SOME ELEMENTARY ALGEBRAIC CONSIDERATIONS INSPIRED BY SMARANDACHE'S FUNCTION (II)

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In this paper we continue the algebraic consideration begun in [2]. As it was sun, two of the proprieties of Smarandache's function are hold:

(1) S is a surjective function;

(2) $S([m,n]) = \max\{S(m), S(n)\}$, where $[m,n]$ is the smallest common multiple of m and n.

That is on $\aleph$ there are considered both of the divisibility order "$\preceq_d$" having the known properties and the total order with the usual order $\leq$ with all its properties. $\aleph$ has also the algebric usual operations "+" and "$\cdot$". For instance:

$$a \leq b \Longleftrightarrow (\exists)\, u \in \aleph \text{ so that } b = a + u.$$

Here we can stand out:

: the universal algebra $(\aleph^*, \Omega)$, the set of operations is $\Omega = \{\vee_d, \varphi_0\}$ where $\vee_d : (\aleph^*)^2 \to \aleph^*$ is given by $m \vee_d n = [m,n]$, and $\varphi_0 : (\aleph^*)^0 \to \aleph^*$ the null operation that fixes 1-unique particular element with the role of neutral element for "$\vee_d$"-that means $\varphi_0(\{\emptyset\}) = 1$ and $1 = e_{\vee_d}$;

: the universal algebra $(\aleph^*, \Omega')$, the set of operations is $\Omega' = \{\vee, \psi_0\}$ where $\vee : \aleph^2 \to \aleph$ is given by $x \vee y = \sup\{x, y\}$ and $\psi_0 : \aleph^0 \to \aleph$ a null operation with $\psi_0(\{\emptyset\}) = 0$ the unique particular element with the role of neutral element for $\vee$, so $0 = e_\vee$.

We observe that the universal algebras $(\aleph^*, \Omega)$ and $(\aleph^*, \Omega')$ are of the same type:

$$\begin{pmatrix} \vee_d & \varphi_0 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} \vee & \psi_0 \\ 2 & 0 \end{pmatrix}$$

and with the similarity (bijective) $\vee_d \Longleftrightarrow \vee$ and $\varphi_0 \Longleftrightarrow \psi_0$, Smarandache's function $S : \aleph^* \to \aleph$ is a morphism surjective between them

$$\begin{aligned} S(x \vee_d y) &= S(x) \vee S(y), \forall x, y \in \aleph^* \text{ from (2) and} \\ S(\varphi_0(\{\emptyset\})) &= \psi_0(\{\emptyset\}) \Longleftrightarrow S(1) = 0. \end{aligned}$$

Problem 3. If $S : \aleph^* \to \aleph$ is Smarandache's function defined as we know by

$$S(n) = m \Longleftrightarrow m = \min\{k : n \text{ divides } k!\}$$

and I is a some set, then there exists an unique $s : (\aleph^*)^I \to \aleph^I$ a surjective morphisme between the universal algebras $\left((\aleph^*)^I, \Omega\right)$ and $\left(\aleph^I, \Omega'\right)$ so that $p_i \circ s = S \circ \tilde{p}_i$, for $i \in I$, where $p_j : \aleph^I \to \aleph$ defined by $a = \{a_i\}_{i \in I} \in \aleph^I$, $p_j(a) = a_j$, for each $j \in I$, p_j are the canonical projections, morphismes between $\left(\aleph^I, \Omega'\right)$ and $(\aleph, \Omega')$-universal algebras of the same kind and $\tilde{p}_j : (\aleph^*)^I \to \aleph^*$ analogously between $\left((\aleph^*)^I, \Omega\right)$ and $(\aleph^*, \Omega)$. We shall go over the following three steps in order to justify the assumption:

Theorem 0.1. *Let by $(\aleph, \Omega)$ is an universal algebra more complexe with*

$$\Omega = \{\vee_d, \wedge_d, \varphi_0, \overline{\varphi}_0\}$$

of the kind $\tau : \Omega \to \aleph$ given by

$$\tau = \begin{pmatrix} \vee_d & \wedge_d & \varphi_0 & \overline{\varphi}_0 \\ 2 & 2 & 0 & 0 \end{pmatrix}$$

where $\vee_d$ and φ_0 are defined as above and $\wedge_d : \aleph^2 \to \aleph$, for each $x, y \in \aleph, x \wedge_d y = (x, y)$ where (x, y) is the biggest common divisor of x and y and $\overline{\varphi}_0 : \aleph^0 \to \aleph$ is the null operation that fixes 0-an unique particular element having the role of the neutral element for "$\wedge_d$" i.e. $\overline{\varphi}_0(\{\emptyset\}) = 0$ so $0 = e_{\wedge_d}$ and I a set. Then $\left(\aleph', \tilde{\Omega}\right)$ with $\tilde{\Omega} = \{\omega_1, \omega_2, \omega_0, \overline{\omega}_0\}$ becomes an universal algebra of the same kind as $(\aleph, \Omega)$ and the canonical projections become surjective morphismes between $\left(\aleph^I, \tilde{\Omega}\right)$ and $(\aleph, \Omega)$, an universal algebra that satisfies the following property of universality:
$(\mathcal{U})$: for every $\left(A, \overline{\Omega}\right)$ with $\overline{\Omega} = \{\top, \perp, \sigma_0, \overline{\sigma}_0\}$ an universal algebra of the same kind

$$\tau = \begin{pmatrix} \top & \perp & \sigma_0 & \overline{\sigma}_0 \\ 2 & 2 & 0 & 0 \end{pmatrix}$$

and $u_i : A \to \aleph$, for each $i \in I$, morphismes between $\left(A, \overline{\Omega}\right)$ and $(\aleph, \Omega)$, exists an unique $u : A \to \aleph^I$ morphism between the universal algebras $\left(A, \overline{\Omega}\right)$ and $\left(\aleph^I, \tilde{\Omega}\right)$ so that $p_j \circ u = u_j$, for each $j \in I$, where $p_j : \aleph^I \to \aleph$ with each $a = \{a_i\}_{i \in I} \in \aleph^I, p_j(a) = a_j$, for each $j \in I$ are the canonical projections morphismes between $\left(\aleph^I, \tilde{\Omega}\right)$ and $(\aleph, \Omega)$.

Proof. Indeed $\left(\aleph^I,\tilde{\Omega}\right)$ with $\tilde{\Omega}=\{\omega_1,\omega_2,\omega_0,\overline{\omega}_0\}$ becomes an universal algebra because we can well define:

ω_1 : $(\aleph^I)^2 \to \aleph^I$ by each $a=\{a_i\}_{i\in I}, b=\{b_i\}_{i\in I}\in\aleph; \omega_1(a,b)=\{a_i \vee_d b_i\}_{i\in I}\in\aleph^I$ and

ω_2 : $(\aleph^I)^2 \to \aleph^I$ by $\omega_2(a,b)=\{a_i \wedge_d b_i\}_{i\in I}\aleph^I$ and also

ω_0 : $(\aleph^I)^0 \to \aleph^I$ with $\omega_0(\{\emptyset\})=\{e_i=1\}_{i\in I}\in\aleph^I$

an unique particular element (the family with all the components equal with 1) fixed by ω_0 and having the role of neutral for the operation ω_1 noted with e_{ω_1} and then $\overline{\omega}_0:(\aleph^I)^0\to\aleph^I$ with $\overline{\omega}_0(\{\emptyset\})=\{\overline{e}_i=0\}_{i\in I}$ an unique particular element fixed by $\overline{\omega}_0$ but hawing the role of neutral for the operation ω_2 noted $\overline{e}_{\omega_2}$ (the verifies are imediate).

The canonical projections $p_j:\aleph^I\to\aleph$, defined as above, become morphismes between $\left(\aleph^I,\tilde{\Omega}\right)$ and $(\aleph,\Omega)$. Indeed the two universal algebras are of the same kind

$$\begin{pmatrix}\omega_1 & \omega_2 & \omega_0 & \overline{\omega}_0\\ 2 & 2 & 0 & 0\end{pmatrix}=\begin{pmatrix}\vee_d & \wedge_d & \varphi_0 & \overline{\varphi}_0\\ 2 & 2 & 0 & 0\end{pmatrix}$$

and with the similairity (bijective) $\omega_1 \Longleftrightarrow \vee_d; \omega_2 \Longleftrightarrow \wedge_d; \omega_0 \Longleftrightarrow \varphi_0; \overline{\omega}_0 \Longleftrightarrow \overline{\varphi}_0$ we observe first that for each $a,b\in\aleph^I, p_j(\omega_1(a,b))=p_j(a)\vee_d p_j(b)$, for each $j\in I$ because $a=\{a_i\}_{i\in I}, b=\{b_i\}_{i\in I}, p_j(\omega_1(a,b))=p_j\left(\{a_i\vee_d b_i\}_{i\in I}\right)=a_j\vee_d b_j$ and $p_j(a)\vee_d p_j(b)=p_j(\{a_i\}_{i\in I})\vee_d p_j\left(\{b_i\}_{i\in I}\right)=a_j\vee_d b_j$ and then $p_j(\omega_0(\{\emptyset\}))=\varphi_0(\{\emptyset\})\Longleftrightarrow p_j\left(\{e_i=1\}_{i\in I}\right)=1\Longleftrightarrow p_j(e_{\omega_1})=e_{\vee_d}$; analogously we prove that p_j, for each $j\in I$ keeps the operations ω_2 and $\overline{\omega}_0$, too. So, it was built the universal algebra $\left(\aleph^I,\tilde{\Omega}\right)$ with $\tilde{\Omega}=\{\omega_1,\omega_2,\omega_0,\overline{\omega}_0\}$ of the kind τ described above.

We prove the property of universality $(\mathcal{U})$.

We observe for this purpose that the u_i morphismes for each $i\in I$, presumes the coditions: for each $x,y\in S, u_i(x\top y)=u_i(x)\vee_d u_i(y); u_i(x\bot y)=u_i(x)\wedge_d u_i(y); u_i(\sigma_0(\{\emptyset\}))=\varphi_0(\{\emptyset\})\Longleftrightarrow u_i(e_\top)=e_{\vee_d}=1$ and $u_i(\overline{\sigma}_0(\{\emptyset\}))=\overline{\varphi}_0(\{\emptyset\})\Longleftrightarrow u_i(\overline{e}_\bot)=e_{\wedge_d}=0$ which show also the similarity (bijective) between $\overline{\Omega}$ and Ω. We also observe that $(S,\overline{\Omega})$ and $\left(\aleph^I,\tilde{\Omega}\right)$ are of the same kind and there is a similarity (bijective) between $\overline{\Omega}$ and $\tilde{\Omega}$ given by $\top\Longleftrightarrow\omega_1;\bot\Longleftrightarrow\omega_2;\sigma_0\Longleftrightarrow\omega_0;\overline{\sigma}_0\Longleftrightarrow\overline{\omega}_0$.

We define the corespondance $u:A\to\aleph^I$ by $u(x)=\{u_i(x)\}_{i\in I}$.

u is the function:

- for each $x\in A, (\exists)\, u_i(x)\in\aleph$ for each $i\in I$ (u_i-functions) so $(\exists)\,\{u_i(x)\}_{i\in I}$ that can be imagines for x;

- $x_1 = x_2 \Longrightarrow u(x_1) = u(x_2)$ because $x_1 = x_2$ and u_i-functions lead to $u_i(x_1) = u_i(x_2)$ for each $i \in I \Longrightarrow \{u_i(x_1)\}_{i\in I} = \{u_i(x_2)\}_{i\in I} \Longrightarrow u(x_1) = u(x_2)$.

u is a morphisme: for each $x, y \in A$, $u(x \top y) = \{u_i(x \top y)\}_{i\in I} = \{u_i(x) \vee_d u_i(y)\}_{i\in I} = \omega_1\left(\{u_i(x)\}_{i\in I}, \{u_i(y)\}_{i\in I}\right) = \omega_1(u(x), u(y))$. Then $u(\sigma_0(\{\emptyset\})) = \omega_0(\{\emptyset\}) \Longleftrightarrow u(e_\top) = e_{\omega_1}$ because for each $\{a_i\}_{i\in I} \in \aleph^I, \omega_1(\{a_i\}_{i\in I}, \{u_i(e_\top)\}_{i\in I}) = \{a_i \vee_d u_i(e_\top)\}_{i\in I} = \{a_i \vee_d 1\}_{i\in I} = \{a_i\}_{i\in I}$.

Analogously we prove that u keeps the operations: $\perp$ and $\overline{\sigma}_0$.

Besides the condition $p_j \circ u = u_j$, for each $j \in I$ is verified (by the definition: for each $x \in S$, $(p_j \circ u)(x) = p_j(u(x)) = p_j(\{u_i(x)\}_{i\in I}) = u_j(x)$).

For the singleness of u we consider u and $\overline{u}$, two morphismes so that $p_j \circ u = u_j$ (1) and $p_j \circ \overline{u} = u_j$ (2), for every $j \in I$. Then for every $x \in A$, if $u(x) = \{u_i(x)\}_{i\in I}$ and $\overline{u}(x) = \{z_i\}_{i\in I}$ we can see that $y_j = u_j(x) = (p_j \circ \overline{u})(x) = p_j(\{z_i\}_{i\in I}) = z_j$, for every $j \in I$ i.e. $u(x) = \overline{u}(x)$, for every $x \in A \Longleftrightarrow u = \overline{u}$.

Consequence . Particularly, taking $A = \aleph^I$ and $u_i = p_i$ we obtain: the morphisme $u : \aleph^I \to \aleph^i$ verifies the condition $p_j \circ u = p_j$, for every $j \in I$, if and only if, $u = 1_{\aleph^I}$.

The property of universality establishes the universal algebra $\left(\aleph^I, \tilde{\Omega}\right)$ until an isomorphisme as it results from:

Theorem 0.2. *If (P, Ω) is an universal algebra of the same kind as $(\aleph, \Omega)$ and $p'_i : P \to \aleph$, $i \in I$ a family of morphismes between (P, Ω) and $(\aleph, \Omega)$ so that for every universal algebra $\left(A, \overline{\Omega}\right)$ and every morphisme $u_i : A \to \aleph$, for every $i \in I$ between $\left(A, \overline{\Omega}\right)$ and $(\aleph, \Omega)$ it exists an unique morphisme $u : A \to P$ with $p'_j \circ u = u_i$, for every $i \in I$, then it exists an unique isomorphisme $f : P \to \aleph^I$ with $p_i \circ f = p'_i$, for every $i \in I$.*

Proof. From the property of universality of $\left(\aleph^I, \tilde{\Omega}\right)$ it results an unique $f : P \to \aleph^I$ so that for every $i \in I, p_i \circ f = p'_i$ with f morphisme between (P, Ω) and $\left(\aleph^I, \tilde{\Omega}\right)$. Applying now the same property of universality to $(P, \Omega) \Longrightarrow$exists an unique $\overline{f} : \aleph^I \to P$ so that $p'_i \circ \overline{f} = p_i$, for every $i \in I$ with $\overline{f}$ morphisme between $\left(\aleph^I, \tilde{\Omega}\right)$ and (P, Ω). Then $p'_j \circ \overline{f} = p_j \Longleftrightarrow p_j \circ \left(f \circ \overline{f}\right) = p_j$, using the last consequence, we get $f \circ \overline{f} = 1_{\aleph^I}$. Analogously, we prove that $f \circ \overline{f} = 1_P$ from where $\overline{f} = f^{-1}$ and the morphisme f becomes isomorphisme.

We could emphasize other properties (a family of finite support or the case $I-$filter) but we remain at these which are strictly necessary to prove the proposed assertion (Problem 3).

b) Firstly it was built $\left(\aleph^I, \tilde{\Omega}\right)$ being an universal algebra more complexe (with four operations). We try now a similar construction starting from $(\aleph, \Omega^*)$ with $\Omega^* =$

$(\vee, \wedge, \psi_0)$ with "$\vee$" and "ψ_0" defined as above and $\wedge : \aleph^2 \to \aleph$ with $x \wedge y = \inf\{x, y\}$ for every $x, y \in \aleph$. ∎

Theorem 0.3. *Let by $(\aleph, \Omega^*)$ the above universal algebra and I a set. Then:*
(i) $\left(\aleph^I, \theta\right)$ with $\theta = \{\theta_1, \theta_2, \theta_0\}$ becomes an universal algebra of the same kind τ as $(\aleph, \Omega^*)$ so $\tau : \theta \to \aleph$ is

$$\tau = \begin{pmatrix} \theta_1 & \theta_2 & \theta_0 \\ 2 & 2 & 0 \end{pmatrix};$$

(ii) For every $j \in I$ the canonical projection $p_j : \aleph^I \to \aleph$ defined by every $a = \{a_i\}_{i \in I} \in \aleph^I, p_j(a) = a_j$ is a surjective morphisme between $\left(\aleph^I, \theta\right)$ and $(\aleph, \Omega^*)$ and $\ker p_j = \left\{a \in \aleph^I : a = \{a_i\}_{i \in I} \text{ and } a_j = 0\right\}$ where by definition we have $\ker p_j = \left\{a \in \aleph^I : p_j(a) = e_\vee\right\}$;
(iii) For every $j \in I$ the canonical injection $q_j : \aleph \to \aleph^I$ for every $x \in \aleph, q_j(x) = \{a_i\}_{i \in I}$ where $a_i = 0$ if $i \neq j$ and $a_j = x$ is an injective morphisme between $(\aleph, \Omega^*)$ and $\left(\aleph^I, \theta\right)$ and $q_j(\aleph) = \left\{\{a_i\}_{i \in I} : a_i = 0, \forall i \in I - \{j\}\right\}$;
(iv) If $j, k \in I$ then:

$$p_j \circ q_k = \begin{cases} O\text{-the null morphisme} & \text{for } j \neq k, \\ 1_\aleph\text{-the identical morphisme} & \text{for } j = k. \end{cases}$$

Proof. (*i*) We well define the operations $\theta_1 : \left(\aleph^I\right)^2 \to \aleph^I$ by $\forall a = \{a_i\}_{i \in I} \in \aleph^I$ and $b = \{b_i\}_{i \in I} \in \aleph^I, \theta_1(a, b) = \{a_i \vee b_i\}_{i \in I}; \theta_2 : \left(\aleph^I\right)^2 \to \aleph^I$ by $\theta_2(a, b) = \{a_i \wedge b_i\}_{i \in I}$ and $\theta_0 : \left(\aleph^I\right)^0 \to \aleph^I$ by $\theta_0(\{\emptyset\}) = \{e_i = 0\}_{i \in I}$ an unique particular element fixed by θ_0, but with the role of neutral element for θ_1 and noted e_{θ_1} (the verifications are immediate).

(*ii*) The canonical projections are proved to be morphismes (see the step a)), they keep all the operations and

$$\ker p_j = \left\{a = \{a_i\}_{i \in I} \in \aleph^I : p_j(a) = e_\vee\right\} = \left\{a \in \aleph^I : a_j = 0\right\}.$$

(*iii*) For every $x, y \in \aleph, q_j(x \vee y) = \{c_i\}_{i \in I}$ where $c_i = 0$ for every $i \neq j$ and $c_j = x \vee y$ and

$$\theta_1\left(\begin{Bmatrix} a_i = 0, & \forall i \neq j \\ a_j = x & \end{Bmatrix}, \begin{Bmatrix} b_i = 0, & \forall i \neq j \\ b_j = y & \end{Bmatrix}\right) = \begin{Bmatrix} c_i = 0 & , \forall i \neq j \\ c_j = x \vee y & \end{Bmatrix}$$

i.e. $q_j(x \vee y) = \theta_1(q_j(x), q_j(y))$ with $j \in I$, therefore q_j keeps the operation "$\vee$" for every $j \in I$. Then $q_j(\psi(\{\emptyset\}) = \theta_0(\{\emptyset\}) \iff q_j(e_\vee) = \{e_i = 0\}_{i \in I} \iff q_j(0) = \{e_i = 0\}_{i \in I} = e_{\theta_1}$ because $\forall a = \{a_i\}_{i \in I} \in \aleph^I, \theta_1(q_j(0), a) = \theta_1\left(\{e_i = 0\}_{i \in I}, \{a_i\}_{i \in I}\right) =$

$\{e_i \vee a_i\}_{i\in I} = \{a_i\}_{i\in I} = a$ enough for $q_j(0) = e_{\theta_1}$ because θ_1 is obviously comutative -this observation refers to all the similar situations met before. Analogously we also prove that θ_2 is kept by q_j and this one for every $j \in I$.

(*iv*) For every $x \in \aleph$, $(p_j \circ q_k)(x) = p_j(q_k(x)) = p_j\left(\left\{\begin{array}{ll} a_i = 0 & ,\forall i \neq k \\ a_k = x & \end{array}\right\}\right) = 0 \Longrightarrow p_j \circ q_k = \mathcal{O}$ for $j \neq k$ and $(p_j \circ q_j)(x) = p_j(q_j(x)) = p_j\left(\left\{\begin{array}{ll} a_i = 0 & ,\forall i \neq j \\ a_j = x & \end{array}\right\}\right) = x \Longrightarrow p_j \circ q_k = 1_{\mathbf{N}}$ for $j = k$. ■

The universal algebra $\left(\aleph^I, \theta\right)$ satisfies the following property of universality:

Theorem 0.4. *For every* $\left(A, \bar{\theta}\right)$ *with* $\bar{\theta} = \{\top, \bot, \theta_0\}$ *an universal algebra of the some kind* $\tau : \bar{\theta} \to \aleph$

$$\tau = \left(\begin{array}{ccc} \top & \bot & \theta_0 \\ 2 & 2 & 0 \end{array}\right)$$

as $\left(\aleph^I, \theta\right)$ *and* $u_i : A \to \aleph$ *for every* $i \in I$ *morphismes between* $\left(A, \bar{\theta}\right)$ *and* $(\aleph, \Omega^*)$, *exists an unique* $u : A \to \aleph^I$ *morphisme between the universal algebras* $\left(A, \bar{\theta}\right)$ *and* $\left(\aleph^I, \theta\right)$ *so that* $p_j \circ u = u_j$, *for every* $j \in I$ *with* $p_j : \aleph^I \to \aleph, \forall \mathrm{a} = \{a_i\}_{i\in I} \in \aleph^I, p_j(a) = a_j$ *the canonical projections morphismes between* $\left(\aleph^I, \theta\right)$ *and* $(\aleph, \Omega^*)$.

Proof. The proof repeats the other one from the Theorem 1, step a). ■

The property of universality establishes the universal algebra $\left(\aleph^I, \theta\right)$ until an isomorphisme, which we can state by:

If (P, Ω^*) it is an universal algebra of the same kind as $(\aleph, \Omega^*)$ and $p'_i : P \to \aleph$ for every $i \in I$ a family of morphismes between (P, Ω^*) and $(\aleph, \Omega^*)$ so that for every universal algebra $\left(A, \bar{\theta}\right)$ and every morphismes $u_i : A \to \aleph, \forall i \in I$ between $\left(A, \bar{\theta}\right)$ and $(\aleph, \Omega^*)$ exists an unique morphisme $u : A \to P$ with $p'_i \circ u = u_i$, for every $i \in I$ then it exists an unique isomorphisme $f : P \to \aleph^I$ with $p_i \circ f = p'_i$, for every $i \in I$.

c) This third step contains the proof of the stated proposition (Problem 3).

As $(\aleph^*, \Omega)$ with $\Omega = (V_d, l_0\}$ is an universal algebra, in accordance with step a) it exists an universal algebra$\left((\aleph^*)^I, \Omega\right)$ with $\Omega = \{\omega_1, \omega_0\}$ defined by:

$$\begin{array}{rcl} \omega_1 & : & ((\aleph^*)^I)^2 \to (\aleph^*)^I \text{ by every } a = \{a_i\}_{i\in I} \text{ and } b = \{b_i\}_{i\in I} \in (\aleph^*)^I, \\ \omega_1(a, b) & = & \{a_i V_d b_i\}_{i\in I} \end{array}$$

and

$$\omega_0 : ((\aleph^*)^I)^0 \to (\aleph^*)^I \text{ by } \omega_0(\{\emptyset\}) = \{e_i = 1\}_{i\in I} = e_{\omega_1},$$

the canonical projections being certainly morphismes between $\left((\aleph^*)^I, \Omega\right)$ and $(\aleph^*, \Omega)$.

As $\left(\aleph, \Omega'\right)$ with $\Omega' = \{V, \Psi_0\}$ is an universal algebra, in accordance with step b) it exists an universal algebra $\left(\aleph^I, \Omega'\right)$ with $\Omega' = \{\theta_1, \theta_0\}$ defined by:

$$\theta_1 : (\aleph^I)^2 \to \aleph^I \text{ by every } a = \{a_i\}_{i\in I}, b = \{b_i\}_{i\in I} \in \aleph^I, \theta_1(a,b) = \{a_i V_d b_i\}_{i\in I}$$

and

$$\theta_0 : (\aleph^I)^0 \to \aleph^I \text{ by } \theta_0(\{\emptyset\}) = \{e_i = 0\}_{i\in I} = e_{\theta_1}$$

The universal algebras $\left((\aleph^*)^I, \Omega\right)$ and $\left(\aleph^I, \Omega'\right)$ are of the same kind

$$\begin{matrix}\omega_1 & \omega_2 \\ 2 & 0\end{matrix} = \begin{matrix}\theta_1 & \theta_0 \\ 2 & 0\end{matrix}$$

We use the property of universality for universal algebra $\left(\aleph', \Omega'\right)$: an universal algebra (A, Ω) can be $\left((\aleph^*)^I, \Omega\right)$ because they are the same kind; the morphismes $u_i : A \to \aleph$ from the assumption will be $\bar{s}_i : (\aleph^*)^I \to \aleph^*$ by every $a = \{a_i\}_{i\in I} \in (\aleph^*)^I, \bar{s}_j(a) = \bar{s}_j\left(\{a_i\}_{i\in I}\right) = s(a_j) \Longleftrightarrow \bar{s}_j = s \circ \bar{p}_j$ for every $j \in I$ where $s : \aleph^* \to \aleph$ is Smarandache's function and $\bar{p}_j : (\aleph^*)^I \to \aleph^*$ the canonical projections, morphismes between $\left((\aleph^*)^I, \Omega\right)$ and $(\aleph^*, \Omega)$. As s is a morphisme berween $(\aleph^*, \Omega)$ and $\left(\aleph, \Omega'\right)$, $\bar{s}_j$ are morphismes (as a composition of morphismes) for every $j \in I$. The assumptions of the property of universality being provided $\Longrightarrow$ exists an unique $\bar{s} : (\aleph^*)^I \to \aleph^I$ morphism between $\left((\aleph^*)^I, \Omega\right)$ and $\left(\aleph^I, \Omega\right)$ so that $p_j \circ s = \bar{s}_j \Longleftrightarrow p_j \circ s = S \circ \bar{p}_j$, for every $j \in I$. We finish the proof noticing that s is also surjection: $p_j \circ S$ surjection (as a composition of surjections) $\Longrightarrow s$ surjection.

Remark: The proof of the step 3 can be done directly. As the universal algebras from the statement are built, we can define a correspondence $s : (\aleph^*)^I \to (\aleph^*)^I$ by every $a = \{a_i\}_{i\in I} \in (\aleph^*)^I, s(a) = \{S(a_i)\}_{i\in I}$, which is a function, then morphisme between the universal algebra of the same kind $\left((\aleph^*)^I, \Omega\right)$ and $\left(\aleph^I, \Omega'\right)$ and is also surjective, the required conditions being satisfied evidently.

The stated Problem finds a prolongation s of the Smarandache function S to more comlexe sets (for $I = \{1\} \Rightarrow s = S$). The properties of the function s for the limitation to $\aleph^*$ could bring new properties for the Smarandache function.

1. References

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