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Smarandache Directionally n-Signed Graphs — A Survey

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Abstract: Let $G = (V, E)$ be a graph. By *directional labeling (or d-labeling)* of an edge $x = uv$ of G by an ordered n-tuple $(a_1, a_2, \cdots, a_n)$, we mean a labeling of the edge x such that we consider the label on uv as $(a_1, a_2, \cdots, a_n)$ in the direction from u to v, and the label on x as $(a_n, a_{n-1}, \cdots, a_1)$ in the direction from v to u. In this survey, we study graphs, called *(n, d)-sigraphs*, in which every edge is d-labeled by an n-tuple $(a_1, a_2, \cdots, a_n)$, where $a_k \in \{+, -\}$, for $1 \leq k \leq n$. Several variations and characterizations of directionally n-signed graphs have been proposed and studied. These include the various notions of balance and others.

Key Words: Signed graphs, directional labeling, complementation, balance.

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§1. Introduction

For graph theory terminology and notation in this paper we follow the book [3]. All graphs considered here are finite and simple. There are two ways of labeling the edges of a graph by an ordered n-tuple $(a_1, a_2, \cdots, a_n)$ (See [10]).

1. *Undirected labeling* or *labeling*. This is a labeling of each edge uv of G by an ordered n-tuple $(a_1, a_2, \cdots, a_n)$ such that we consider the label on uv as $(a_1, a_2, \cdots, a_n)$ irrespective of the direction from u to v or v to u.

2. *Directional labeling* or *d-labeling*. This is a labeling of each edge uv of G by an ordered n-tuple $(a_1, a_2, \cdots, a_n)$ such that we consider the label on uv as $(a_1, a_2, \cdots, a_n)$ in the direction from u to v, and $(a_n, a_{n-1}, \cdots, a_1)$ in the direction from v to u.

Note that the d-labeling of edges of G by ordered n-tuples is equivalent to labeling the symmetric digraph $\overrightarrow{G} = (V, \overrightarrow{E})$, where uv is a symmetric arc in $\overrightarrow{G}$ if, and only if, uv is an edge in G, so that if $(a_1, a_2, \cdots, a_n)$ is the d-label on uv in G, then the labels on the arcs $\overrightarrow{uv}$ and $\overrightarrow{vu}$ are $(a_1, a_2, \cdots, a_n)$ and $(a_n, a_{n-1}, \cdots, a_1)$ respectively.

Let H_n be the *n-fold sign group*, $H_n = \{+, -\}^n = \{(a_1, a_2, \cdots, a_n) : a_1, a_2, \cdots, a_n \in \{+, -\}\}$ with co-ordinate-wise multiplication. Thus, writing $a = (a_1, a_2, \cdots, a_n)$ and $t =$

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$(t_1, t_2, \cdots, t_n)$ then $at := (a_1t_1, a_2t_2, \cdots, a_nt_n)$. For any $t \in H_n$, the *action* of t on H_n is $a^t = at$, the co-ordinate-wise product.

Let $n \geq 1$ be a positive integer. An *n-signed graph (n-signed digraph)* is a graph $G = (V, E)$ in which each edge (arc) is labeled by an ordered n-tuple of signs, i.e., an element of H_n. A *signed graph* $G = (V, E)$ is a graph in which each edge is labeled by $+$ or $-$. Thus a 1-signed graph is a signed graph. Signed graphs are well studied in literature (See for example [1, 4-7, 13-21, 23, 24].

In this survey, we study graphs in which each edge is labeled by an ordered n-tuple $a = (a_1, a_2, \cdots, a_n)$ of signs (i.e, an element of H_n) in one direction but in the other direction its label is the reverse: $a^r = (a_n, a_{n-1}, \cdots, a_1)$, called *directionally labeled n-signed graphs* (or *(n, d)-signed graphs*).

Note that an n-signed graph $G = (V, E)$ can be considered as a symmetric digraph $\overrightarrow{G} = (V, \overrightarrow{E})$, where both $\overrightarrow{uv}$ and $\overrightarrow{vu}$ are arcs if, and only if, uv is an edge in G. Further, if an edge uv in G is labeled by the n-tuple $(a_1, a_2, \cdots, a_n)$, then in $\overrightarrow{G}$ both the arcs $\overrightarrow{uv}$ and $\overrightarrow{vu}$ are labeled by the n-tuple $(a_1, a_2, \cdots, a_n)$.

In [1], the authors study voltage graph defined as follows: A *voltage graph* is an ordered triple $\overrightarrow{G} = (V, \overrightarrow{E}, M)$, where V and $\overrightarrow{E}$ are the vertex set and arc set respectively and M is a group. Further, each arc is labeled by an element of the group M so that if an arc $\overrightarrow{uv}$ is labeled by an element $a \in M$, then the arc $\overrightarrow{vu}$ is labeled by its inverse, a^{-1}.

Since each n-tuple $(a_1, a_2, \cdots, a_n)$ is its own inverse in the group H_n, we can regard an n-signed graph $G = (V, E)$ as a voltage graph $\overrightarrow{G} = (V, \overrightarrow{E}, H_n)$ as defined above. Note that the d-labeling of edges in an (n, d)-signed graph considering the edges as symmetric directed arcs is different from the above labeling. For example, consider a $(4, d)$-signed graph in **F**igure 1. As mentioned above, this can also be represented by a symmetric 4-signed digraph. Note that this is not a voltage graph as defined in [1], since for example; the label on $\overrightarrow{v_2v_1}$ is not the (group) inverse of the label on $\overrightarrow{v_1v_2}$.

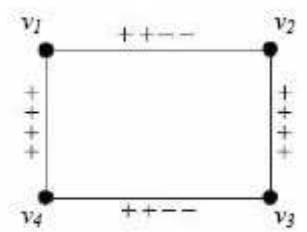

Fig.1

In [8-9], the authors initiated a study of $(3, d)$ and $(4, d)$-Signed graphs. Also, discussed some applications of $(3, d)$ and $(4, d)$-Signed graphs in real life situations.

In [10], the authors introduced the notion of complementation and generalize the notion of balance in signed graphs to the directionally n-signed graphs. In this context, the authors look upon two kinds of complementation: complementing some or all of the signs, and reversing the order of the signs on each edge. Also given some motivation to study (n, d)-signed graphs in connection with relations among human beings in society.

In [10], the authors defined complementation and isomorphism for (n, d)-signed graphs as follows: For any $t \in H_n$, the *t-complement* of $a = (a_1, a_2, \cdots, a_n)$ is: $a^t = at$. The *reversal* of $a = (a_1, a_2, \cdots, a_n)$ is: $a^r = (a_n, a_{n-1}, \cdots, a_1)$. For any $T \subseteq H_n$, and $t \in H_n$, the *t-complement* of T is $T^t = \{a^t : a \in T\}$.

For any $t \in H_n$, the *t-complement* of an (n, d)-signed graph $G = (V, E)$, written G^t, is the same graph but with each edge label $a = (a_1, a_2, \cdots, a_n)$ replaced by a^t. The *reversal* G^r is the same graph but with each edge label $a = (a_1, a_2, \cdots, a_n)$ replaced by a^r.

Let $G = (V, E)$ and $G' = (V', E')$ be two (n, d)-signed graphs. Then G is said to be *isomorphic* to G' and we write $G \cong G'$, if there exists a bijection $\phi : V \to V'$ such that if uv is an edge in G which is d-labeled by $a = (a_1, a_2, \cdots, a_n)$, then $\phi(u)\phi(v)$ is an edge in G' which is d-labeled by a, and conversely.

For each $t \in H_n$, an (n, d)-signed graph $G = (V, E)$ is *t-self complementary*, if $G \cong G^t$. Further, G is *self reverse*, if $G \cong G^r$.

Proposition 1.1(E. Sampathkumar et al. [10]) *For all $t \in H_n$, an (n, d)-signed graph $G = (V, E)$ is t-self complementary if, and only if, G^a is t-self complementary, for any $a \in H_n$.*

For any cycle C in G, let $\mathcal{P}(\overrightarrow{C})$ [10] denotes the product of the n-tuples on C given by $(a_{11}, a_{12}, \cdots, a_{1n})(a_{21}, a_{22}, \cdots, a_{2n}) \cdots (a_{m1}, a_{m2}, \cdots, a_{mn})$ and

$\mathcal{P}(\overleftarrow{C}) = (a_{mn}, a_{m(n-1)}, \cdots, a_{m1})(a_{(m-1)n}, a_{(m-1)(n-1)}, \cdots, a_{(m-1)1}) \cdots (a_{1n}, a_{1(n-1)}, \cdots, a_{11})$.

Similarly, for any path P in G, $\mathcal{P}(\overrightarrow{P})$ denotes the product of the n-tuples on P given by $(a_{11}, a_{12}, \cdots, a_{1n})(a_{21}, a_{22}, \cdots, a_{2n}) \cdots (a_{m-1,1}, a_{m-1,2}, \cdots, a_{m-1,n})$ and

$$\mathcal{P}(\overleftarrow{P}) = (a_{(m-1)n}, a_{(m-1)(n-1)}, \cdots, a_{(m-1)1}) \cdots (a_{1n}, a_{1(n-1)}, \cdots, a_{11}).$$

An n-tuple $(a_1, a_2, \cdots, a_n)$ is *identity n-tuple*, if each $a_k = +$, for $1 \leq k \leq n$, otherwise it is a *non-identity n-tuple*. Further an n-tuple $a = (a_1, a_2, \cdots, a_n)$ is *symmetric*, if $a^r = a$, otherwise it is a *non-symmetric n-tuple*. In (n, d)-signed graph $G = (V, E)$ an edge labeled with the identity n-tuple is called an *identity edge*, otherwise it is a *non-identity edge*.

Note that the above products $\mathcal{P}(\overrightarrow{C})$ ($\mathcal{P}(\overrightarrow{P})$) as well as $\mathcal{P}(\overleftarrow{C})$ ($\mathcal{P}(\overleftarrow{P})$) are n-tuples. In general, these two products need not be equal.

§2. Balance in an (n, d)-Signed Graph

In [10], the authors defined two notions of balance in an (n, d)-signed graph $G = (V, E)$ as follows:

Definition 2.1 *Let $G = (V, E)$ be an (n, d)-sigraph. Then,*

(i) G is identity balanced *(or* i-balanced*), if $P(\overrightarrow{C})$ on each cycle of G is the identity n-tuple, and*

(ii) G is balanced*, if every cycle contains an even number of non-identity edges.*

Note: An i-balanced (n, d)-sigraph need not be balanced and conversely. For example, consider the $(4, d)$-sigraphs in Figure.2. In Figure.2(a) G is an i-balanced but not balanced, and in Figure.2(b) G is balanced but not i-balanced.

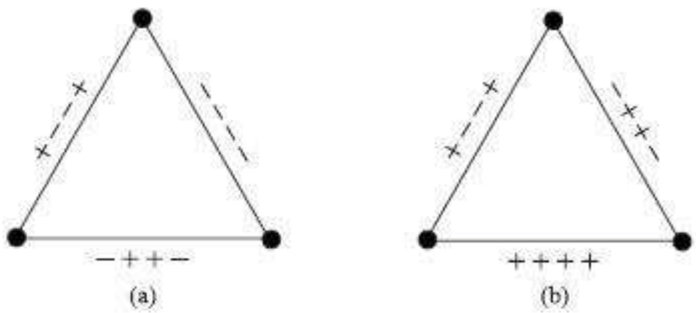

Fig.2

2.1 Criteria for balance

An (n, d)-signed graph $G = (V, E)$ is i-balanced if each non-identity n-tuple appears an even number of times in $P(\overrightarrow{C})$ on any cycle of G.

However, the converse is not true. For example see Figure.3(a). In Figure.3(b), the number of non-identity 4-tuples is even and hence it is balanced. But it is not i-balanced, since the 4-tuple $(++--)$ (as well as $(--++)$) does not appear an even number of times in $P(\overrightarrow{C})$ of 4-tuples.

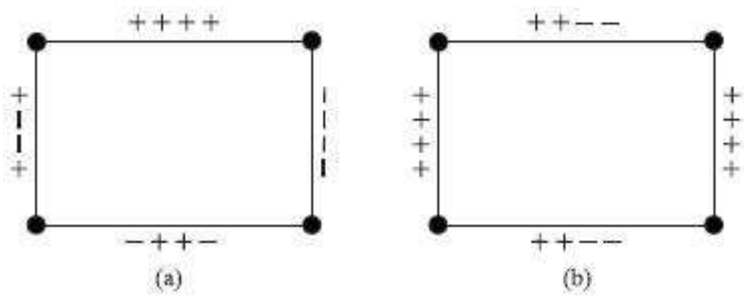

Fig.3

In [10], the authors obtained following characterizations of balanced and i-balanced (n, d)-sigraphs:

Proposition 2.2(E.Sampathkumar et al. [10]) *An (n, d)-signed graph $G = (V, E)$ is balanced if, and only if, there exists a partition $V_1 \cup V_2$ of V such that each identity edge joins two vertices in V_1 or V_2, and each non-identity edge joins a vertex of V_1 and a vertex of V_2.*

As earlier discussed, let $P(C)$ denote the product of the n-tuples in $P(\overrightarrow{C})$ on any cycle C in an (n, d)-sigraph $G = (V, E)$.

Theorem 2.3(E.Sampathkumar et al. [10]) *An (n, d)-signed graph $G = (V, E)$ is i-balanced if, and only if, for each k, $1 \leq k \leq n$, the number of n-tuples in $P(C)$ whose k^{th} co-ordinate is $-$ is even.*

In H_n, let S_1 denote the set of non-identity symmetric n-tuples and S_2 denote the set of non-symmetric n-tuples. The product of all n-tuples in each $S_k, 1 \leq k \leq 2$ is the identity n-tuple.

Theorem 2.4(E.Sampathkumar et al. [10]) *An (n, d)-signed graph $G = (V, E)$ is i-balanced, if both of the following hold:*

(i) In $P(C)$, each n-tuple in S_1 occurs an even number of times, or each n-tuple in S_1 occurs odd number of times (the same parity, or equal mod 2).

(ii) In $P(C)$, each n-tuple in S_2 occurs an even number of times, or each n-tuple in S_2 occurs an odd number of times.

In [11], the authors obtained another characterization of i-balanced (n, d)-signed graphs as follows:

Theorem 2.5(E.Sampathkumar et al. [11]) *An (n, d)-signed graph $G = (V, E)$ is i-balanced if, and only if, any two vertices u and v have the property that for any two edge distinct $u - v$ paths $\overrightarrow{P_1} = (u = u_0, u_1, \cdots, u_m = v$ and $\overrightarrow{P_2} = (u = v_0, v_1, \cdots, v_n = v)$ in G, $\mathcal{P}(\overrightarrow{P_1}) = (\mathcal{P}(\overrightarrow{P_2}))^r$ and $\mathcal{P}(\overrightarrow{P_2}) = (\mathcal{P}(\overrightarrow{P_1}))^r$.*

From the above result, the following are the easy consequences:

Corollary 2.6 *In an i-balanced (n, d)-signed graph G if two vertices are joined by at least 3 paths then the product of n tuples on any paths joining them must be symmetric.*

A graph $G = (V, E)$ is said to be k-connected for some positive integer k, if between any two vertices there exists at least k disjoint paths joining them.

Corollary 2.7 *If the underlying graph of an i-balanced (n, d)-signed graph is 3-connected, then all the edges in G must be labeled by a symmetric n-tuple.*

Corollary 2.8 *A complete (n, d)-signed graph on $p \geq 4$ is i-balanced then all the edges must be labeled by symmetric n-tuple.*

2.2 Complete (n, d)-Signed Graphs

In [11], the authors defined: an (n, d)-sigraph is *complete*, if its underlying graph is complete. Based on the complete (n, d)-signed graphs, the authors proved the following results: An (n, d)-signed graph is *complete*, if its underlying graph is complete.

Proposition 2.9(E.Sampathkumar et al. [11]) *The four triangles constructed on four vertices $\{a, b, c, d\}$ can be directed so that given any pair of vertices say (a, b) the product of the edges of these 4 directed triangles is the product of the n-tuples on the arcs $\overrightarrow{ab}$ and $\overrightarrow{ba}$.*

Corollary 2.10 *The product of the n-tuples of the four triangles constructed on four vertices $\{a, b, c, d\}$ is identity if at least one edge is labeled by a symmetric n-tuple.*

The *i-balance base* with axis a of a complete (n, d)-signed graph $G = (V, E)$ consists list of the product of the n-tuples on the triangles containing a [11].

Theorem 2.11(E.Sampathkumar et al. [11]) *If the i-balance base with axis a and n-tuple of an* *edge adjacent to a is known, the product of the n-tuples on all the triangles of G can be deduced from it.*

In the statement of above result, it is not necessary to know the n-tuple of an edge incident at a. But it is sufficient that an edge incident at a is a symmetric n-tuple.

Theorem 2.12(E.Sampathkumar et al. [11]) *A complete (n,d)-sigraph $G=(V,E)$ is i-balanced if, and only if, all the triangles of a base are identity.*

Theorem 2.13(E.Sampathkumar et al. [11]) *The number of i-balanced complete (n,d)-sigraphs of m vertices is p^{m-1}, where $p=2^{\lceil n/2\rceil}$.*

§3. Path Balance in (n,d)-Signed Graphs

In [11], E.Sampathkumar et al. defined the path balance in an (n,d)-signed graphs as follows:

Let $G=(V,E)$ be an (n,d)-sigraph. Then G is

1. *Path i-balanced*, if any two vertices u and v satisfy the property that for any $u-v$ paths P_1 and P_2 from u to v, $\mathcal{P}(\overrightarrow{P}_1)=\mathcal{P}(\overrightarrow{P}_2)$.
2. *Path balanced* if any two vertices u and v satisfy the property that for any $u-v$ paths P_1 and P_2 from u to v have same number of non identity n-tuples.

Clearly, the notion of path balance and balance coincides. That is an (n,d)-signed graph is balanced if, and only if, G is path balanced.

If an (n,d) signed graph G is i-balanced then G need not be path i-balanced and conversely.

In [11], the authors obtained the characterization path i-balanced (n,d)-signed graphs as follows:

Theorem 3.1(Characterization of path i-balanced $(n;d)$ signed graphs) *An (n,d)-signed graph is path i-balanced if, and only if, any two vertices u and v satisfy the property that for any two vertex disjoint $u-v$ paths P_1 and P_2 from u to v, $\mathcal{P}(\overrightarrow{P}_1)=\mathcal{P}(\overrightarrow{P}_2)$.*

§4. Local Balance in (n,d)-Signed Graphs

The notion of local balance in signed graph was introduced by F. Harary [5]. A signed graph $S=(G,\sigma)$ is locally at a vertex v, or S is *balanced at* v, if all cycles containing v are balanced. A cut point in a connected graph G is a vertex whose removal results in a disconnected graph. The following result due to Harary [5] gives interdependence of local balance and cut vertex of a signed graph.

Theorem 4.1(F.Harary [5]) *If a connected signed graph $S=(G,\sigma)$ is balanced at a vertex u. Let v be a vertex on a cycle C passing through u which is not a cut point, then S is balanced at v.*

In [11], the authors extend the notion of local balance in signed graph to (n, d)-signed graphs as follows: Let $G = (V, E)$ be a (n, d)-signed graph. Then for any vertices $v \in V(G)$, G is *locally i-balanced at v* (*locally balanced at v*) if all cycles in G containing v is i-balanced (balanced).

Analogous to the above result, in [11] we have the following for an (n, d) signed graphs:

Theorem 4.2 *If a connected (n, d)-signed graph $G = (V, E)$ is locally i-balanced (locally balanced) at a vertex u and v be a vertex on a cycle C passing through u which is not a cut point, then S is locally i-balanced(locally balanced) at v.*

§5. Symmetric Balance in (n, d)-Signed Graphs

In [22], P.S.K.Reddy and U.K.Misra defined a new notion of balance called *symmetric balance* or *s-balanced* in (n, d)-signed graphs as follows:

Let $n \geq 1$ be an integer. An *n-tuple* $(a_1, a_2, \cdots, a_n)$ is *symmetric*, if $a_k = a_{n-k+1}, 1 \leq k \leq n$. Let $H_n = \{(a_1, a_2, \cdots, a_n) : a_k \in \{+, -\}, a_k = a_{n-k+1}, 1 \leq k \leq n\}$ be the set of all symmetric n-tuples. Note that H_n is a group under coordinate wise multiplication, and the order of H_n is 2^m, where $m = \lceil n/2 \rceil$. Let $G = (V, E)$ be an (n, d)-signed graph. Then G is *symmetric balanced* or *s-balanced* if $P(\overrightarrow{C})$ on each cycle C of G is symmetric n-tuple.

Note: If an (n, d)-signed graph $G = (V, E)$ is i-balanced then clearly G is s-balanced. But a s-balanced (n, d)-signed graph need not be i-balanced. For example, the $(4, d)$-signed graphs in Figure 4. G is an s-balanced but not i-balanced.

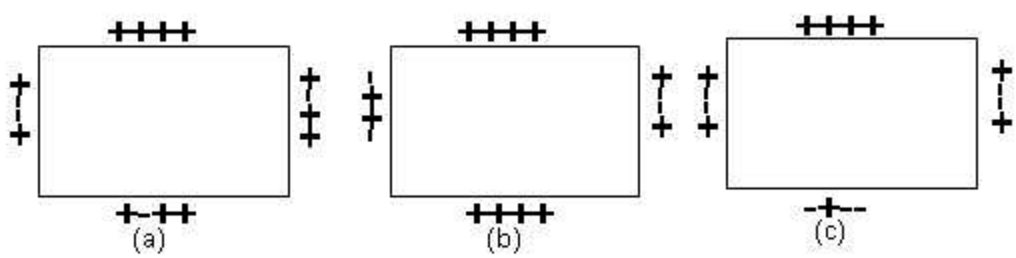

Fig.4

In [22], the authors obtained the following results based on symmetric balance or s-balanced in (n, d)-signed graphs.

Theorem 5.1(P.S.K.Reddy and U.K.Mishra [22]) *A (n, d)-signed graph is s-balanced if and only if every cycle of G contains an even number of non-symmetric n-tuples.*

The following result gives a necessary and sufficient condition for a balanced (n, d)-signed graph to be s-balanced.

Theorem 5.2(P.S.K.Reddy and U.K.Mishra [22]) *A balanced (n, d) signed graph $G = (V, E)$ is s-balanced if and only if every cycle of G contains even number of non identity symmetric n*

tuples.

In [22], the authors obtained another characterization of s-balanced (n, d)-signed graphs, which is analogous to the partition criteria for balance in signed graphs due to Harary [4].

Theorem 5.3(Characterization of s-balanced (n, d)-sigraph) *An (n, d)-signed graph $G = (V, E)$ is s balanced if and only if the vertex set $V(G)$ of G can be partitioned into two sets V_1 and V_2 such that each symmetric edge joins the vertices in the same set and each non-symmetric edge joins a vertex of V_1 and a vertex of V_2.*

An n-marking $\mu : V(G) \to H_n$ of an (n, d)-signed graph $G = (V, E)$ is an assignment n-tuples to the vertices of G. In [22], the authors given another characterization of s-balanced (n, d)-signed graphs which gives a relationship between the n-marking and s-balanced (n, d)-signed graphs.

Theorem 5.4(P.S.K.Reddy and U.K.Mishra [22]) *An (n, d)-signed graph $G = (V, E)$ is s-balanced if and only if there exists an n-marking μ of vertices of G such that if the n-tuple on any arc $\overrightarrow{uv}$ is symmetric or nonsymmetric according as the n-tuple $\mu(u)\mu(v)$ is.*

§6. Directionally 2-Signed Graphs

In [12], E.Sampathkumar et al. proved that the directionally 2-signed graphs are equivalent to bidirected graphs, where each end of an edge has a sign. A bidirected graph implies a signed graph, where each edge has a sign. Signed graphs are the special case $n = 1$, where directionality is trivial. Directionally 2-signed graphs (or $(2, d)$-signed graphs) are also special, in a less obvious way. A bidirected graph $\mathrm{B} = (G, \beta)$ is a graph $G = (V, E)$ in which each end (e, u) of an edge $e = uv$ has a sign $\beta(e, u) \in \{+, -\}$. G is the underlying graph and β is the bidirection. (The + sign denotes an arrow on the u-end of e pointed into the vertex u; a $-$ sign denotes an arrow directed out of u. Thus, in a bidirected graph each end of an edge has an independent direction. Bidirected graphs were defined by Edmonds [2].) In view of this, E.Sampathkumar et al. [12] proved the following result:

Theorem 6.1(E.Sampathkumar et al. [12]) *Directionally 2-signed graphs are equivalent to bidirected graphs.*

§7. Conclusion

In this brief survey, we have described directionally n-signed graphs (or (n, d)-signed graphs) and their characterizations. Many of the characterizations are more recent. This in an active area of research. We have included a set of references which have been cited in our description. These references are just a small part of the literature, but they should provide a good start for readers interested in this area.

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