CONSTRUCTION OF ELEMENTS OF THE SMARANDACHE SQUARE-PARTIAL-DIGITAL SUBSEQUENCE

by Lamarr Widmer

The Smarandache Square-Partial-Digital Subsequence (SPDS) is the sequence of square integers which admit a partition for which each segment is a square integer. An example is $506^2 = 256036$ which has partition 256|0|36 . Ashbacher considers these numbers on page 44 of [1] and quickly shows that the SPDS is infinite by exhibiting two infinite "families" of elements. We will extend his results by showing how to construct infinite families of elements of SPDS containing desired patterns of digits.

Theorem 1: Let c be any concatenation of square numbers. There are infinitely many elements of SPDS which contain the sequence c.

proof: If c forms an even integer, let N = c. Otherwise, let N be c with a digit 4 added at the right. So N is an even number. Find any factorization $N = 2ab$. Consider the number

$m = a\cdot 10^n + b$ for sufficiently large n. (Sufficiently large means that $10^n > b^2$ and $10^n > N$.) Then $m^2 = a^2 10^{2n} + N\cdot 10^n + b^2 \in$ SPDS .

Q.E.D.

For example, let us construct elements of SPDS containing the string c = 2514936 . In the notation of our proof, we have ab = 1257468 and we can use a = 6 and b = 209578 (among many possibilities). This gives us the numbers

$600000209578^2 = 360000251493643922938084$

$6000000209578^2 = 36000002514936043922938084$

$60000000209578^2 = 3600000025149360043922938084$

etc., which all belong to SPDS.

This allows us to imbed any sequence of squares in the interior of an element of SPDS. What about the ends? Clearly we cannot put all such sequences at the end of an element of SPDS. No perfect square ends in the digits 99, for example. Our best result in this respect is the following.

Theorem 2: Let a be any positive integer. There are infinitely many elements of SPDS which begin and end with a^2.

proof: For large enough n (ie. $10^n > 225a^2$), consider

$$m = a\cdot 10^{2n} + \frac{a}{2} 10^n + a = a\cdot 10^{2n} + 5a\cdot 10^{n-1} + a$$

Then

$$m^2 = a^2\cdot 10^{4n} + a^2\cdot 10^{3n} + \frac{9}{4} a^2\cdot 10^{2n} + a^2\cdot 10^n + a^2$$

$$= a^2\cdot 10^{4n} + a^2\cdot 10^{3n} + (15a)^2\cdot 10^{2n-2} + a^2\cdot 10^n + a^2$$

belongs to SPDS.

Q.E.D.

We illustrate for $a = 8$. For successive values of n beginning with 5, we have the following elements of SPDS.

$80000400008^2 = 6400064001440006400064$

$8000004000008^2 = 64000064000144000064000064$

$800000040000008^2 = 640000064000014400000640000064$

etc.

We have a number of observations concerning this last result. First, an obvious debt is owed to Ashbacher's work [1], in which he gives the family $212^2 = 44944$, $20102^2 = 404090404$, ... Second, we actually have exhibited an infinite family of elements of SPDS in which a^2 appears *four* times. And finally, we note that an alternate proof can be given using $m = a \cdot 10^{2n+1} + \frac{a}{2} 10^n + a$ for which $m^2 = a^2 \cdot 10^{4n+2} + a^2 \cdot 10^{3n+1} + (45a)^2 \cdot 10^{2n-2} + a^2 \cdot 10^n + a^2$.

This concludes our results emphasizing the infinitude of SPDS. In addition we wish to note an instance of the square of an element of SPDS which also belongs to SPDS, namely $441^2 = 194481$. Can an example be found of integers m, m^2, m^4 all belonging to SPDS? It is relatively easy to find two consecutive squares in SPDS. One example is $12^2 = 144$ and $13^2 = 169$. Does SPDS also contain a sequence of three or more consecutive squares?

Reference:

[1] Charles Ashbacher, Collection of Problems On Smarandache Notions, Erhus University Press, Vail, 1996.

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