On the Difference $S(Z(n)) - Z(S(n)$

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Abstract: In this paper, we prove that there exist infinitely many positive integers n satisfying $S(Z(n)) > Z(S(n))$ or $S(Z(n)) < Z(S(n))$.

Key words: Smarandache function, Pseudo-Smarandache function, composite function, difference.

For any positive integer n, let $S(n)$, $Z(n)$ denote the Smarandache function and the Pseudo-Smarandache function of n respectively. In this paper, we prove the following results:

Theorem 1: There exist infinitely many n satisfying $S(Z(n)) > Z(S(n))$.

Theorem 2: There exist infinitely many n satisfying $S(Z(n)) < Z(S(n))$.

The above mentioned results solve Problem 21 of [1].

Proof of Theorem 1.
Let p be an odd prime. If $n = (1/2)p(p+1)$, then we have

(1) $S(Z(n)) = S(Z((1/2)p(p+1))) = S(p) = p$

and

(2) $Z(S(n)) = Z(S((1/2)p(p+1))) = Z(p) = p-1$.

We see from (1) and (2) that $S(Z(n)) > Z(S(n))$ for any odd prime p. It is a well-known fact that there exist infinitely many odd primes p. Thus, the theorem is proved.

Proof of Theorem 2.
If $n = p$, where p is an odd prime, then we have

(3) $S(Z(n)) = S(Z(p)) = S(p-1) < p-1$

and

(4) $Z(S(n)) = Z(S(p)) = Z(p) = p-1$.

By (3) and (4), we get $S(Z(n)) < Z(S(n))$ for any p. Thus, the theorem is proved.

Reference

[1] C. Ashbacher, Problems, **Smarandache Notions Journal**, 9(1998), 144-151.

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