About The $S(n) = S(n - S(n))$ Equation

Mihaly Bencze
Str. Harmanului 6, 2212 Sacele 3
Jud. Brasov, Romania

Theorem 1: (M. Bencze, 1997) There exists infinitely many $n \in N$ such that $S(n) = S(n - S(n))$, where S is the Smarandache function.

Proof: Let r be a positive integer and $p > r$ a prime number. Then

$$S(pr) = S(p) = S((r-1)p) = S(pr - p) = S(pr - S(pr)).$$

Remark 1.1 There exists infinitely many $n \in N$ such that

$S(n) = S(n - S(n)) = S(n - S(n - S(n))) = \ldots$

Theorem 2: There exists infinitely many $n \in N$ such that

$$S(n) = S(n + S(n)).$$

Proof:

$$S(pr) = S(p) = S((r+1)p) = S(pr+p) = S(pr + S(pr)).$$

Remark 2.1 There exists infinitely many $n \in N$ such that

$S(n) = S(n + S(n)) = S(n + S(n + S(n))) = \ldots$

Theorem 3 There exists infinitely many $n \in N$ such that

$$S(n) = S(n \pm kS(n)).$$

Proof: See theorems 1 and 2.