitions $\mathbf{Z}p$ into some equivalence classes. The equivalence class

$D_{sup(Zp)} = \{r \in \mathbf{Z}p : r \Delta C(r) = Sup(\mathbf{Z}p)\}$ is defined as D - form. Properties of SMARANDACHE GROUPOID and D - form are discussed here.

Key Words : SMARANDACHE GROUPOID, complement element and D - form.

1. Introduction :

Let m be a positive integer greater than one. Then every positive integer r can be written uniquely in the form $r = a_{n-1}m^{n-1} + a_{n-2}m^{n-2} + ---- + a_1m + a_0$ where $n \geq 0$, a_i is an integer, $0 \leq a_i < m$, m is called the base of r, which is denoted by $(a_{n-1}a_{n-2} ---- a_1a_0)_m$. The absolute difference of two integers $r = (a_{n-1}a_{n-2} ----- a_1a_0)_m$ and $s = (b_{n-1}b_{n-2} ----- b_1b_0)_m$ denoted by $r \Delta s$ and defined as

$$r \Delta s = (|a_{n-1} - b_{n-1}|\, |a_{n-2} - b_{n-2}| ----- |a_1 - b_1|\, |a_0 - b_0|)_m$$
$$= (c_{n-1}c_{n-2} ------- c_1c_0)_m, \quad \text{where} \quad c_i = |a_i - b_i| \quad \text{for} \quad i = 0, 1, 2 ----- n-1.$$

In this operation, $r \Delta s$ is not necessarily equal to $|r - s|$ i.e. absolute difference of r and s.

If the equivalence classes of $\mathbf{Z}p$ are expressed as m - adic numbers, then $\mathbf{Z}p$ with binary operation Δ is a groupoid, which contains some non-trivial groups. This groupoid is defined as SMARANDACHE GROUPOID. Some properties of this groupoid are established here.

2. Preliminaries :

We recall the following definitions and properties to introduce SMARANDACHE GROUPOID.

Definition 2.1 (2)

Let p be a fixed integer greater than one. If a and b are integers such that a-b is divisible by p, then a is congruent to b modulo p and indicate this by writting $a \equiv b \pmod{p}$. This congruence relation is an equivalence relation on the set of all integers.

The set of p different equivalence classes is $\mathbb{Z}p = \{ 0, 1, 2, 3, ------ p-1 \}$

Proposition 2.2 (1)

If $a \equiv b \pmod{p}$ and $c \equiv d \pmod{p}$

Then i) $a +_p c = b +_p d$

ii) $a \times_p c = b \times_p d$

Proposition 2.3 (2)

Let m be a positive integer greater than one. Then every integer r can be written uniquely in the form

$$r = a_{n-1}m^{n-1} + a_{n-2}m^{n-2} + ----- + a_1 m + a_0$$

$$= \sum_{i=0}^{n-1} a_i m^i \quad \text{for} \quad i = 0, 1, 2, -----, m-1 ;$$

Where $n \geq 0$, a_i is an integer $0 \leq a_i < m$. Here m is called the base of r, which is denoted by $(a_{n-1}a_{n-2} \cdots \cdots a_1 a_0)_m$.

Proposition 2.4

If $r = (a_{n-1}a_{n-2} \cdots \cdots a_1a_0)_m$ and $s = (b_{n-1}b_{n-2} \cdots \cdots b_1b_0)_m$, then

i) $r = s$ if and only if $a_i = b_i$ for $i = 0, 1, 2, -----, n-1$.

ii) $r < s$ if and only if $(a_{n-1}a_{n-2} \cdots \cdots a_1a_0)_m < (b_{n-1}b_{n-2} \cdots \cdots b_1b_0)_m$

iii) $r > s$ if and only if $(a_{n-1}a_{n-2} \cdots \cdots a_1a_0)_m > (b_{n-1}b_{n-2} \cdots \cdots b_1b_0)_m$

3. Smarandache groupoid :

Definition 3.1

Let $r = (a_{n-1}a_{n-2} \cdots a_i \cdots a_1a_0)_m$ and $s = (b_{n-1}b_{n-2} \cdots b_i \cdots b_1b_0)_m$, then the absolute difference denoted by Δ of r and s is defined as

$r \Delta s = (c_{n-1}c_{n-2} --- c_i --- c_1c_0)_m$, where $c_i = |a_i - b_i|$ for $i = 0, 1, 2 ----- n-1$.

Here, $r \Delta s$ is not necessarily equal to $|r - s|$. For example

$5 = (101)_2$ and $6 = (110)_2$ and $5 \Delta 6 = (011)_2 = 3$ but $|5 - 6| = 1$.

In this paper, we shall consider $5 \Delta 6 = 3$, not $5 \Delta 6 = 1$.

Definition 3.2

Let $(\mathbb{Z}p, +p)$ be a commulative group of order $p = m^n$. If the elements of $\mathbb{Z}p$ are expressed as m - adic numbers as shown below :

$$0 = (00 \quad \ldots \quad \ldots \quad 00)_m$$
$$1 = (00 \quad \ldots \quad \ldots \quad 01)_m$$
$$2 = (00 \quad \ldots \quad \ldots \quad 02)_m$$
$$\ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots$$
$$m - 1 = (00 \quad \ldots \quad \ldots \quad 0\ m\text{-}1)_m$$
$$m = (00 \quad \ldots \quad \ldots \quad 1\ 0)_m$$
$$m + 1 = (00 \quad \ldots \quad \ldots \quad 1\ 1)_m$$
$$\ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots$$
$$m^2 - 1 = (00 \quad \ldots \quad \ldots \quad m\text{-}1\ m\text{-}1)_m$$
$$m^2 = (00 \quad \ldots \quad \ldots \quad 1\ 0\ 0)_m$$
$$\ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots$$
$$m^n - 1 = (m\text{-}1\ m\text{-}1 \quad \ldots \quad \ldots \quad m\text{-}1\ m\text{-}1)_m$$

The set $\mathbf{Z}p$ is closed under binary operation Δ. Thus ($\mathbf{Z}p$, Δ) is a groupoid. The elements $(00 \quad \ldots \quad \ldots \quad 00)_m$ and $(m\text{-}1\ m\text{-}1 \quad \ldots \quad \ldots \quad m\text{-}1\ m\text{-}1)_m$ are called infimum and supremum of $\mathbf{Z}p$.

The set H_1 of the elements noted below :

$$0 = (00 \quad \ldots \quad \ldots \quad 00)_m$$
$$1 = (00 \quad \ldots \quad \ldots \quad 01)_m$$
$$m = (00 \quad \ldots \quad \ldots \quad 1\ 0)_m$$
$$m + 1 = (00 \quad \ldots \quad \ldots \quad 1\ 1)_m$$
$$\ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots$$

$$\frac{m^{n-1} - m}{m - 1} = (0\ 1 \quad \ldots \quad \ldots \quad 1\ 0)_m = \alpha \text{ (say)}$$

$$\frac{m^{n-1} - 1}{m - 1} = (0\ 1 \quad \ldots \quad \ldots \quad 1\ 1)_m = \beta \text{ (say)}$$

$$\frac{m^n - m}{m - 1} = (1\ 1 \quad \ldots \quad \ldots \quad 1\ 0)_m = \gamma \text{ (say)}$$

$$\frac{m^n - 1}{m - 1} = (1\ 1 \quad \ldots \quad \ldots \quad 1\ 1)_m = \delta \text{ (say)}$$

is a proper subset of $\mathbf{Z}p$.

(H_1, Δ) is a group of order 2^n and its group table is as follows :

Δ	0	1	m	m+1	...	...	α	β	γ	δ
0	0	1	m	m+1	...	...	α	β	γ	δ
1	1	0	m+1	m	...	...	β	α	δ	γ
m	m	m+1	0	1	...	...	γ	δ	α	β
m+1	m+1	m	1	0	...	...	δ	γ	β	α
....	...	...	...	...	...	...	...	...	...	...
....	...	...	...	...	...	...	...	...	...	...
α	α	β	γ	δ	...	...	0	1	m	m+1
β	β	α	δ	γ	...	...	1	0	m+1	m
γ	γ	δ	α	β	...	...	m	m+1	0	1
δ	δ	γ	β	α	...	...	m+1	m	1	0

Table - 1

Similarly the proper sub-sets

$H_2 = \{ 0, 2, 2m, 2(m+1) \quad \dots \quad \dots \quad 2\alpha, 2\beta, 2\gamma, 2\delta \}$

$H_3 = \{ 0, 3, 3m, 3(m+1) \quad \dots \quad \dots \quad 3\alpha, 3\beta, 3\gamma, 3\delta \}$

... ...

$H_{m-1} = \{ 0, m-1, m(m-1), m^2-1 \quad \dots \quad \dots \quad (m-1)\alpha, (m-1)\beta, (m-1)\gamma, (m-1)\delta \}$

are groups of order 2^n under the operation absolute difference. So the groupoid (Zp, Δ) contains mainly the groups (H_1, Δ), (H_2, Δ), (H_3, Δ) .... (H_{m-1}, Δ) and this groupoid is defined as SMARANDACHE GROUPOID . Here we use S.Gd. in place of SMARANDACHE GROUPOID.

Remarks 3.2

i) Let (**Z**p, +p) be a commutative group of order p, where $m^{n-1} < p < m^n$, then (**Z**p, Δ) is not groupoid.

For example ($\mathbf{Z}_5$, +5) is a commutative group of order 5, where $2^2 < p < 2^3$.

Here $\mathbf{Z}_5 = \{ 0, 1, 2, 3, 4 \}$ and

$0 = (0\ 0\ 0)_2$	$4 = (1\ 0\ 0)_2$
$1 = (0\ 0\ 1)_2$	$5 = (1\ 0\ 1)_2$
$2 = (0\ 1\ 0)_2$	$6 = (1\ 1\ 0)_2$
$3 = (0\ 1\ 1)_2$	$7 = (1\ 1\ 1)_2$

A composition table of $\mathbb{Z}_5$ is given below :

Δ	0	1	2	3	4
0	0	1	2	3	4
1	1	0	3	2	5
2	2	3	0	1	6
3	3	2	1	0	7
4	4	5	6	7	0

Table - 2

Hence $\mathbb{Z}_5$ is not closed under the operation Δ, i.e. $(\mathbb{Z}_5, \Delta)$ is not a groupoid.

ii) S. Gd. is not necessarily associative.

Let $1 = (00 \; \ldots \; \ldots \; 01)_m$

$2 = (00 \; \ldots \; \ldots \; 02)_m$ and

$3 = (00 \; \ldots \; \ldots \; 03)_m$ be three elements of $(\mathbb{Z}p, \Delta)$, then

$(1 \Delta 2) \Delta 3 = 2$ and

$1 \Delta (2 \Delta 3) = 0$

i.e. $(1 \Delta 2) \Delta 3 \neq 1 \Delta (2 \Delta 3)$.

iii) S. Gd. is commutative.

iv) S. Gd. has identity element $0 = (00 \; \ldots \; \ldots \; 0)_m$

v) Each element of S. Gd. is self inverse i.e. $\forall \, p \in \mathbb{Z}p, \quad p \Delta p = 0$.

Proposition 3.3

If (H, Δ) and (K, Δ) be two groups of order 2^n contained in S. Gd. $(\mathbb{Z}p, \Delta)$, then H is isomorphic to K.

Proof is obvious.

4. Complement element in S. Gd. $(\mathbb{Z}p, \Delta)$.

Definition 4.1

Let $(\mathbb{Z}p, \Delta)$ be a S. Gd., then the complement of any element $p \in \mathbb{Z}p$ is equal to $p \Delta \mathrm{Sup}(\mathbb{Z}p) = p \Delta m^n\text{-}1$ i.e. $C(p) = m^n\text{-}1 \Delta p$. This function is known as complement function and it satisfies the following properties.

i) $C(0) = m^n - 1$

ii) $C(m^n - 1) = 0$

ii) $C(C(p)) = p \quad \forall \quad p \in \mathbb{Z}p$

iv) If $p \leq q$ then $C(p) \geq C(q)$.

Definition 4.2

An element p of a S. Gd. $\mathbb{Z}p$ is said to be self complement if $p \Delta \sup(\mathbb{Z}p) = p$ i.e. $C(p) = p$.

If m is an odd integer greater than one, then $\frac{m^n - 1}{2}$ is the self complement of $(\mathbb{Z}p, \Delta)$.

If m is an even integer, then there exists no self complement in $(\mathbb{Z}p, \Delta)$.

Remarks 4.3

i) The complement of an element belonging to a S. Gd. is unique.

ii) The S. Gd. is closed under complement operation.

5. A binary relation in S. Gd.

Definition 5.1

Let $(\mathbb{Z}p, \Delta)$ be a S. Gd. An element p of $\mathbb{Z}p$ is said to be related to $q \in \mathbb{Z}p$ iff $p \Delta C(p) = q \Delta C(q)$ and written as $p \equiv q \Leftrightarrow p \Delta C(p) = q \Delta C(q)$.

Proposition 5.2

For the elements p and q of S. Gd. $(\mathbb{Z}p, \Delta)$, $p \equiv q \Leftrightarrow C(p) \equiv C(q)$.

Proof : By definition

$$
\begin{aligned}
p \equiv q &\Leftrightarrow p \Delta C(p) = q \Delta C(q). \\
&\Leftrightarrow C(p) \Delta p = C(q) \Delta q \\
&\Leftrightarrow C(p) \Delta C(C(p)) = C(q) \Delta C(C(q)) \\
&\Leftrightarrow C(p) \equiv C(q)
\end{aligned}
$$

Proposition 5.3

Let $(\mathbb{Z}p, \Delta)$ be a S. Gd., then a binary relation $p \equiv q \Leftrightarrow p \Delta C(p) = q \Delta C(q)$ for $p, q \in \mathbb{Z}p$, is an equivalence relation.

Proof : Let $(\mathbb{Z}p, \Delta)$ be a S. Gd. and for any two elements p and q of $\mathbb{Z}p$, let us define a binary relation $p \equiv q \Leftrightarrow p \Delta C(p) = q \Delta C(q)$.

This relation is

i) reflexive for if p is an arbitrary element of $\mathbb{Z}p$, we get $p \Delta C(p) = p \Delta C(p)$ for all $p \in \mathbb{Z}p$. Hence $p \equiv p \Leftrightarrow p \Delta C(p) = p \Delta C(p) \quad \forall \quad p \in \mathbb{Z}p$.

ii) Symmetric, for if p and q are any elements of $\mathbb{Z}p$ such that $p \equiv q$, then

$$
\begin{aligned}
p \equiv q &\Leftrightarrow p \Delta C(p) = q \Delta C(q) \\
&\Leftrightarrow q \Delta C(q) = p \Delta C(p) \\
&\Leftrightarrow q \equiv p
\end{aligned}
$$

iii) transitive, for p, q, r are any three elements of $\mathbb{Z}p$ such that

$p \equiv q$ and $q \equiv r$, then

$p \equiv q \Leftrightarrow p \Delta C(p) = q \Delta C(q)$ and

$q \equiv r \Leftrightarrow q \Delta C(q) = r \Delta C(r)$.

Thus $p \Delta C(p) = r \Delta C(r) \Leftrightarrow p \equiv r$

Hence $p \equiv q$ and $q \equiv r$ implies $p \equiv r$

6. D - Form of S. Gd.

Let $(\mathbb{Z}p, \Delta)$ be a S. Gd. of order m^n. Then the equivalence relation referred in the proposition 5.3 partitions $\mathbb{Z}p$ into mutually disjoint classes.

Definition 6.1

If r be any element of S. Gd. $(\mathbb{Z}p, \Delta)$ such that $r \Delta C(r) = x$, then the equivalence class generated by x is denoted by Dx and defined by

$$Dx = \{ r \in \mathbb{Z}p : r \Delta C(r) = x \}$$

The equivalence class generated by $\sup(\mathbb{Z}p)$ and defined by

$D_{\sup(\mathbb{Z}p)} = \{ r \in \mathbb{Z}p : r \Delta C(r) = \sup(\mathbb{Z}p) \}$ is called the D - form of $(\mathbb{Z}p, \Delta)$.

Example 6.2

Let $(\mathbb{Z}_9, +9)$ be a commutative group, then $\mathbb{Z}_9 = \{ 0, 1, 2, 3, 4, 5, 6, 7, 8 \}$. If the elements of $\mathbb{Z}_9$ are written as 3-adic numbers, then

$\mathbb{Z}_9 = \{(00)_3, (01)_3, (02)_3, (10)_3, (11)_3, (12)_3, (20)_3, (21)_3, (22)_3\}$ and

$(\mathbb{Z}_9, \Delta)$ is a S. Gd. of order $3^2 = 9$. Its composition table is as follows :

Δ	0	1	2	3	4	5	6	7	8
0	0	1	2	3	4	5	6	7	8
1	1	0	1	4	3	4	7	6	7
2	2	1	0	5	4	3	8	7	6
3	3	4	5	0	1	2	3	4	5
4	4	3	4	1	0	1	4	3	4
5	5	4	3	2	1	0	5	4	3
6	6	7	8	3	4	5	0	1	2
7	7	6	7	4	3	4	1	0	1
8	8	7	6	5	4	3	2	1	0

Table - 3

Here $0 \Delta C(0) = 0 \Delta 8 = 8$

$1 \Delta C(1) = 1 \Delta 7 = 6$

$2 \Delta C(2) = 2 \Delta 6 = 8$

$3 \Delta C(3) = 3 \Delta 5 = 2$

$4 \Delta C(4) = 4 \Delta 4 = 0$

$5 \Delta C(5) = 5 \Delta 3 = 2$

$6 \Delta C(6) = 6 \Delta 2 = 8$

$7 \Delta C(7) = 7 \Delta 1 = 6$

$8 \Delta C(8) = 8 \Delta 0 = 8$

Hence $D_8 = \{ 0, 2, 6, 8 \} = \{(00)_3, (02)_3, (20)_3, (22)_3 \}$

$D_6 = \{ 1, 7 \}$

$D_2 = \{ 3, 5 \}$

$D_0 = \{ 4 \}$

The self complement element of $(\mathbb{Z}_9, \Delta)$ is 4 and D- form of this S. Gd. is $\{0, 2, 6, 8\} = D_8$

Here $\mathbb{Z}_9 = D_0 \cup D_2 \cup D_6 \cup D_8$.

Proposition 6.3

Any two equivalence classes in a S. Gd. $(\mathbb{Z}p, \Delta)$ are either disjoint or identical.

Proof is obvious.

Proposition 6.4

Every S. Gd. $(\mathbb{Z}p, \Delta)$ is equal to the union of its equivalence classes.

Proof is obvious.

Proposition 6.5

Every D- form of a S. Gd. $(\mathbb{Z}p, \Delta)$ is a commutative group.

Proof : Let $(\mathbb{Z}p, \Delta)$ be a S. Gd. of order $P = m^n$. The elements of D- form of this groupoid are as follows.

$0 = (00 \; \dots \; \dots \; 00)_m$

$m - 1 = (00 \; \dots \; \dots \; 0\ m\text{-}1)_m$

$m^2 - m = (00 \; \dots \; \dots \; m\text{-}1\ 0)_m$

$m^2 - 1 = (00 \; \dots \; \dots \; m\text{-}1\ m\text{-}1)_m$

… … … …

… … … …

$m^{n-1} - m = (0\ m\text{-}1 \; \dots \; \dots \; m\text{-}1\ 0)_m$

$m^{n-1} - 1 = (0\ m\text{-}1 \; \dots \; \dots \; m\text{-}1\ m\text{-}1)_m$

$m^n - m = (m\text{-}1\ m\text{-}1 \; \dots \; \dots \; m\text{-}1\ 0)_m$

$m^n - 1 = (m\text{-}1\ m\text{-}1 \; \dots \; \dots \; m\text{-}1\ m\text{-}1)_m$

$\therefore D_{m^n-1} = \{ 0, m-1, m^2-m, m^2-1, \text{- - - - - - -}, m^{n-1}-m, m^{n-1}-1, m^n-m, m^n-1 \}$

Here (D_{m^n-1}, Δ) is a commutative group and its table is given below :

Δ	0	$m-1$	m^2-m	m^2-1	...	$m^{n-1}-m$	$m^{n-1}-1$	m^n-m	m^n-1
0	0	$m-1$	m^2-m	m^2-1	...	$m^{n-1}-m$	$m^{n-1}-1$	m^n-m	m^n-1
$m-1$	$m-1$	0	m^2-1	m^2-m	...	$m^{n-1}-1$	$m^{n-1}-m$	m^n-1	m^n-m
m^2-m	m^2-m	m^2-1	0	$m-1$	...	m^n-m	m^n-1	$m^{n-1}-m$	$m^{n-1}-1$
m^2-1	m^2-1	m^2-m	$m-1$	0	...	m^n-1	m^n-m	$m^{n-1}-1$	$m^{n-1}-m$
- - -		- - -			...	- - -		- - -	
$m^{n-1}-m$	$m^{n-1}-m$	$m^{n-1}-1$	m^n-m	m^n-1	...	0	$m-1$	m^2-m	m^2-1
$m^{n-1}-1$	$m^{n-1}-1$	$m^{n-1}-m$	m^n-1	m^n-m	...	$m-1$	0	m^2-1	m^2-m
m^n-m	m^n-m	m^n-1	$m^{n-1}-m$	$m^{n-1}-1$	...	m^2-m	m^2-1	0	$m-1$
m^n-1	m^n-1	m^n-m	$m^{n-1}-1$	$m^{n-1}-m$	...	m^2-1	m^2-m	$m-1$	0

Table - 4

Remarks 6.6

Let $(\mathbf{Z}p, \Delta)$ be a S. Gd. of order m^n.

The equivalence relation $p \equiv q \Leftrightarrow p \,\Delta\, C(p) = q \,\Delta\, C(q)$ partitions $\mathbf{Z}p$ into some equivalence classes.

i) If m is odd integer, then the number of elements belonging to the equivalence classes are not equal. In the example 6.2, the number of elements belonging to the equivalence classes D_0, D_2, D_6, D_8 are not equal due to $m = 3$.

ii) If m is even integer, then the number of elements belonging to the equivalence classes are equal.

For example, $\mathbf{Z}_{16} = \{ 0, 1, 2, \text{- - - - - - -}, 15 \}$ be a commutative group. If the elements of $\mathbf{Z}_{16}$ are expressed as 4- adic numbers, then $(\mathbf{Z}_{16}, \Delta)$ is a S. Gd. The composition table of $(\mathbf{Z}_{16}, \Delta)$ is given below :

Δ	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	0	1	2	5	4	5	6	9	8	9	10	13	12	13	14
2	2	1	0	1	6	5	4	5	10	9	8	9	14	13	12	13
3	3	2	1	0	7	6	5	4	11	10	9	8	15	14	13	12
4	4	5	6	7	0	1	2	3	4	5	6	7	8	9	10	11
5	5	4	5	6	1	0	1	2	5	4	5	6	9	8	9	10
6	6	5	4	5	2	1	0	1	10	5	6	7	8	9	10	11
7	7	6	5	4	3	2	1	0	7	6	5	4	11	10	9	8
8	8	9	10	11	4	5	6	7	0	1	2	3	4	5	6	7
9	9	8	9	10	5	4	5	6	1	0	1	2	5	4	5	6
10	10	9	8	9	6	5	4	5	2	1	0	1	6	5	4	5
11	11	10	9	8	7	6	5	4	3	2	1	0	7	6	5	4
12	12	13	14	15	8	9	10	11	4	5	6	7	0	1	2	3
13	13	12	13	14	9	8	9	10	5	4	5	6	1	0	1	2
14	14	13	12	13	10	9	8	9	6	5	4	5	2	1	0	1
15	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0

Table - 5

Here $0 \Delta C(0) = 15 = 15 \Delta C(15)$

$1 \Delta C(1) = 13 = 14 \Delta C(14)$

$2 \Delta C(2) = 13 = 13 \Delta C(13)$

$3 \Delta C(3) = 15 = 12 \Delta C(12)$

$4 \Delta C(4) = 7 = 11 \Delta C(11)$

$5 \Delta C(5) = 5 = 10 \Delta C(10)$

$6 \Delta C(6) = 5 = 9 \Delta C(9)$

$7 \Delta C(7) = 7 = 8 \Delta C(8)$

Hence $D_{15} = \{ 0, 3, 12, 15 \}$, $D_{13} = \{ 1, 2, 13, 14 \}$

$D_7 = \{ 4, 8, 7, 11 \}$, $D_5 = \{ 5, 6, 9, 10 \}$

The number of elements of the equivalence classes are equal due to m = 4, which is even integer.

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