Expressions of the Smarandache Coprime Function

Sebastian Martin Ruiz, Avda de Regla, 43 Chipiona, 11550 Cadiz, Spain

Smarandache Coprime Function is defined this way:

$$C_k(n_1, n_2, \cdots, n_k) = \begin{cases} 0 & if \quad n_1, n_2, \cdots, n_k \ are\ coprime\ numbers \\ 1 & otherwise \end{cases}$$

We see two expressions of the Smarandache Coprime Function for k=2.
EXPRESSION 1:

$$C_2(n_1, n_2) = -E\left[-\frac{n_1 n_2 - lcm(n_1, n_2)}{n_1 n_2}\right]$$

$E(x)$ = the biggest integer number smaller or equal than x.
If n_1, n_2 are coprime numbers:

$$lcm(n_1, n_2) = n_1 n_2 \quad \Rightarrow C_2(n_1, n_2) = -E\left[\frac{0}{n_1 n_2}\right] = 0$$

If n_1, n_2 aren't coprime numbers:

$$lcm(n_1, n_2) < n_1 n_2 \quad \Rightarrow 0 < \frac{n_1 n_2 - lcm(n_1, n_2)}{n_1 n_2} < 1 \Rightarrow C_2(n_1, n_2) = 1$$

EXPRESSION 2:

$$C_2(n_1,n_2) = 1 - E\left[-\frac{\displaystyle\prod_{\substack{d \mid n_1 \\ d>1}} \prod_{\substack{d' \mid n_2 \\ d'>1}} \mid d-d' \mid}{\displaystyle\prod_{d|n_1} \prod_{d|n_2} (d+d')} \right]$$

If n_1, n_2 are coprime numbers then $d \neq d' \quad \forall\, d, d' \neq 1$

$$\Rightarrow 0 < \frac{\displaystyle\prod_{\substack{d \mid n_1 \\ d>1}} \prod_{\substack{d' \mid n_2 \\ d'>1}} \mid d-d' \mid}{\displaystyle\prod_{d|n_1} \prod_{d|n_2} (d+d')} < 1 \Rightarrow C_2(n_1,n_2) = 0$$

If n_1, n_2 aren't coprime numbers $\exists d = d' \quad d > 1, d' > 1 \Rightarrow C_2(n_1,n_2) = 1$
Smarandache coprime function for $k \geq 2$.

$$C_k(n_1,n_2,\cdots,n_k) = -E\left[\frac{1}{GCD(n_1,n_2,\cdots,n_k)} - 1\right]$$

If $n_1, n_2, \cdots, n_k$ are coprime numbers:

$$GCD(n_1,n_2,\cdots,n_k) = 1 \Rightarrow C_k(n_1,n_2,\cdots,n_k) = 0$$

If $n_1, n_2, \cdots, n_k$ aren't coprime numbers: $GCD(n_1,n_2,\cdots,n_k) > 1$

$$0 < \frac{1}{GCD} < 1 \Rightarrow -E\left[\frac{1}{GCD} - 1\right] = 1 = C_k(n_1,n_2,\cdots,n_k)$$

References:

1. E. Burton, "Smarandache Prime and Coprime Functions"
2. F. Smarandache, "Collected Papers", Vol. II, 200 p.,p. 137, Kishinev University Press.