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Smarandache inversion sequence

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Abstract We study the Smarandache inversion sequence which is a new concept, related sequences, conjectures, properties, and problems. This study was conducted by using (Maple 8)–a computer Algebra System.

Keywords Smarandache inversion, Smarandache reverse sequence.

Introduction

In [1], C.Ashbacher, studied the Smarandache reverse sequence:

$$1, 21, 321, 4321, 54321, 654321, 7654321, 87654321, 987654321, 10987654321, 1110987654321, \quad (1)$$

and he checked the first 35 elements and no prime were found. I will study sequence (1), from different point of view than C. Ashbacher. The importance of this sequence is to consider the place value of digits for example the number 1110987654321, to be considered with its digits like this : 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, and so on. (This consideration is the soul of this study because our aim is to study all relations like this (without loss of generality): $11 > 10 > 9 > 8 > 7 > 6 > 5 > 4 > 3 > 2 > 1$).

Definition. The value of the Smarandache Inversions (SI) of a positive integers, is the number of the relations $i > j$ (i and j are the digits of the positive integer that we concern with it), where i always in the left of j as the case of all numbers in (1). I will study the following cases of above equation.

Examples. The number 1234 has no inversions ((SI) = 0, or zero inversion), also the number 1, while the number 4321 has 6 inversions, because $4 > 3 > 2 > 1$, $3 > 2 > 1$, and $2 > 1$. The number 1110987654321 has 55 inversions, and 1342 has two inversions. So our interest will be of the numbers in Smarandache reverse sequence i.e. (1), because it has mathematical patterns and interesting properties.

Theorem. The values of SI of (1), is given by the following formula:

$$SI(n) = \frac{n(n-1)}{2}, \quad (2)$$

n is the number of inversions.

Proof. For $n = 1$, $SI(1) = \frac{1(1-1)}{2} = 0$, this is clearly true.
Now suppose that $SI(k) = \frac{k(k-1)}{2}$ is true, then $SI(k+1) = \frac{k+1(k+1-1)}{2} = \frac{k(k+1)}{2}$, thus the assertion is true for $n = k+1$, if it is true for $n = k$.

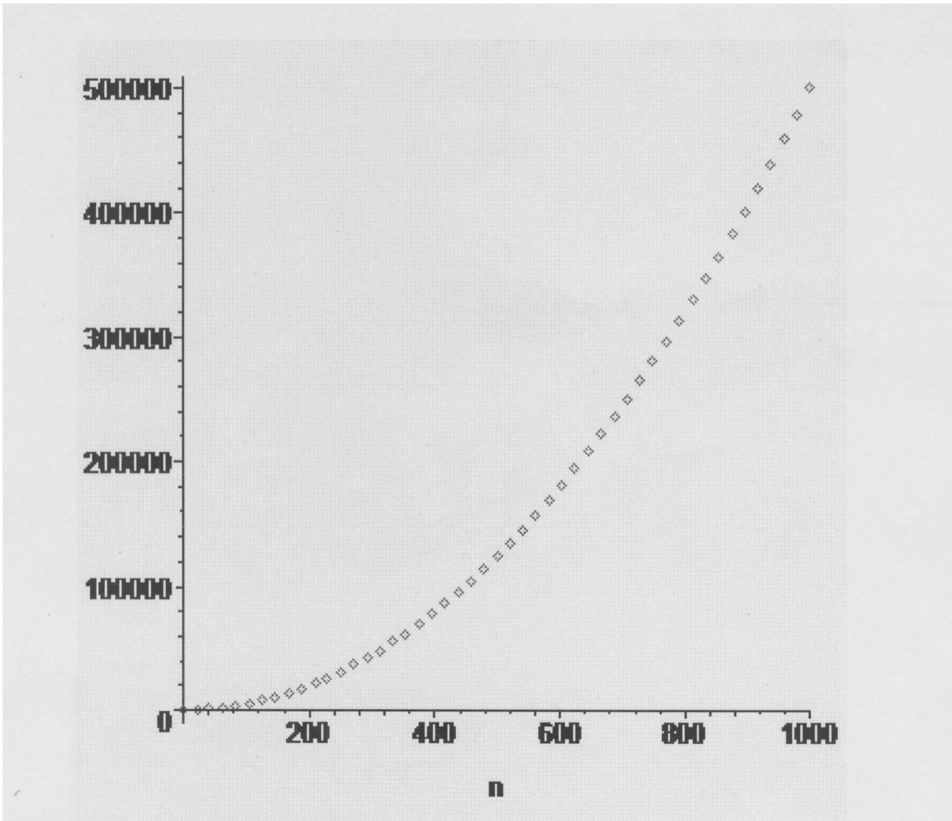

Figure 1: Plot of function $SI(n) = \frac{n(n-1)}{2}$

From the above figure, we can see although n is small, $SI(n) = \frac{n(n-1)}{2}$ it has big values. For example, if $n = 1000$, then $SI(1000) = 499500$.

Using Maple 8 programming language [2], verifying the first 100 terms of $SI(n)$:

$$SI(1) = 0, \quad SI(2) = 1, \quad SI(3) = 3, \quad SI(4) = 6,$$

$$SI(5) = 10, \quad SI(6) = 15, \quad SI(7) = 21, \quad SI(8) = 28,$$

$$SI(7) = 21, \quad SI(8) = 28, \quad SI(9) = 36, \quad SI(10) = 45,$$

$$SI(11) = 55, \quad SI(12) = 66, \quad SI(13) = 78, \quad SI(14) = 91,$$

$$SI(15) = 105, \quad SI(16) = 120, \quad SI(17) = 136,$$

$$SI(18) = 153, \quad SI(19) = 171, \quad SI(20) = 190,$$

$$SI(21) = 210, \quad SI(22) = 231, \quad SI(23) = 253,$$

$SI(24) = 276, \quad SI(25) = 300, \quad SI(26) = 325,$

$SI(27) = 351, \quad SI(28) = 378, \quad SI(29) = 406,$

$SI(30) = 435, \quad SI(31) = 465, \quad SI(32) = 496,$

$SI(33) = 528, \quad SI(34) = 561, \quad SI(35) = 595,$

$SI(36) = 630, \quad SI(37) = 666, \quad SI(38) = 703,$

$SI(39) = 741, \quad SI(40) = 780, \quad SI(41) = 820,$

$SI(42) = 861, \quad SI(43) = 903, \quad SI(44) = 946,$

$SI(45) = 990, \quad SI(46) = 1035, \quad SI(47) = 1081,$

$SI(48) = 1128, \quad SI(49) = 1176, \quad SI(50) = 1225,$

$SI(51) = 1275, \quad SI(52) = 1326, \quad SI(53) = 1378,$

$SI(54) = 1431, \quad SI(55) = 1485, \quad SI(56) = 1540,$

$SI(57) = 1596, \quad SI(58) = 1653, \quad SI(59) = 1711,$

$SI(60) = 1770, \quad SI(61) = 1830, \quad SI(62) = 1891,$

$SI(63) = 1953, \quad SI(64) = 2016, \quad SI(65) = 2080,$

$SI(66) = 2145, \quad SI(67) = 2211, \quad SI(68) = 2278,$

$SI(69) = 2346, \quad SI(70) = 2415, \quad SI(71) = 2485,$

$SI(72) = 2556, \quad SI(73) = 2628, \quad SI(74) = 2701,$

$SI(75) = 2775, \quad SI(76) = 2850, \quad SI(77) = 2926,$

$SI(78) = 3003, \quad SI(79) = 3081, \quad SI(80) = 3160,$

$SI(81) = 3240, \quad SI(82) = 3321, \quad SI(83) = 3403,$

$SI(84) = 3486, \quad SI(85) = 3570, \quad SI(86) = 3655,$

$SI(87) = 3741, \quad SI(88) = 3828, \quad SI(89) = 3916,$

$SI(90) = 4005, \quad SI(91) = 4095, \quad SI(92) = 4186,$

$SI(93) = 4278, \quad SI(94) = 4371, \quad SI(95) = 4465,$

$SI(96) = 4560, \quad SI(97) = 4656, \quad SI(98) = 4753,$

$SI(99) = 4851, \quad SI(100) = 4950.$

Summation of $SI(n) = \frac{n(n-1)}{2}$, we have

$$\sum_{i=1}^{n} \frac{i(i-1)}{2} = \frac{n(n^2-1)}{6}. \tag{3}$$

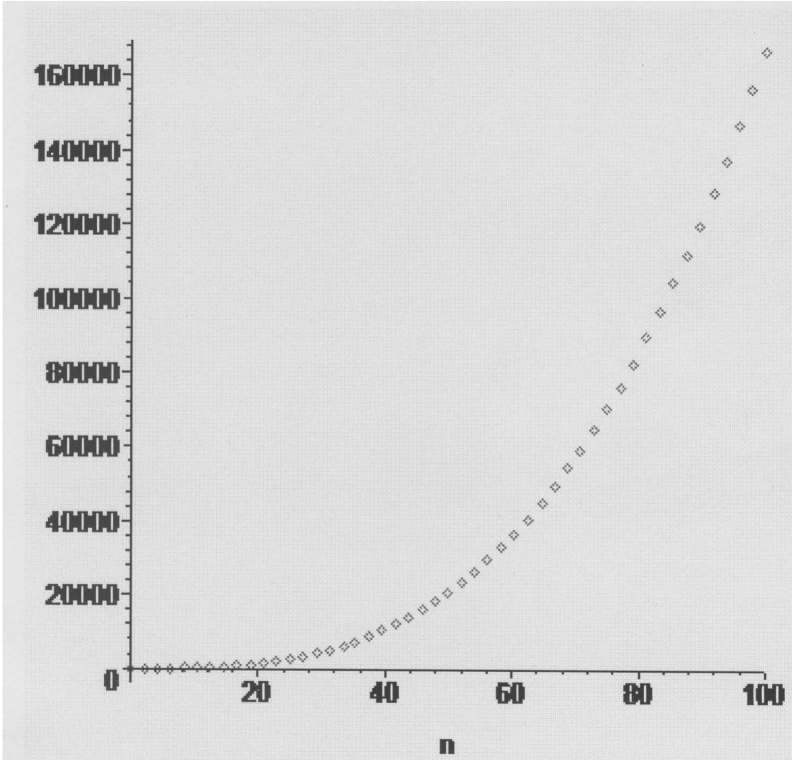

Figure 2: Plot of summation of $\sum_{i=1}^{n}\frac{i(i-1)}{2}=\frac{n(n^2-1)}{6}$

Proof. For $n = 1$, the assertion of (3) is that

$$\sum_{i=1}^{n}\frac{i(i-1)}{2}=0=\frac{1(1^2-1)}{6},$$

and this is clearly true.

Now suppose that

$$\sum_{i=1}^{k}\frac{i(i-1)}{2}=\frac{k(k^2-1)}{6},$$

then adding $\frac{k(k-1)}{2}$ to both sides of this equation, we obtain

$$\sum_{i=1}^{k+1}\frac{i(i-1)}{2}=\frac{k(k^2-1)}{6}+\frac{k(k-1)}{2}=\frac{k^3+3k^2+2k}{6}=\frac{k(k+1)(k+2)}{6}.$$

Thus the assertion is true for $n = k + 1$ if it is true for $n = k$.

Using Maple 8 programming language, verifying the first 73 terms of $\sum_{i=1}^{n}\frac{i(i-1)}{2}=\frac{n(n^2-1)}{6}$:

$$\sum SI(1)=0, \qquad \sum SI(2)=1, \qquad \sum SI(3)=4,$$

$$\sum SI(4) = 10, \qquad \sum SI(5) = 20, \qquad \sum SI(6) = 35,$$
$$\sum SI(7) = 56, \qquad \sum SI(8) = 84, \qquad \sum SI(9) = 120,$$
$$\sum SI(10) = 165, \qquad \sum SI(11) = 220, \qquad \sum SI(12) = 286,$$
$$\sum SI(13) = 364, \qquad \sum SI(14) = 455, \qquad \sum SI(15) = 560,$$
$$\sum SI(16) = 680, \qquad \sum SI(17) = 816, \qquad \sum SI(18) = 969,$$
$$\sum SI(19) = 1140, \qquad \sum SI(20) = 1330, \qquad \sum SI(21) = 1540,$$
$$\sum SI(22) = 1771, \qquad \sum SI(23) = 2024, \qquad \sum SI(24) = 2300,$$
$$\sum SI(25) = 2600, \qquad \sum SI(26) = 2925, \qquad \sum SI(27) = 3276,$$
$$\sum SI(28) = 3654, \qquad \sum SI(29) = 4060, \qquad \sum SI(30) = 4495,$$
$$\sum SI(31) = 4960, \qquad \sum SI(32) = 5456, \qquad \sum SI(33) = 5984,$$
$$\sum SI(34) = 6545, \qquad \sum SI(35) = 7140, \qquad \sum SI(36) = 7770,$$
$$\sum SI(37) = 8436, \qquad \sum SI(38) = 9139, \qquad \sum SI(39) = 9880,$$
$$\sum SI(40) = 10660, \qquad \sum SI(41) = 11480,$$
$$\sum SI(42) = 12341, \qquad \sum SI(43) = 13244,$$
$$\sum SI(44) = 14190, \qquad \sum SI(45) = 15180,$$
$$\sum SI(46) = 16215, \qquad \sum SI(47) = 17296,$$
$$\sum SI(48) = 18424, \qquad \sum SI(49) = 19600,$$
$$\sum SI(50) = 20825, \qquad \sum SI(51) = 22100,$$
$$\sum SI(52) = 23426, \qquad \sum SI(53) = 24804,$$
$$\sum SI(54) = 26235, \qquad \sum SI(55) = 27720,$$
$$\sum SI(56) = 29260, \qquad \sum SI(57) = 30856,$$
$$\sum SI(58) = 32509, \qquad \sum SI(59) = 34220,$$
$$\sum SI(60) = 35990, \qquad \sum SI(61) = 37820,$$
$$\sum SI(62) = 39711, \qquad \sum SI(63) = 41664,$$
$$\sum SI(64) = 43680, \qquad \sum SI(65) = 45760,$$
$$\sum SI(66) = 47905, \qquad \sum SI(67) = 50116,$$
$$\sum SI(68) = 52394, \qquad \sum SI(69) = 54740,$$

$$\sum SI(70) = 57155, \qquad \sum SI(71) = 59640,$$
$$\sum SI(72) = 62196, \qquad \sum SI(73) = 64824.$$

Properties of $SI(n) = \frac{n(n-1)}{2}$:
1).

$$SI(n) + SI(n-1) = (n-1)^2. \tag{4}$$

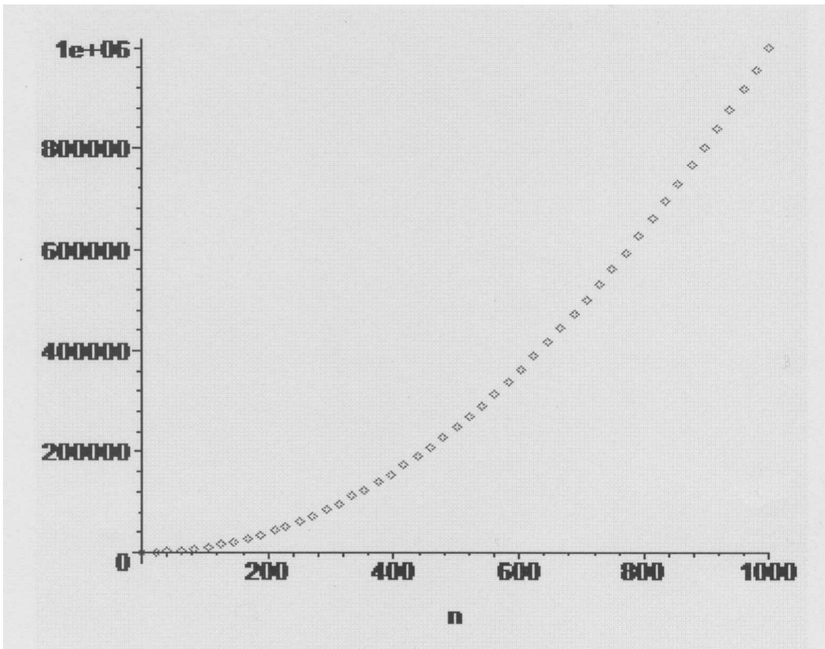

Fig 3: Plot of function $SI(n) + SI(n-1) = (n-1)^2$

Proof.

$$\begin{aligned} SI(n) + SI(n-1) &= \frac{n(n-1)}{2} + \frac{(n-1)(n-1-1)}{2} \\ &= \frac{n(n-1) + (n-1)(n-2)}{2} \\ &= \frac{(n-1)(2n-2)}{2} \\ &= (n-1)^2. \end{aligned}$$

Using Maple 8 programming language, verifying the first 40 terms of $SI(n) + SI(n-1) = (n-1)^2$:

$$SI(1) + SI(0) = [SI(0)]^2, \qquad SI(2) + SI(1) = [SI(1)]^2,$$
$$SI(3) + SI(2) = [SI(2)]^2, \qquad SI(4) + SI(3) = [SI(3)]^2,$$
$$SI(5) + SI(4) = [SI(4)]^2, \qquad SI(6) + SI(5) = [SI(5)]^2,$$

$$SI(7)+SI(6)=[SI(0)]^2, \quad SI(8)+SI(7)=[SI(7)]^2,$$
$$SI(9)+SI(8)=[SI(0)]^2, \quad SI(10)+SI(9)=[SI(9)]^2,$$
$$SI(11)+SI(10)=[SI(10)]^2, \quad SI(12)+SI(11)=[SI(11)]^2,$$
$$SI(13)+SI(12)=[SI(12)]^2, \quad SI(14)+SI(13)=[SI(13)]^2,$$
$$SI(15)+SI(14)=[SI(14)]^2, \quad SI(16)+SI(15)=[SI(15)]^2,$$
$$SI(17)+SI(16)=[SI(16)]^2, \quad SI(18)+SI(17)=[SI(17)]^2,$$
$$SI(19)+SI(18)=[SI(18)]^2, \quad SI(20)+SI(19)=[SI(19)]^2,$$
$$SI(21)+SI(20)=[SI(20)]^2, \quad SI(22)+SI(19)=[SI(21)]^2,$$
$$SI(23)+SI(22)=[SI(22)]^2, \quad SI(24)+SI(23)=[SI(23)]^2,$$
$$SI(25)+SI(24)=[SI(24)]^2, \quad SI(26)+SI(25)=[SI(25)]^2,$$
$$SI(27)+SI(26)=[SI(26)]^2, \quad SI(28)+SI(27)=[SI(27)]^2,$$
$$SI(29)+SI(28)=[SI(28)]^2, \quad SI(30)+SI(29)=[SI(29)]^2,$$
$$SI(31)+SI(30)=[SI(30)]^2, \quad SI(32)+SI(31)=[SI(31)]^2,$$
$$SI(33)+SI(32)=[SI(32)]^2, \quad SI(34)+SI(33)=[SI(33)]^2,$$
$$SI(35)+SI(34)=[SI(34)]^2, \quad SI(36)+SI(35)=[SI(35)]^2,$$
$$SI(37)+SI(36)=[SI(36)]^2, \quad SI(38)+SI(37)=[SI(37)]^2,$$
$$SI(39)+SI(38)=[SI(38)]^2, \quad SI(40)+SI(39)=[SI(39)]^2.$$

From the above values we can notes the following important conjecture.

Conjecture. There are other values on n such that

$SI2(4)+SI2(5)=SI(6)$, (three consecutive positive number)

$SI2(7)+SI2(9)=SI(11)$, (three odd consecutive positive number)

$SI2(6)+SI2(13)=SI(14)$, $\cdots$ etc.

2).

$$SI(n)^2-SI(n-1)^2=(n-1)^3. \tag{5}$$

Proof.

$$\begin{aligned}
SI(n)^2-SI(n-1)^2 &= \left[\frac{n(n-1)}{2}\right]^2-\left[\frac{(n-1)(n-1-1)}{2}\right]^2\\
&= \left[\frac{n(n-1)}{2}\right]^2-\left[\frac{(n-1)(n-2)}{2}\right]^2\\
&= \frac{(n-1)^2(4n-4)}{4}\\
&= (n-1)^3.
\end{aligned}$$

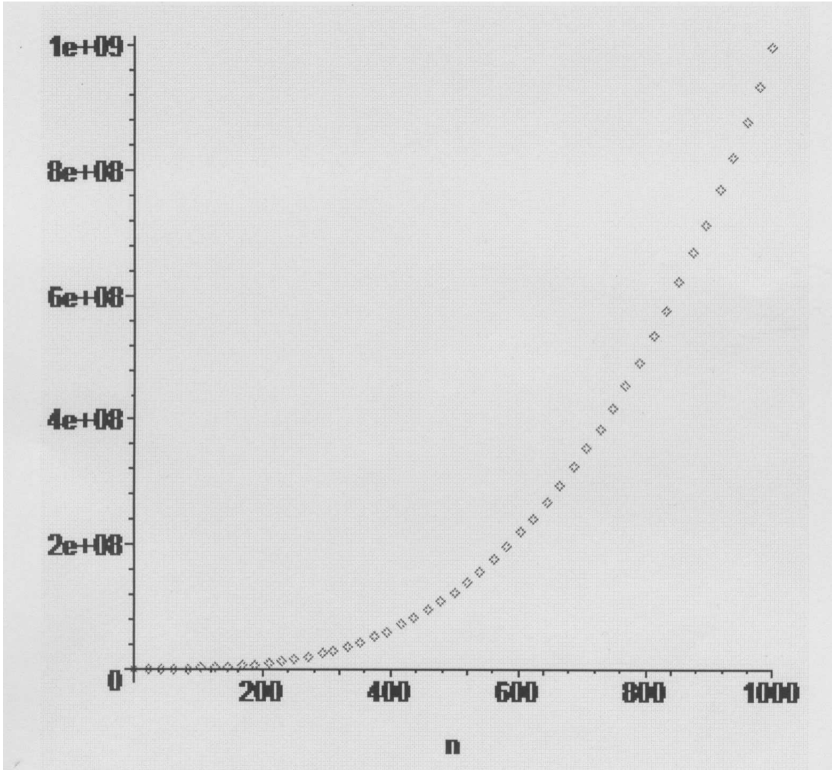

Fig 4: Plot of function $SI(n)^2 - SI(n-1)^2 = (n-1)^3$

Using Maple 8 programming language, verifying the first 28 terms of $SI(n)^2 - SI(n-1)^2 = SI(n-1)^3$:

$$SI(1)^2 - SI(0)^2 = [SI(0)]^3, \quad SI(2)^2 - SI(1)^2 = [SI(1)]^3,$$

$$SI(3)^2 - SI(2)^2 = [SI(2)]^3, \quad SI(4)^2 - SI(3)^2 = [SI(3)]^3,$$

$$SI(5)^2 - SI(4)^2 = [SI(4)]^3, \quad SI(6)^2 - SI(5)^2 = [SI(5)]^3,$$

$$SI(7)^2 - SI(6)^2 = [SI(0)]^3, \quad SI(8)^2 - SI(7)^2 = [SI(7)]^3,$$

$$SI(9)^2 - SI(8)^2 = [SI(0)]^3, \quad SI(10)^2 - SI(9)^2 = [SI(9)]^3,$$

$$SI(11)^2 - SI(10)^2 = [SI(10)]^3, \quad SI(12)^2 - SI(11)^2 = [SI(11)]^3,$$

$$SI(15)^2 - SI(14)^2 = [SI(14)]^3, \quad SI(16)^2 - SI(15)^2 = [SI(15)]^3,$$

$$SI(17)^2 - SI(16)^2 = [SI(16)]^3, \quad SI(18)^2 - SI(17)^2 = [SI(17)]^3,$$

$$SI(19)^2 - SI(18)^2 = [SI(18)]^3, \quad SI(20)^2 - SI(19)^2 = [SI(19)]^3,$$

$$SI(21)^2 - SI(20)^2 = [SI(20)]^3, \quad SI(22)^2 - SI(19)^2 = [SI(21)]^3,$$

$$SI(23)^2 - SI(22)^2 = [SI(22)]^3, \quad SI(24)^2 - SI(23)^2 = [SI(23)]^3,$$

$$SI(25)^2 - SI(24)^2 = [SI(24)]^3, \quad SI(26)^2 - SI(25)^2 = [SI(25)]^3,$$

$$SI(27)^2 - SI(26)^2 = [SI(26)]^3, \quad SI(28)^2 - SI(27)^2 = [SI(27)]^3.$$

Adding (4) and (5) , and only with slight modifications , we could have:
3).

$$(n^2-1)^2+(n^2-1)^3=[n(n-1)(n+1)]^2. \tag{6}$$

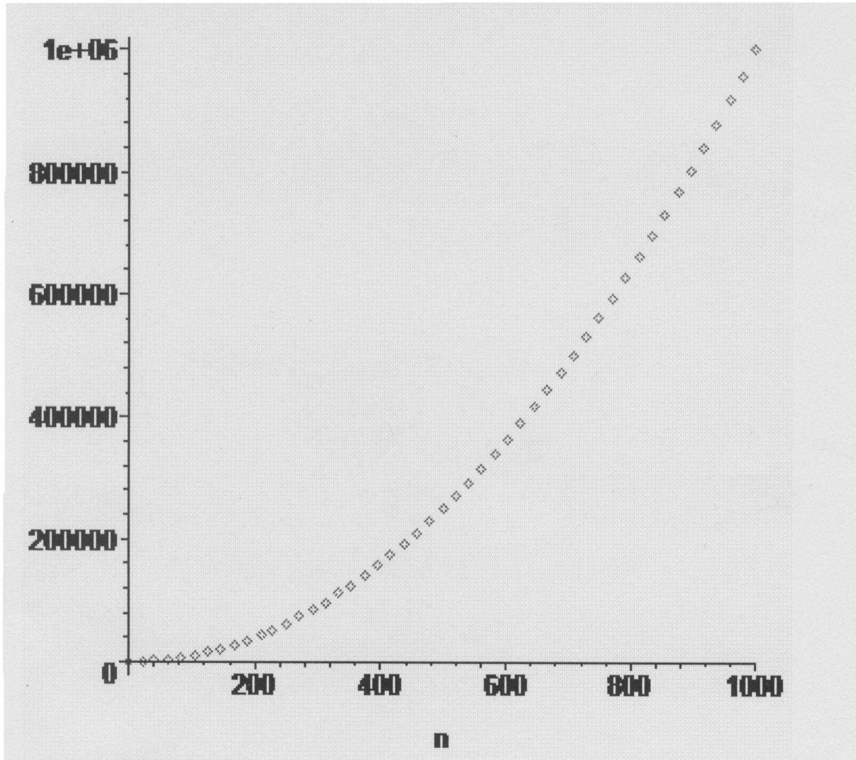

Fig 5: Plot of function $(n^2-1)^2+(n^2-1)^3=[n(n-1)(n+1)]^2$

By direct factorizations and calculations we can easily prove (6).
Using Maple 8 programming language, verifying the first 1680 terms of (6):

$$[0]^2+[0]^3=[0]^2, \qquad [3]^2+[3]^3=[6]^2,$$

$$[8]^2+[8]^3=[24]^2, \qquad [15]^2+[15]^3=[60]^2,$$

$$[24]^2+[24]^3=[120]^2, \qquad [35]^2+[35]^3=[210]^2,$$

$$[48]^2+[48]^3=[336]^2, \qquad [63]^2+[63]^3=[504]^2,$$

$$[80]^2+[80]^3=[720]^2, \qquad [99]^2+[99]^3=[990]^2,$$

$$[120]^2+[120]^3=[1320]^2, \qquad [143]^2+[143]^3=[1716]^2,$$

$$[168]^2+[168]^3=[2184]^2, \qquad [195]^2+[195]^3=[2730]^2,$$

$$[224]^2+[224]^3=[3360]^2, \qquad [255]^2+[255]^3=[4080]^2,$$

$$[288]^2+[288]^3=[4896]^2, \qquad [323]^2+[323]^3=[5814]^2,$$

$$[360]^2+[360]^3=[6840]^2, \qquad [399]^2+[399]^3=[7980]^2,$$

$$[440]^2 + [440]^3 = [9240]^2, \quad [483]^2 + [483]^3 = [10626]^2,$$
$$[528]^2 + [528]^3 = [12144]^2, \quad [575]^2 + [575]^3 = [13800]^2,$$
$$[624]^2 + [624]^3 = [15600]^2, \quad [675]^2 + [675]^3 = [17550]^2,$$
$$[728]^2 + [728]^3 = [19656]^2, \quad [783]^2 + [783]^3 = [21924]^2,$$
$$[840]^2 + [840]^3 = [24360]^2, \quad [899]^2 + [899]^3 = [26970]^2,$$
$$[960]^2 + [960]^3 = [29760]^2, \quad [1023]^2 + [1023]^3 = [32736]^2,$$
$$[1088]^2 + [1088]^3 = [35904]^2, \quad [1155]^2 + [1155]^3 = [39270]^2,$$
$$[1224]^2 + [1224]^3 = [42840]^2, \quad [1295]^2 + [1295]^3 = [46620]^2,$$
$$[1368]^2 + [1368]^3 = [50616]^2, \quad [1443]^2 + [1443]^3 = [54834]^2,$$
$$[1520]^2 + [1520]^3 = [59280]^2, \quad [1599]^2 + [1599]^3 = [63960]^2,$$
$$[1680]^2 + [1680]^3 = [68880]^2.$$

4). Subtracting (4) from (5), and only with slight modifications, we could have:

$$(n^2 + 1)^3 - (n^2 + 1)^2 = n^2(n^2 + 1)^2. \tag{7}$$

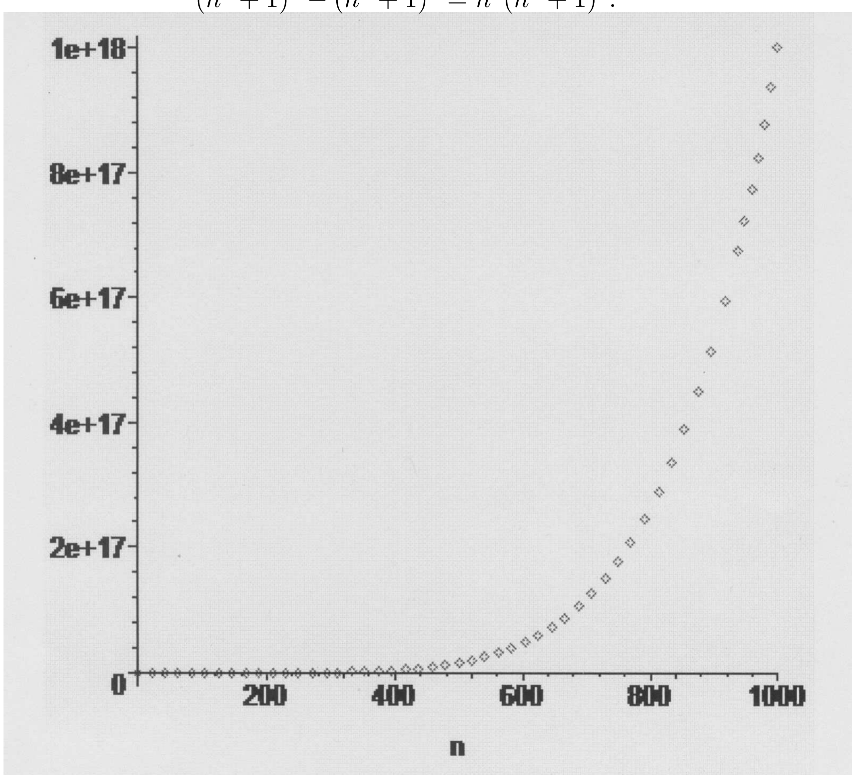

Fig 6: Plot of function $(n^2 + 1)^3 - (n^2 + 1)^2 = n^2(n^2 + 1)^2$

By direct factorizations and calculations we can easily prove (7).

Using Maple 8 programming language, verifying the first 2026 terms of (7):

$$[82]^3 - [82]^2 = [738]^2, \quad [101]^3 - [101]^2 = [1010]^2,$$

$$[122]^3 - [122]^2 = [1342]^2, \quad [145]^3 - [145]^2 = [1740]^2,$$

$$[170]^3 - [170]^2 = [2210]^2, \quad [197]^3 - [197]^2 = [2758]^2,$$

$$[226]^3 - [226]^2 = [3390]^2, \quad [257]^3 - [257]^2 = [4112]^2,$$

$$[290]^3 - [290]^2 = [4930]^2, \quad [325]^3 - [325]^2 = [5850]^2,$$

$$[362]^3 - [362]^2 = [6878]^2, \quad [401]^3 - [401]^2 = [8020]^2,$$

$$[442]^3 - [442]^2 = [9282]^2, \quad [485]^3 - [485]^2 = [10670]^2,$$

$$[530]^3 - [530]^2 = [12190]^2, \quad [577]^3 - [577]^2 = [13848]^2,$$

$$[626]^3 - [626]^2 = [15650]^2, \quad [677]^3 - [677]^2 = [17602]^2,$$

$$[730]^3 - [730]^2 = [19710]^2, \quad [785]^3 - [785]^2 = [21980]^2,$$

$$[842]^3 - [842]^2 = [24418]^2, \quad [901]^3 - [901]^2 = [27030]^2,$$

$$[962]^3 - [962]^2 = [29822]^2, \quad [1025]^3 - [1025]^2 = [32800]^2,$$

$$[1090]^3 - [1090]^2 = [35970]^2, \quad [1157]^3 - [1157]^2 = [39338]^2,$$

$$[1226]^3 - [1226]^2 = [42910]^2, \quad [1297]^3 - [1297]^2 = [46692]^2,$$

$$[1370]^3 - [1370]^2 = [50690]^2, \quad [1445]^3 - [1445]^2 = [54910]^2,$$

$$[1522]^3 - [1522]^2 = [59358]^2, \quad [1601]^3 - [1601]^2 = [64040]^2,$$

$$[1682]^3 - [1682]^2 = [68962]^2, \quad [1765]^3 - [1765]^2 = [74130]^2,$$

$$[1850]^3 - [1850]^2 = [79550]^2, \quad [1937]^3 - [1937]^2 = [85228]^2,$$

$$[2026]^3 - [2026]^2 = [91170]^2.$$

5).

$$SI(n) - SI(n-1) = n - 1. \tag{8}$$

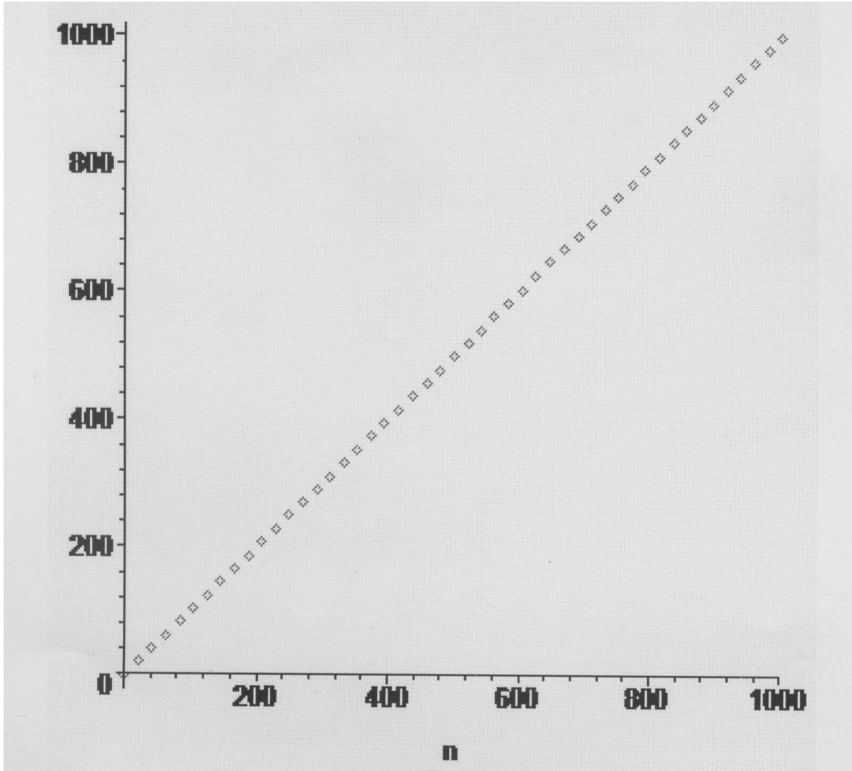

Fig 7: Plot of function $SI(n) - SI(n-1) = n - 1$

Proof.

$$\begin{aligned} SI(n) - SI(n-1) &= \frac{n(n-1)}{2} - \frac{(n-1)(n-1-1)}{2} \\ &= \frac{n(n-1)-(n-1)(n-2)}{2} \\ &= \frac{2n-2}{2} \\ &= n-1. \end{aligned}$$

Using Maple 8 programming language, verifying the first 80 terms of $SI(n) - SI(n-1) = n - 1$:

$$SI(1) - SI(0) = 0, \qquad SI(2) - SI(1) = 1,$$

$$SI(3) - SI(2) = 2, \qquad SI(4) - SI(3) = 3,$$

$$SI(5) - SI(4) = 4, \qquad SI(6) - SI(5) = 5,$$

$$SI(7) - SI(6) = 6, \qquad SI(8) - SI(7) = 7,$$

$$SI(9) - SI(8) = 8, \qquad SI(10) - SI(9) = 9,$$

$$SI(11) - SI(10) = 10, \qquad SI(12) - SI(11) = 11,$$

$$SI(13) - SI(12) = 12, \qquad SI(14) - SI(13) = 13,$$

$$SI(15) - SI(14) = 14, \quad SI(16) - SI(15) = 15,$$
$$SI(17) - SI(16) = 16, \quad SI(18) - SI(17) = 17,$$
$$SI(19) - SI(18) = 18, \quad SI(20) - SI(19) = 19,$$
$$SI(21) - SI(20) = 20, \quad SI(22) - SI(21) = 21,$$
$$SI(23) - SI(22) = 22, \quad SI(24) - SI(23) = 23,$$
$$SI(25) - SI(24) = 24, \quad SI(26) - SI(25) = 25,$$
$$SI(27) - SI(26) = 26, \quad SI(28) - SI(27) = 27,$$
$$SI(29) - SI(28) = 28, \quad SI(30) - SI(29) = 29,$$
$$SI(31) - SI(30) = 30, \quad SI(32) - SI(31) = 31,$$
$$SI(33) - SI(32) = 32, \quad SI(34) - SI(33) = 33,$$
$$SI(35) - SI(34) = 34, \quad SI(36) - SI(35) = 35,$$
$$SI(37) - SI(36) = 36, \quad SI(38) - SI(37) = 37,$$
$$SI(39) - SI(38) = 38, \quad SI(40) - SI(39) = 39,$$
$$SI(41) - SI(40) = 40, \quad SI(42) - SI(41) = 41,$$
$$SI(43) - SI(42) = 42, \quad SI(44) - SI(43) = 43,$$
$$SI(45) - SI(44) = 44, \quad SI(46) - SI(45) = 45,$$
$$SI(47) - SI(46) = 46, \quad SI(48) - SI(47) = 47,$$
$$SI(49) - SI(48) = 48, \quad SI(50) - SI(49) = 49,$$
$$SI(51) - SI(50) = 50, \quad SI(52) - SI(51) = 51,$$
$$SI(53) - SI(52) = 52, \quad SI(54) - SI(53) = 53,$$
$$SI(55) - SI(54) = 54, \quad SI(56) - SI(55) = 55,$$
$$SI(57) - SI(56) = 56, \quad SI(58) - SI(57) = 57,$$
$$SI(59) - SI(58) = 58, \quad SI(60) - SI(59) = 59,$$
$$SI(61) - SI(60) = 60, \quad SI(62) - SI(61) = 61,$$
$$SI(63) - SI(62) = 62, \quad SI(64) - SI(63) = 63,$$
$$SI(65) - SI(64) = 64, \quad SI(66) - SI(65) = 65,$$
$$SI(67) - SI(66) = 68, \quad SI(68) - SI(67) = 67,$$
$$SI(69) - SI(68) = 68, \quad SI(70) - SI(69) = 69,$$
$$SI(71) - SI(70) = 70, \quad SI(72) - SI(71) = 71,$$
$$SI(73) - SI(72) = 72, \quad SI(74) - SI(73) = 73,$$
$$SI(75) - SI(74) = 74, \quad SI(76) - SI(75) = 75,$$

$$SI(77) - SI(76) = 76, \quad SI(78) - SI(77) = 77,$$
$$SI(79) - SI(78) = 78, \quad SI(80) - SI(79) = 79.$$

6).

$$SI(n+1)SI(n-1) + SI(n) = SI(n)^2.$$

Proof.

$$\begin{aligned} SI(n+1)SI(n-1) + SI(n) &= \frac{n(n+1)}{2} \cdot \frac{(n-2)(n-1)}{2} + \frac{n(n-1)}{2} \\ &= \left[\frac{n(n-1)}{2}\right]^2. \end{aligned}$$

7).

$$SI(n)^2 + SI(n-1)^2 = k^2. \tag{9}$$

In this case I find the following two solutions:
i) $SI(8)^2 + SI(7)^2 = (35)^2$, i.e. $(28)^2 + (21)^2 = (35)^2$,
ii) $SI(42)^2 + SI(41)^2 = (1189)^2$, i.e.$(861)^2 + (820)^2 = (1189)^2$.

8). General SI identities given by numbers:
$SI(0) + SI(1) + SI(2) = 1$,
$SI(1) + SI(2) + SI(3) = 2^2$,
$SI(6) + SI(7) + SI(8) = 2^6$,
$SI(15) + SI(16) + SI(17) = 19^2$,
$SI(64) + SI(65) + SI(66) = 79^2$,
$SI(153) + SI(154) + SI(155) = 2^4 \cdot 47^2$,
$SI(0) + SI(1) + SI(2) + SI(3) = 2^2$,
$SI(6) + SI(7) + SI(8) + SI(9) = 2^2 \cdot 5^2$,
$SI(40) + SI(41) + SI(42) + SI(43) = 2^2 \cdot 29^2$,
$SI(238) + SI(239) + SI(240) + SI(241) = 2^2 \cdot 13^2$,
$SI(19) + SI(20) + SI(21) + SI(22) + SI(23) + SI(24) = 11^3$,
$SI(4) + SI(5) + SI(6) + SI(7) + SI(8) + SI(9) + SI(10) + SI(11) = 2^3 \cdot 3^3$,
$SI(1) - SI(0) + SI(2) = 1$,
$SI(7) - SI(6) + SI(8) = 2^3 \cdot 5^3$,
$SI(1) + SI(0) + SI(2) = 1$,
$SI(5) + SI(4) + SI(6) = 19^2$.

References

[1] Ashbacher.C, Plucking from the tree of Smarandache Sequences and Functions, http://www.gallup.unm.edu/~Smarandache.

[2] M.B. Monagan and others, Maple 7, Programming Guide, Waterloo Maple Inc.