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Smarandache Partitions

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Abstract I study Smarandache numbers partitions, and the partitions set of these numbers. This study conducted by Computer Algebra System namely, Maple 8.

Keywords Smarandache numbers s(n); Partitions P(n); Partitions sets; Smarandache numbers partitions P(s(n)).

§1.1 The procedure

Using the following procedure, we can verify the number of unrestricted partitions of the Smarandache numbers n is denoted by $P(s(n))$. With the Maple (V. 8)[see, 2] definitions.

```
S:= proc(n::nonnegint)
option remember;
local i, j, fact:
fact:=1:
for i from 2 while irem ( fact, n)<> 0 do
fact := fact *i:
od :
return i − 1:
end proc:
b:= proc(n::nonnegint)
option remember;
with (combstruct):
count (Partition(n));
end proc:
```

This procedure can verify the number of partitions, very fast, for example, it can verify the number of partitions of 200 in 0.2 second, while George Andrews said that "Actual enumeration of the $P(200) = 3972999029388$ would certainly take more than a lifetime, [1, p 150]."

Below the first 100 Smarandache numbers verifying by the above procedure:

§1.2 Partition counting of Smarandache numbers

By using the above procedure, we can got the first 100 partitions of Smarandache numbers as follows:

$P(s(1)) = 1 \quad P(s(2)) = 2 \quad P(s(3)) = 3 \quad P(s(4)) = 5$
$P(s(5)) = 7 \quad P(s(6)) = 3 \quad P(s(7)) = 15 \quad P(s(8)) = 5$
$P(s(9)) = 11 \quad P(s(10)) = 7 \quad P(s(11)) = 56 \quad P(s(12)) = 5$
$P(s(13)) = 101 \quad P(s(14)) = 15 \quad P(s(15)) = 7 \quad P(s(16)) = 11$
$P(s(17)) = 297 \quad P(s(18)) = 11 \quad P(s(19)) = 490 \quad P(s(20)) = 7$
$P(s(21)) = 15 \quad P(s(22)) = 56 \quad P(s(23)) = 1255 \quad P(s(24)) = 5$
$P(s(25)) = 42 \quad P(s(26)) = 101 \quad P(s(27)) = 30 \quad P(s(28)) = 15$
$P(s(29)) = 4565 \quad P(s(30)) = 7 \quad P(s(31)) = 6842 \quad P(s(32)) = 22$
$P(s(33)) = 56 \quad P(s(35)) = 297 \quad P(s(35)) = 15 \quad P(s(36)) = 11$
$P(s(37)) = 21637 \quad P(s(38)) = 490 \quad P(s(39)) = 101 \quad P(s(40)) = 7$
$P(s(41)) = 44583 \quad P(s(42)) = 15 \quad P(s(43)) = 63261 \quad P(s(44)) = 56$
$P(s(45)) = 11 \quad P(s(46)) = 1255 \quad P(s(47)) = 124754 \quad P(s(48)) = 11$
$P(s(49)) = 135 \quad P(s(50)) = 42 \quad P(s(51)) = 297 \quad P(s(52)) = 101$
$P(s(53)) = 329931 \quad P(s(54)) = 30 \quad P(s(55)) = 56 \quad P(s(56)) = 15$
$P(s(57)) = 490 \quad P(s(58)) = 4565 \quad P(s(59)) = 831820 \quad P(s(60)) = 7$
$P(s(61)) = 1121505 \quad P(s(62)) = 6842 \quad P(s(63)) = 15 \quad P(s(64)) = 22$
$P(s(65)) = 101 \quad P(s(66)) = 56 \quad P(s(67)) = 2679689 \quad P(s(68)) = 297$
$P(s(69)) = 1255 \quad P(s(70)) = 15 \quad P(s(71)) = 4697205 \quad P(s(72)) = 11$
$P(s(73)) = 6185689 \quad P(s(74)) = 21637 \quad P(s(75)) = 42 \quad P(s(76)) = 490$
$P(s(77)) = 56 \quad P(s(78)) = 101 \quad P(s(79)) = 13848650 \quad P(s(80)) = 11$
$P(s(81)) = 30 \quad P(s(82)) = 44583 \quad P(s(83)) = 23338469 \quad P(s(84)) = 15$
$P(s(85)) = 297 \quad P(s(86)) = 63261 \quad P(s(87)) = 4565 \quad P(s(88)) = 56$
$P(s(89)) = 49995925 \quad P(s(90)) = 11 \quad P(s(91)) = 101 \quad P(s(92)) = 1255$
$P(s(93)) = 6842 \quad P(s(94)) = 124754 \quad P(s(95)) = 490 \quad P(s(96)) = 22$
$P(s(97)) = 133230930 \quad P(s(98)) = 135 \quad P(s(99)) = 56 \quad P(s(100)) = 42$

We can not (without lose of generality)that: $P(s(4)) = P(s(8)) = P(s(12)) = P(s(24))$, this is because $s(4) = s(8) = s(12) = s(24) = 4$, and so on.

§2.1 The procedure of partitions sets

Now, the following procedure, we can verify the unrestricted partitions of the Smarandache numbers. With the Maple (V. 8) definitions.

S:= proc (n::nonnegint)
option remember;
local i, j, $fact$:

```
fact := 1:
for i from 2 while irem ( fact, n)<> 0 do
fact:= fact *i:
od :
return i − 1:
end proc:
b:= proc (n::nonnegint)
option remember;
with (combstruct):
allstructs (Partition(n));
end proc:
```

§2.2 Partition Sets of Smarandache numbers

By using the above procedure, we can got the first 15 partition sets of Smarandache numbers as follows:

PartitionSetOf $(s(1)) = [[1]]$,

PartitionSetOf $(s(2)) = [[1, 1], [2]]$,

PartitionSetOf $(s(3)) = [[[1, 1, 1], [1, 2], [3]]$,

PartitionSetOf $(s(4)) = [[1, 1, 1, 1], [1, 1, 2], [2, 2], [1, 3], [4]]$,

PartitionSetOf $(s(5)) = \{[\ [1, 1, 1, 1, 1], [1, 1, 1, 2], [1, 2, 2], [1, 1, 3], [2, 3], [1, 4], [5]\ ]\}$,

PartitionSetOf $(s(6)) = \{[\ [1, 1, 1], [1, 2], [3]\ ]\}$,

PartitionSetOf $(s(7)) = \{[\ [1, 1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 2], [1, 1, 1, 2, 2], [1, 2, 2, 2], [1, 1, 1, 1, 3],$
$[1, 1, 2, 3], [2, 2, 3], [1, 3, 3], [1, 1, 1, 1, 4], [1, 2, 4], [3, 4], [1, 1, 5], [2, 5], [1, 6], [7]\ ]\}$,

PartitionSetOf $(s(8)) = \{[\ [1, 1, 1, 1], [1, 1, 2], [2, 2], [1, 3], [4]\ ]\}$,

PartitionSetOf $(s(9)) = \{[\ [1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 2], [1, 1, 2, 2], [2, 2, 2], [1, 1, 1, 3], [1, 2, 3],$
$[3, 3], [1, 1, 4], [2, 4], [1, 5], [6]\ ]\}$,

PartitionSetOf $(s(10)) = \{[\ [1, 1, 1, 1, 1], [1, 1, 1, 2], [1, 2, 2], [1, 1, 3], [2, 3], [1, 4], [5]\ ]\}$,

PartitionSetOf $(s(11)) = \{[\ [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1, 1, 1, 1, 2], [1, 1, 1, 1, 1, 1, 1, 2, 2],$
$[1, 1, 1, 1, 1, 2, 2, 2], [1, 1, 1, 2, 2, 2, 2], [1, 2, 2, 2, 2, 2], [1, 1, 1, 1, 1, 1, 1, 1, 3], [1, 1, 1, 1, 1, 1, 2, 3],$
$[1, 1, 1, 1, 2, 2, 3], [1, 1, 2, 2, 2, 3], [2, 2, 2, 2, 3], [1, 1, 1, 1, 1, 3, 3], [1, 1, 1, 2, 3, 3], [1, 2, 2, 3, 3,],$
$[1, 1, 3, 3, 3], [2, 3, 3, 3], [1, 1, 1, 1, 1, 1, 1, 4], [1, 1, 1, 1, 1, 2, 4], [1, 1, 1, 2, 2, 4], [1, 2, 2, 2, 4],$
$[1, 1, 1, 1, 3, 4], [1, 1, 2, 3, 4], [2, 2, 3, 4], [1, 3, 3, 4], [1, 1, 1, 4, 4], [1, 2, 4, 4], [3, 4, 4], [1, 1, 1, 1, 1, 5],$
$[1, 1, 1, 1, 2, 5], [1, 1, 2, 2, 5], [2, 2, 2, 5], [1, 1, 1, 3, 5], [1, 2, 3, 5], [3, 3, 5], [1, 1, 4, 5], [2, 4, 5], [1, 5, 5,],$
$[1, 1, 1, 1, 1, 6], [1, 1, 1, 2, 6], [1, 2, 2, 6], [1, 1, 3, 6], [2, 3, 6], [1, 4, 6], [5, 6], [1, 1, 1, 1, 7], [1, 1, 2, 7], [2, 2, 7],$
$[1, 3, 7], [4, 7], [1, 1, 1, 8], [1, 2, 8], [3, 8], [1, 1, 9], [2, 9], [1, 10], [11]\ ]\}$,

PartitionSetOf $(s(12)) = \{[\ [1, 1, 1, 1], [1, 1, 2], [2, 2], [1, 3], [4]\ ]\}$,

PartitionSetOf $(s(13)) = \{[\ [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1], [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2],$
$[1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 2], [1, 1, 1, 1, 1, 1, 1, 2, 2, 2], [1, 1, 1, 1, 1, 2, 2, 2, 2], [1, 1, 1, 2, 2, 2, 2, 2],$
$[1, 2, 2, 2, 2, 2, 2], [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 3], [1, 1, 1, 1, 1, 1, 1, 1, 2, 3], [1, 1, 1, 1, 1, 1, 2, 2, 3],$
$[1, 1, 1, 1, 2, 2, 2, 3], [1, 1, 2, 2, 2, 2, 3], [2, 2, 2, 2, 2, 3], [1, 1, 1, 1, 1, 1, 1, 3, 3], [1, 1, 1, 1, 1, 2, 3, 3],$
$[1, 1, 1, 2, 2, 3, 3], [1, 2, 2, 2, 3, 3], [1, 1, 1, 1, 3, 3, 3], [1, 1, 2, 3, 3, 3], [2, 2, 3, 3, 3], [1, 3, 3, 3, 3],$

$[1,1,1,1,1,1,1,1,1,4]$, $[1,1,1,1,1,1,1,2,4]$, $[1,1,1,1,1,2,2,4]$, $[1,1,1,2,2,2,4]$, $[1,2,2,2,2,4]$, $[1,1,1,1,1,1,3,4]$, $[1,1,1,1,2,3,4]$, $[1,1,2,2,3,4]$, $[2,2,2,3,4]$, $[1,1,1,3,3,4]$, $[1,2,3,3,4]$, $[3,3,3,4]$, $[1,1,1,1,1,4,4]$, $[1,1,1,2,4,4]$, $[1,2,2,4,4]$, $[1,1,3,4,4]$, $[2,3,4,4]$, $[1,4,4,4]$, $[1,1,1,1,1,1,1,1,5]$, $[1,1,1,1,1,1,2,5]$, $[1,1,1,1,2,2,5]$, $[1,1,2,2,2,5]$, $[2,2,2,2,5]$, $[1,1,1,1,1,3,5]$, $[1,1,1,2,3,5]$, $[1,2,2,3,5]$, $[1,1,3,3,5][2,3,3,5]$, $[1,1,1,1,4,5]$, $[1,1,2,4,5]$, $[2,2,4,5]$, $[1,3,4,5]$, $[4,4,5]$, $[1,1,1,5,5]$, $[1,2,5,5]$, $[3,5,5]$], $[1,1,1,1,1,1,1,6]$, $[1,1,1,1,1,2,6]$, $[1,1,1,2,2,6]$, $[1,2,2,2,6]$, $[1,1,1,1,3,6]$, $[1,1,2,3,6]$, $[2,2,3,6,]$, $[1,3,3,6]$, $[1,1,1,4,6]$, $[1,2,4,6]$, $[3,4,6]$, $[1,1,5,6]$, $[2,5,6]$, $[1,6,6]$, $[1,1,1,1,1,1,7]$, $[1,1,1,1,2,7]$, $[1,1,2,2,7]$, , $[2,2,2,7]$, $[1,1,1,3,7][1,2,3,7]$, $[3,3,7]$, $[1,1,4,7]$, $[2,4,7]$, $[1,5,7]$, $[6,7]$, $[1,1,1,1,1,8]$, $[1,1,1,2,8]$, $[1,2,2,8]$, $[1,1,3,8]$, $[2,3,8]$, $[1,4,8]$, $[5,8]$, $[1,1,1,1,9]$, $[1,1,2,9]$, $[2,2,9]$, $[1,3,9]$, $[4,9]$, $[1,1,1,10]$, $[1,2,10]$, $[3,10]$, $[1,1,11]$, $[2,11]$, $[1,12]$, $[13]]\}$,

PartitionSetOf $(s(14)) = \{[\ [1,1,1,1,1,1,1], [1,1,1,1,1,2], [1,1,1,2,2], [1,2,2,2], [1,1,1,1,3], [1,1,2,3], [2,2,3], [1,3,3], [1,1,1,4], [1,2,4], [3,4], [1,1,5], [2,5], [1,6], [7]\ ]\}$,

PartitionSetOf $(s(15)) = \{[\ [1,1,1,1,1], [1,1,1,2], [1,2,2], [1,1,3], [2,3], [1,4], [5]\ ]\}$,

We can not (without lose of generality) that: Partitions of $P(s(4)) = P(s(8)) = P(s(12)) = P(s(24))$, this is because all of them have the same Smarandache numbers and the same partitions sets, and so on.

References

[1] G. E. Andrews, Number Theory, Drover, New York , 1971.

[2] M.B. Monagan etc., Maple 7, Programming Guide, Waterloo Maple Inc., 2001, pp. 36.