The Smarandache adjacent number sequences and its asymptotic property

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Abstract The main purpose of this paper is using the elementary method to study the Smarand -ache adjacent number sequences, and give several interesting asymptotic formula for it.

Keywords Smarandache adjacent number sequences, elementary method, asymptotic formula.

§1. Introduction

For any positive integer n, the famous Smarandache adjacent number sequences $\{a(n,m)\}$ are defined as the number of such set, making the number of each set can be divided into several same parts, where m represent the bits of n. For example, Smarandache $a(1,1) = 1$, $a(2,1) = 22$, $a(3,1) = 333$, $a(4,1) = 4444$, $a(5,1) = 55555$, $a(6,1) = 666666$, $a(7,1) = 7777777$, $a(8,1) = 88888888$, $a(9,1) = 999999999$, $a(10,2) = 10101010101010101010,\ldots$, $a(100,3) = \underbrace{100\cdots100}_{100},\ldots$, and so on.

In the reference [1], Professor F. Smarandache asked us to study the properties of this sequence. About this problem, it seems that none had studied them before, at least we couldn't find any reference about it.

The problem of this sequence's first n items summation is meaningful. After a simple deduction and calculation, we can get a complex formula, but it's not ideal. So we consider the asymptotic problem of the average $\ln a(n,1) + \ln a(n,2) + \cdots + \ln a(N,M)$. We use the elementary method and the property of integral nature of the carrying to prove the following conclusion:

Theorem. If m is the bits of n, for any positive integer N, we have the asymptotic formula:

$$\sum_{n \leq N} \ln a(n,m) = N \cdot \ln N + O(N).$$

But the two asymptotic formulas is very rough, we will continue to study the precise asymptotic formulas.

§2. Proof of the theorem

In this section, we shall use the elementary methods to prove our theorems directly. First, we give one simple lemma which is necessary in the proof of our theorem. The proof of this lemma can be found in the reference [8].

Lemma 1. If f has a continuous derivative f' on the interval $[x, y]$, where $0 < y < x$,

$$\sum_{y < k \leq x} f(n) = \int_y^x f(t)dt + \int_y^x (t - [t])f'(t)dt + f(x)([x] - x) - f(y)([y] - y).$$

Then, we consider the structure of $\{a(n, m)\}$. We will get the following equations:

$a(1, 1) = 1,$

$a(2, 1) = 2 \cdot 10^1 + 2 \cdot 10^0,$

$a(3, 1) = 3 \cdot 10^2 + 3 \cdot 10^1 + 3 \cdot 10^0,$

$a(4, 1) = 4 \cdot 10^3 + 4 \cdot 10^2 + 4 \cdot 10^1 + 4 \cdot 10^0,$

$\cdots,$

$a(9, 1) = 9 \cdot 10^8 + 9 \cdot 10^7 + \cdots + 9 \cdot 10^2 + 9 \cdot 10^1 + 9 \cdot 10^0,$

$a(10, 2) = 10 \cdot 10^{18} + 10 \cdot 10^{16} + \cdots + 10 \cdot 10^2 + 10 \cdot 10^0$

$\cdots,$

$a(100, 3) = 100 \cdot 10^{297} + 100 \cdot 10^{294} + \cdots + 100 \cdot 10^3 + 100 \cdot 10^0$

$\cdots,$

$a(n, m) = n \cdot n^{297} + n \cdot 10^{294} + \cdots + n \cdot 10^m + n \cdot 10^0.$

If we analysis the above equations, we can get :

$$\prod_{1 \leq n \leq N} a(n, M) = \left(\prod_{n=1}^9 a(1, n) \right) \cdots \left(\prod_{n=10^{M-1}-1}^{10^{(10^{M-1}-1) \cdot (M-1)}} a(n, M-1) \right) \cdot \left(\prod_{n=10^{M-1}}^N a(N, M) \right)$$

$$= N! \frac{(10-1) \cdot (10^2 - 1) \cdots (10^{(10^{M-1}-1) \cdot (M-1)} - 1) \cdot (10^{(10^M-1) \cdot M} - 1)}{(10-1)^9 \cdot (10^2 - 1)^{90} \cdots (10^{M-1} - 1)^{9 \cdot 10^{M-2}} \cdot (10^M - 1)^{9 \cdot 10^{M-1}}}. (1)$$

When $x \to 0$, we note that the estimation $\ln(1 + x) = x + O(x^2)$, so we have

$$\sum_{k=1}^M \ln (10^k + 1)^{9 \cdot 10^{k-1}} = 9 \cdot \sum_{k=1}^M 10^{k-1} \cdot \left(k \cdot \ln 10 + \frac{1}{10^k} + O\left(\frac{1}{10^{2k}} \right) \right)$$

$$= \sum_{k=1}^M k \cdot 10^{k-1} \cdot 9 \ln 10 + \frac{9}{10} M + O(1)$$

$$= M \cdot 10^M \cdot \ln 10 + \frac{1}{9}(1 - 10^M) \cdot \ln 10 + \frac{9}{10} M + O(1)$$

$$= M \cdot 10^M \cdot \ln 10 + O(N). \tag{2}$$

$$\sum_{k=1}^{M} \ln(10^{(10^k-1)\cdot k} - 1)$$

$$= \sum_{k=1}^{M} \left(10^k - 1\right) \cdot k \ln 10 - \sum_{k=1}^{M} \frac{1}{10^{(10^k-1)\cdot k}} + O\left(\frac{1}{10^{10^{2\cdot(10^k-1)\cdot k}}}\right)$$

$$= \frac{M \cdot 10^{M+1} \cdot \ln 10}{9} + M \cdot 10^M \cdot \ln 10 + O(N). \tag{3}$$

Applying the Lemma 1, we obtain

$$\ln(N!) = \sum_{1 \le n \le N} \ln n = N \cdot \ln N - N + O(1). \tag{4}$$

Combining the equation (1), asymptotic formulas (2), (3) and (4), we obtain the asymptotic formula

$$\sum_{1 \le n \le N} \ln a(n, m) = \sum_{1 \le n \le N} \ln n + \sum_{k=1}^{M} \ln\left(10^k + 1\right)^{9\cdot 10^{k-1}} - \sum_{k=1}^{M} \ln(10^{(10^k-1)\cdot k} - 1)$$

$$= N \cdot \ln N + O(N).$$

Thus, we have accomplished the proof of the theorem.

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