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Solution of a Conjecture on Skolem Mean Graph of Stars $K_{1,l}\bigcup K_{1,m}\bigcup K_{1,n}$

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Abstract: In this paper, we prove a conjecture that the three stars $K_{1,l}\bigcup K_{1,m}\bigcup K_{1,n}$ is a skolem mean graph if $|m-n|<4+l$ for integers $l,m\geq 1$ and $l\leq m<n$.

Key Words: Smarandachely edge m-labeling f_S^*, Smarandachely super m-mean graph, skolem mean labeling, Skolem mean graph, star.

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§1. Introduction

All graphs in this paper are finite, simple and undirected. Terms not defined here are used in the sense of Harary [4]. A vertex labeling of G is an assignment $f:V(G)\to\{1,2,3,\ldots,p+q\}$ be an injection. For a vertex labeling f, the induced Smarandachely edge m-labeling f_S^* for an edge $e=uv$, an integer $m\geq 2$ is defined by $f_S^*(e)=\left\lceil\frac{f(u)+f(v)}{m}\right\rceil$. Then f is called a Smarandachely super m-mean labeling if $f(V(G))\cup\{f^*(e):e\in E(G)\}=\{1,2,3,\ldots,p+q\}$. Particularly, in the case of $m=2$, we know that

$$f^*(e)=\begin{cases}\frac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even;}\\ \frac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd.}\end{cases}$$

Such a labeling is usually called a mean labeling. A graph that admits a Smarandachely super mean m-labeling is called a Smarandachely super m-mean graph, particularly, a skolem mean graph if $m=2$ in [1]. It was proved that any path is a skolem mean graph, $K_{1,m}$ is not a skolem mean graphif $m\geq 4$, and the two stars $K_{1,m}\bigcup K_{1,n}$ is a skolem mean graph if and only if $|m-n|\leq 4$. In [2], it was proved that the three star $K_{1,l}\bigcup K_{1,m}\bigcup K_{1,n}$ is a skolem mean graph if $|m-n|=4+l$ for $l=1,2,3,\cdots$, $m=1,2,3,\cdots$ and $\leq m<n$. It is also shown in [2] that the three star $K_{1,l}\bigcup K_{1,m}\bigcup K_{1,n}$ is not a skolem mean graph if $|m-n|>4+l$ for $l=1,2,3,\cdots$, $m=1,2,3,\cdots$, $n\geq l+m+5$ and $l\leq m<n$, the four star $K_{1,l}\bigcup K_{1,l}\bigcup K_{1,m}\bigcup K_{1,n}$ is a skolem mean graph if $|m-n|=4+2l$ for $l=2,3,4,\cdots$, $m=2,3,4,\cdots$, $n=2l+m+4$ and $l\leq m<n$; the four star $K_{1,l}\bigcup K_{1,l}\bigcup K_{1,m}\bigcup K_{1,n}$ is not a skolem mean graph if $|m-n|>4+2l$ for $l=2,3,4,\cdots$, $m=2,3,4,\cdots$, $n\geq 2l+m+5$ and $l\leq m<n$; the four star $K_{1,1}\bigcup K_{1,1}\bigcup K_{1,m}\bigcup K_{1,n}$ is a skolem mean graph if $|m-n|=7$ for

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$m = 1, 2, 3, \cdots$, $n = m + 7$, $1 \leq m < n$, and the four star $K_{1,1} \bigcup K_{1,1} \bigcup K_{1,m} \bigcup K_{1,n}$ is not a skolem mean graph if $|m - n| > 7$ for $m = 1, 2, 3, \cdots$, $n \geq m + 8$ and $1 \leq m < n$. In [3], the condition for a graph to be skolem mean is that $p \geq q + 1$.

§2. Main Theorem

Definition 2.1 *The three star is the disjoint union of $K_{1,l}$, $K_{1,m}$ and $K_{1,n}$ for integers $l, m, n \geq 1$. Such a graph is denoted by $K_{1,l} \bigcup K_{1,m} \bigcup K_{1,n}$.*

Theorem 2.2 *If $l \leq m < n$, the three star $K_{1,l} \bigcup K_{1,m} \bigcup K_{1,n}$ is a skolem mean graph if $|m - n| < 4 + l$ for integers $l, m \geq 1$.*

Proof Consider the graph $G = K_{1,l} \bigcup K_{1,m} \bigcup K_{1,n}$. Let $\{u\} \bigcup \{u_i : 1 \leq i \leq l\}$, $\{v\} \bigcup \{v_j : 1 \leq j \leq m\}$ and $\{w\} \bigcup \{w_k : 1 \leq k \leq n\}$ be the vertices of G. Then G has $l+m+n+3$ vertices and $l + m + n$ edges. We have $V(G) = \{u, v, w\} \bigcup \{u_i : 1 \leq i \leq l\} \bigcup \{v_j : 1 \leq j \leq m\} \bigcup \{w_k : 1 \leq k \leq n\}$. The proof id divided into four cases following.

Case 1 Let $l \leq m < n$ where $n = l + m + 3$ for integers $l, m \geq 1$. We prove such graph G is a skolem mean graph. The required vertex labeling $f : V(G) \to \{1, 2, 3, \cdots, l + m + n + 3\}$ is defined as follows:

$$\begin{aligned}
&f(u) = 1, \quad f(v) = 3;\\
&f(w) = l + m + n + 3;\\
&f(u_i) = 2i + 3 \text{ for } 1 \leq i \leq l;\\
&f(v_j) = 2l + 2j + 3 \text{ for } 1 \leq j \leq m;\\
&f(w_k) = 2k \text{ for } 1 \leq k \leq n - 1 \text{ and}\\
&f(w_n) = l + m + n + 2.
\end{aligned}$$

The corresponding edge labels are as follows:

The edge labels of uu_i is $i + 2$ for $1 \leq i \leq l$, vv_j is $l + j + 3$ for $1 \leq j \leq m$ and ww_k is $\dfrac{2k + l + m + n + 3}{2}$ for $1 \leq k \leq n - 1$. Also, the edge label of ww_n is $l + m + n + 3$. Therefore, the induced edge labels of G are distinct. Hence G is a skolem mean graph.

Case 2 Let $l \leq m < n$ where $n = l + m + 2$ for integers $l, m \geq 1$. We prove that G is a skolem mean graph. The required vertex labeling $f : V(G) \to \{1, 2, 3, \cdots, l + m + n + 3\}$ is defined as follows:

$$\begin{aligned}
&f(u) = 1;\ f(v) = 2;\ f(w) = l + m + n + 3;\\
&f(u_i) = 2i + 2 \text{ for } 1 \leq i \leq l;\\
&f(v_j) = 2l + 2j + 2 \text{ for } 1 \leq j \leq m;\\
&f(w_k) = 2k + 1 \text{ for } 1 \leq k \leq n - 1 \text{ and}\\
&f(w_n) = l + m + n + 2.
\end{aligned}$$

The corresponding edge labels are as follows:

The edge labels of uu_i is $i+2$ for $1 \leq i \leq l$; vv_j is $l+j+2$ for $1 \leq j \leq m$ and ww_k is $\frac{2k+l+m+n+4}{2}$ for $1 \leq k \leq n-1$. Also, the edge label of ww_n is $l+m+n+3$. Therefore, the induced edge labels of G are distinct. Hence the graph G is a skolem mean graph.

Case 3 Let $l \leq m < n$ where $n = l+m+1$ for integers $l, m \geq 1$. In this case, the required vertex labeling $f : V(G) \rightarrow \{1, 2, 3, \cdots, l+m+n+3\}$ is defined as follows:

$$f(u) = 1;\ f(v) = 2;\ f(w) = l+m+n+3;$$
$$f(u_i) = 2i+1 \text{ for } 1 \leq i \leq l;$$
$$f(v_j) = 2l+2j+1 \text{ for } 1 \leq j \leq m;$$
$$f(w_k) = 2k+2 \text{ for } 1 \leq k \leq n-1 \text{ and}$$
$$f(w_n) = l+m+n+2.$$

The corresponding edge labels are as follows:

The edge labels of uu_i is $i+1$ for $1 \leq i \leq l$; vv_j is $l+j+2$ for $1 \leq j \leq m$ and ww_k is $\frac{2k+l+m+n+5}{2}$ for $1 \leq k \leq n-1$. Also, the edge label of ww_n is $l+m+n+3$. Therefore, the induced edge labels of G are distinct. Therefore, G is a skolem mean graph.

Case 4 Let $l \leq m < n$ where $n = l+m$ for integers $l, m \geq 1$. We prove such graph G is a skolem mean graph. In this case, the required vertex labeling $f : V(G) \rightarrow \{1, 2, 3, \cdots, l+m+n+3\}$ is defined as follows:

$$f(u) = 1;\ f(v) = 3;\ f(w) = l+m+n+3;$$
$$f(u_i) = 2i \text{ for } 1 \leq i \leq l;$$
$$f(v_j) = 2l+2j \text{ for } 1 \leq j \leq m;$$
$$f(w_k) = 2k+3 \text{ for } 1 \leq k \leq n-1 \text{ and}$$
$$f(w_n) = l+m+n+2.$$

Calculation shows the corresponding edge labels are as follows:

The edge labels of uu_i is $i+1$ for $1 \leq i \leq l$; vv_j is $l+j+2$ for $1 \leq j \leq m$ and ww_k is $\frac{2k+l+m+n+6}{2}$ for $1 \leq k \leq n-1$. Also, the edge label of ww_n is $l+m+n+3$. Therefore, the induced edge labels of G are distinct and G is a skolem mean graph.

Combining these discussions of Cases $1-4$, we know that G is a skolem mean graph. □

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