International J.Math. Combin. Vol.3 (2009), 82-96

Some Results on Super Mean Graphs

R. Vasuki[1] and A. Nagarajan[2]

[1] Department of Mathematics, Dr.Sivanthi Aditanar College of Engineering,
Tiruchendur - 628 215, Tamil Nadu, India.

[2] Department of Mathematics, V.O. Chidambaram College,
Thoothukudi - 628 002, Tamil Nadu, India.

E-mail: vasukisehar@yahoo.co.in, nagarajan.voc@gmail.com

Abstract: Let G be a graph and $f : V(G) \to \{1, 2, 3, \ldots, p + q\}$ be an injection. For each edge $e = uv$ and an integer $m \geq 2$, the induced *Smarandachely edge m-labeling* f^*_S is defined by

$$f^*_S(e) = \left\lceil \frac{f(u) + f(v)}{m} \right\rceil.$$

Then f is called a *Smarandachely super m-mean labeling* if $f(V(G)) \cup \{f^*(e) : e \in E(G)\} = \{1, 2, 3, \ldots, p + q\}$. Particularly, in the case of $m = 2$, we know that

$$f^*(e) = \begin{cases} \frac{f(u)+f(v)}{2} & \text{if } f(u) + f(v) \text{ is even;} \\ \frac{f(u)+f(v)+1}{2} & \text{if } f(u) + f(v) \text{ is odd.} \end{cases}$$

Such a labeling is usually called a *super mean labeling*. A graph that admits a Smarandachely super mean m-labeling is called *Smarandachely super m-mean graph*. In this paper, we prove that the H-graph, corona of a H-graph, $G \odot S_2$ where G is a H-graph, the cycle C_{2n} for $n \geq 3$, corona of the cycle C_n for $n \geq 3$, mC_n-snake for $m \geq 1, n \geq 3$ and $n \neq 4$, the dragon $P_n(C_m)$ for $m \geq 3$ and $m \neq 4$ and $C_m \times P_n$ for $m = 3, 5$ are super mean graphs, i.e., Smarandachely super 2-mean graphs.

Keywords: Labeling, Smarandachely super mean labeling, Smarandachely super m-mean graph, super mean labeling, super mean graphs

AMS(2000): 05C78

§1. Introduction

Throughout this paper, by a graph we mean a finite, undirected, simple graph. Let $G(V, E)$ be a graph with p vertices and q edges. For notations and terminology we follow [1].

Let G_1 and G_2 be any two graphs with p_1 and p_2 vertices respectively. Then the Cartesian

[1] Received July 24, 2009. Accepted Aug.28, 2009.

product $G_1 \times G_2$ has p_1p_2 vertices which are $\{(u,v)/u \in G_1, v \in G_2\}$. The edges are obtained as follows: (u_1,v_1) and (u_2,v_2) are adjacent in $G_1 \times G_2$ if either $u_1 = u_2$ and v_1 and v_2 are adjacent in G_2 or u_1 and u_2 are adjacent in G_1 and $v_1 = v_2$.

The corona of a graph G on p vertices $v_1, v_2, \ldots, v_p$ is the graph obtained from G by adding p new vertices $u_1, u_2, \ldots, u_p$ and the new edges u_iv_i for $1 \le i \le p$, denoted by $G \odot K_1$. For a graph G, the 2-corona of G is the graph obtained from G by identifying the center vertex of the star S_2 at each vertex of G, denoted by $G \odot S_2$. The baloon of a graph G, $P_n(G)$ is the graph obtained from G by identifying an end vertex of P_n at a vertex of G. $P_n(C_m)$ is called a dragon. The join of two graphs G and H is the graph obtained from $G \cup H$ by joining each vertex of G with each vertex of H by means of an edge and it is denoted by $G + H$.

A path of n vertices is denoted by P_n and a cycle on n vertices is denoted by C_n. $K_{1,m}$ is called a star, denoted by S_m. The bistar $B_{m,n}$ is the graph obtained from K_2 by identifying the center vertices of $K_{1,m}$ and $K_{1,n}$ at the end vertices of K_2 respectively, denoted by $B(m)$. A triangular snake T_n is obtained from a path $v_1v_2 \ldots v_n$ by joining v_i and v_{i+1} to a new vertex w_i for $1 \le i \le n-1$, that is, every edge of a path is replaced by a triangle C_3.

We define the H-graph of a path P_n to be the graph obtained from two copies of P_n with vertices $v_1, v_2, \ldots, v_n$ and $u_1, u_2, \ldots, u_n$ by joining the vertices $v_{\frac{n+1}{2}}$ and $u_{\frac{n+1}{2}}$ if n is odd and the vertices $v_{\frac{n}{2}+1}$ and $u_{\frac{n}{2}}$ if n is even and a cyclic snake mC_n the graph obtained from m copies of C_n by identifying the vertex $v_{(k+2)_j}$ in the j^{th} copy at a vertex $v_{1_{j+1}}$ in the $(j+1)^{th}$ copy if $n = 2k+1$ and identifying the vertex $v_{(k+1)_j}$ in the j^{th} copy at a vertex $v_{1_{j+1}}$ in the $(j+1)^{th}$ copy if $n = 2k$.

A vertex labeling of G is an assignment $f : V(G) \to \{1, 2, 3, \ldots, p+q\}$ be an injection. For a vertex labeling f, the induced *Smarandachely edge m-labeling* f_S^* for an edge $e = uv$, an integer $m \ge 2$ is defined by

$$f_S^*(e) = \left\lceil \frac{f(u)+f(v)}{m} \right\rceil.$$

Then f is called a *Smarandachely super m-mean labeling* if $f(V(G)) \cup \{f^*(e) : e \in E(G)\} = \{1, 2, 3, \ldots, p+q\}$. Particularly, in the case of $m = 2$, we know that

$$f^*(e) = \begin{cases} \frac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even;} \\ \frac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd.} \end{cases}$$

Such a labeling is usually called a *super mean labeling*. A graph that admits a Smarandachely super mean m-labeling is called *Smarandachely super m-mean graph*, particularly, *super mean graph* if $m = 2$. A super mean labeling of the graph P_6^2 is shown in Fig.1.1.

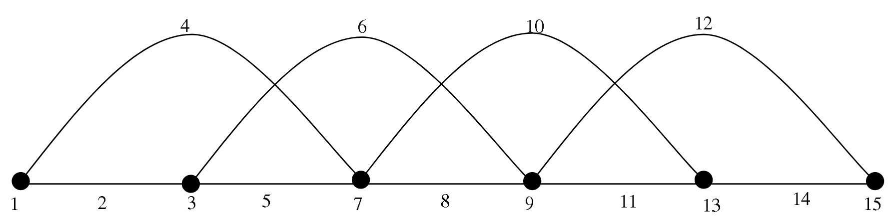


Fig.1.1

The concept of mean labeling was first introduced by S. Somasundaram and R. Ponraj [7]. They have studied in [4,5,7,8] the mean labeling of some standard graphs.

The concept of super mean labeling was first introduced by R. Ponraj and D. Ramya [2]. They have studied in [2,3] the super mean labeling of some standard graphs like $P_n, C_{2n+1}, n \geq 1$, $K_n (n \leq 3)$, $K_{1,n}(n \leq 3)$, T_n, $C_m \cup P_n (m \geq 3, n \geq 1)$, $B_{m,n}(m = n, n+1)$ etc. They have proved that the union of two super mean graph is super mean graph and C_4 is not a super mean graph. Also they determined all super mean graph of order ≤ 5.

In this paper, we establish the super meanness of the graph C_{2n} for $n \geq 3$, the H-graph, Corona of a H- graph, 2-corona of a H-graph, corona of cycle C_n for $n \geq 3$, mC_n-snake for $m \geq 1$, $n \geq 3$ and $n \neq 4$, the dragon $P_n(C_m)$ for $m \geq 3$ and $m \neq 4$ and $C_m \times P_n$ for $m = 3, 5$.

§2. Results

Theorem 2.1 *The H-graph G is a super mean graph.*

Proof Let $v_1, v_2, \ldots, v_n$ and $u_1, u_2, \ldots, u_n$ be the vertices of the graph G. We define a labeling $f : V(G) \to \{1, 2, \ldots, p+q\}$ as follows:

$f(v_i) = 2i - 1, \quad 1 \leq i \leq n$

$f(u_i) = 2n + 2i - 1, \quad 1 \leq i \leq n$

For the vertex labeling f, the induced edge labeling f^* is defined as follows:

$f^*(v_i v_{i+1}) = 2i, \quad 1 \leq i \leq n-1$

$f^*(u_i u_{i+1}) = 2n + 2i, \quad 1 \leq i \leq n-1$

$f^*(v_{\frac{n+1}{2}} u_{\frac{n+1}{2}}) = 2n$ if n is odd

$f^*(v_{\frac{n}{2}+1} u_{\frac{n}{2}}) = 2n$ if n is even

Then clearly it can be verified that the H-graph G is a super mean graph. For example the super mean labelings of H-graphs G_1 and G_2 are shown in Fig.2.1. □

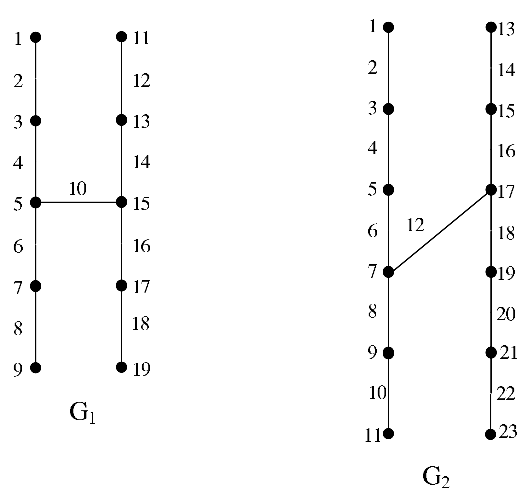


Fig.2.1

Theorem 2.2 *If a H-graph G is a super mean graph, then $G \odot K_1$ is a super mean graph.*

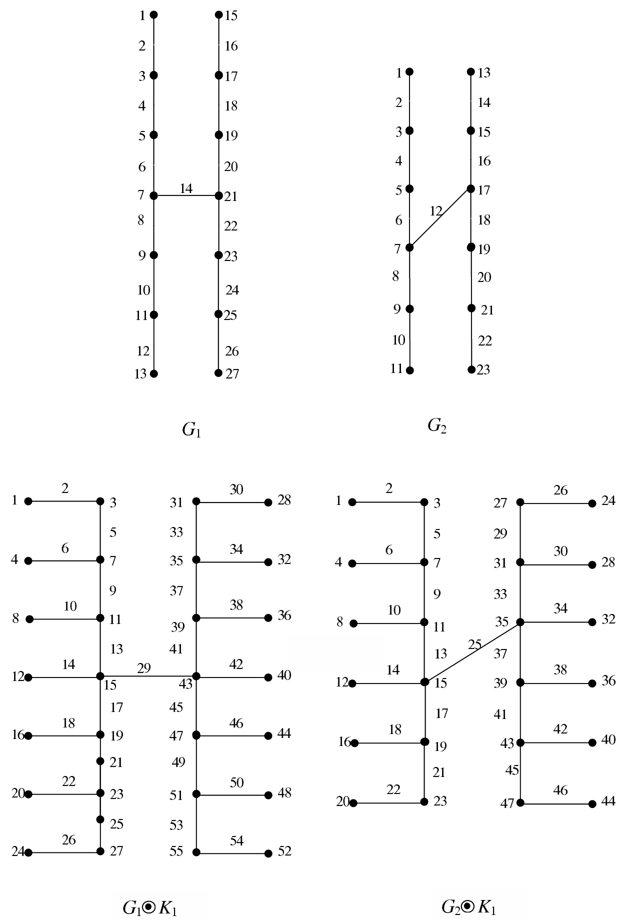


Fig.2.2

Proof Let f be a super mean labeling of G with vertices $v_1, v_2, \ldots, v_n$ and $u_1, u_2, \ldots, u_n$. Let $v_1', v_2', \ldots, v_n'$ and $u_1', u_2', \ldots, u_n'$ be the corresponding new vertices in $G \odot K_1$.

We define a labeling $g : V(G \odot K_1) \to \{1, 2, \ldots, p+q\}$ as follows:

$$\begin{aligned} g(v_i) &= f(v_i) + 2i, & 1 \le i \le n \\ g(u_i) &= f(u_i) + 2n + 2i, & 1 \le i \le n \\ g(v_1') &= f(v_1) & \\ g(v_i') &= f(v_i) + 2i - 3, & 2 \le i \le n \\ g(u_i') &= f(u_i) + 2n + 2i - 3, & 1 \le i \le n \end{aligned}$$

For the vertex labeling g, the induced edge labeling g^* is defined as follows:

$$
\begin{aligned}
g^*(v_iv_{i+1}) &= f^*(v_iv_{i+1}) + 2i + 1, && 1 \le i \le n-1 \\
g^*(u_iu_{i+1}) &= f^*(u_iu_{i+1}) + 2n + 2i + 1, && 1 \le i \le n-1 \\
g^*(v_iv_i') &= f(v_i) + 2i - 1, && 1 \le i \le n \\
g^*(u_iu_i') &= f(u_i) + 2n + 2i - 1, && 1 \le i \le n \\
g^*(v_{\frac{n+1}{2}}u_{\frac{n+1}{2}}) &= 2f^*((v_{\frac{n+1}{2}}u_{\frac{n+1}{2}}) + 1 && \text{if } n \text{ is odd} \\
g^*(v_{\frac{n}{2}+1}u_{\frac{n}{2}}) &= 2f^*((v_{\frac{n}{2}+1}u_{\frac{n}{2}}) + 1 && \text{if } n \text{ is even}
\end{aligned}
$$

It can be easily verified that g is a super mean labeling and hence $G \odot K_1$ is a super mean graph. For example the super mean labeling of H-graphs G_1, G_2, $G_1 \odot K_1$ and $G_2 \odot K_1$ are shown in Fig.2.2. □

Theorem 2.3 *If a H-graph G is a super mean graph, then $G \odot S_2$ is a super mean graph.*

Proof Let f be a super mean labeling of G with vertices $v_1, v_2, \ldots, v_n$ and $u_1, u_2, \ldots, u_n$. Let $v_1', v_2', \ldots, v_n'$, $v_1'', v_2'', \ldots, v_n''$, $u_1', u_2', \ldots, u_n'$ and $u_1'', u_2'', \ldots, u_n''$ be the corresponding new vertices in $G \odot S_2$.

We define $g : V(G \odot S_2) \to \{1, 2, \ldots, p+q\}$ as follows:

$$
\begin{aligned}
g(v_i) &= f(v_i) + 4i - 2, && 1 \le i \le n \\
g(v_i') &= f(v_i) + 4i - 4, && 1 \le i \le n \\
g(v_i'') &= f(v_i) + 4i, && 1 \le i \le n \\
g(u_i) &= f(u_i) + 4n + 4i - 2, && 1 \le i \le n \\
g(u_i') &= f(u_i) + 4n + 4i - 4, && 1 \le i \le n \\
g(u_i'') &= f(u_i) + 4n + 4i, && 1 \le i \le n
\end{aligned}
$$

For the vertex labeling g, the induced edge labeling g^* is defined as follows:

$$
\begin{aligned}
g^*(v_{\frac{n+1}{2}}u_{\frac{n+1}{2}}) &= 3f^*(v_{\frac{n+1}{2}}u_{\frac{n+1}{2}}) && \text{if } n \text{ is odd} \\
g^*(v_{\frac{n}{2}+1}u_{\frac{n}{2}}) &= 3f^*(v_{\frac{n}{2}+1}u_{\frac{n}{2}}) && \text{if } n \text{ is even}
\end{aligned}
$$

$$
\begin{aligned}
g^*(v_iv_{i+1}) &= f^*(v_iv_{i+1}) + 4i, && 1 \le i \le n-1 \\
g^*(v_iv_i') &= f(v_i) + 4i - 3, && 1 \le i \le n \\
g^*(v_iv_i'') &= f(v_i) + 4i - 1, && 1 \le i \le n \\
g^*(u_iu_{i+1}) &= f^*(u_iu_{i+1}) + 4n + 4i && 1 \le i \le n-1 \\
g^*(u_iu_i') &= f(u_i) + 4n + 4i - 3, && 1 \le i \le n \\
g^*(u_iu_i'') &= f(u_i) + 4n + 4i - 1, && 1 \le i \le n
\end{aligned}
$$

It can be easily verified that g is a super mean labeling and hence $G \odot S_2$ is a super mean graph. For example the super mean labelings of $G_1 \odot S_2$ and $G_2 \odot S_2$ are shown in Fig.2.3. □

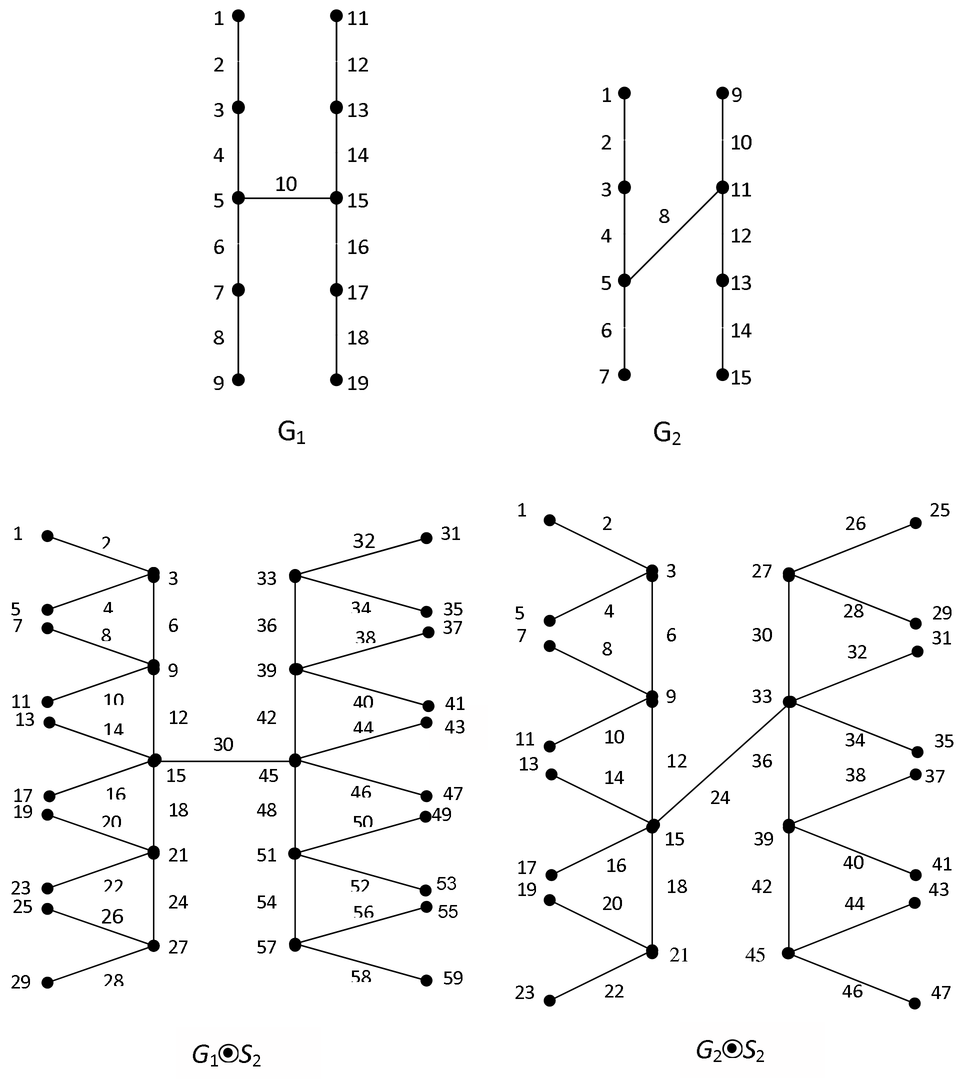


Fig.2.3

Theorem 2.4 *Cycle C_{2n} is a super mean graph for $n \geq 3$.*

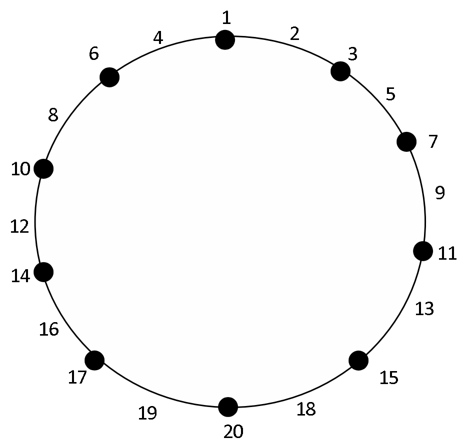
Proof Let C_{2n} be a cycle with vertices $u_1, u_2, \ldots, u_{2n}$ and edges $e_1, e_2, \ldots, e_{2n}$. Define $f : V(C_{2n}) \to \{1, 2, \ldots, p+q\}$ as follows:

$$
\begin{aligned}
f(u_1) &= 1 \\
f(u_i) &= 4i-5, & 2 \leq i \leq n \\
f(u_{n+j}) &= 4n-3j+3, & 1 \leq j \leq 2 \\
f(u_{n+j+2}) &= 4n-4j-2, & 1 \leq j \leq n-2
\end{aligned}
$$

For the vertex labeling f, the induced edge labeling f^* is defined as follows:

$$
\begin{aligned}
f^*(e_1) &= 2 \\
f^*(e_i) &= 4i-3, && 2 \le i \le n-1 \\
f^*(e_n) &= 4n-2, \\
f^*(e_{n+1}) &= 4n-1, \\
f^*(e_{n+j+1}) &= 4n-4j, && 1 \le j \le n-1
\end{aligned}
$$

It can be easily verified that f is a super mean labeling and hence C_{2n} is a super mean graph. For example the super mean labeling of C_{10} is shown in Fig.2.4. □



C_{10}

Fig.2.4

Remark 2.5 In [2], it was proved that $C_{2n+1}, n \ge 1$ is a super mean graph and C_4 is not a super mean graph and hence the cycle C_n is a super mean graph for $n \ge 3$ and $n \ne 4$.

Theorem 2.6 *Corona of a cycle C_n is a super mean graph for $n \ge 3$.*

Proof Let C_n be a cycle with vertices $u_1, u_2, \ldots, u_n$ and edges $e_1, e_2, \ldots, e_n$. Let $v_1, v_2, \ldots, v_n$ be the corresponding new vertices in $C_n \odot K_1$ and E_i be the edges joining $u_iv_i, i = 1$ to n.

Define $f : V(C_n \odot K_1) \to \{1, 2, \ldots, p+q\}$ as follows:

Case i When n is odd, $n = 2m+1, m = 1, 2, 3, \ldots$

$$f(u_1) = 3$$

$$f(u_i) = \begin{cases} 5 + 8(i-2) & 2 \le i \le m+1 \\ 12 + 8(2m+1-i) & m+2 \le i \le 2m+1 \end{cases}$$

$$f(v_1) = 1$$

$$f(v_i) = \begin{cases} 7 + 8(i-2) & 2 \le i \le m+1 \\ 10 + 8(2m+1-i) & m+2 \le i \le 2m+1 \end{cases}$$

For the vertex labeling f, the induced edge labeling f^* is defined as follows:

$$f^*(e_1) = 4$$

$$f^*(e_i) = \begin{cases} 9 + 8(i-2) & 2 \le i \le m+1 \\ 8 + 8(2m+1-i) & m+2 \le i \le 2m+1 \end{cases}$$

$$f^*(E_1) = 2$$

$$f^*(E_i) = \begin{cases} 6 + 8(i-2) & 2 \le i \le m+1 \\ 11 + 8(2m+1-i) & m+2 \le i \le 2m+1 \end{cases}$$

Case ii When n is even, $n = 2m, m = 2, 3, \ldots$

$$\begin{aligned}
f(u_1) &= 3 \\
f(u_i) &= 5 + 8(i-2), && 2 \le i \le m \\
f(u_{m+1}) &= 8m - 2, \\
f(u_i) &= 12 + 8(2m-i), && m+2 \le i \le 2m \\
f(v_1) &= 1 \\
f(v_i) &= 7 + 8(i-2), && 2 \le i \le m \\
f(v_{m+1}) &= 8m, \\
f(v_{m+2}) &= 8m - 7, \\
f(v_i) &= 10 + 8(2m-i), && m+3 \le i \le 2m
\end{aligned}$$

For the vertex labeling f, the induced edge labeling f^* is defined as follows:

$$\begin{aligned}
f^*(e_1) &= 4 \\
f^*(e_i) &= 9 + 8(i-2), && 2 \le i \le m-1 \\
f^*(e_m) &= 8m - 6, \\
f^*(e_{m+1}) &= 8m - 3, \\
f^*(e_i) &= 8 + 8(2m-i), && m+2 \le i \le 2m \\
f^*(E_1) &= 2 \\
f^*(E_i) &= 6 + 8(i-2), && 2 \le i \le m \\
f^*(E_{m+1}) &= 8m - 1 \\
f^*(E_i) &= 11 + 8(2m-i), && m+2 \le i \le 2m
\end{aligned}$$

It can be easily verified that f is a super mean labeling and hence $C_n \odot K_1$ is a super mean graph. For example the super mean labelings of $C_7 \odot K_1$ and $C_8 \odot K_1$ are shown in Fig.2.5.□

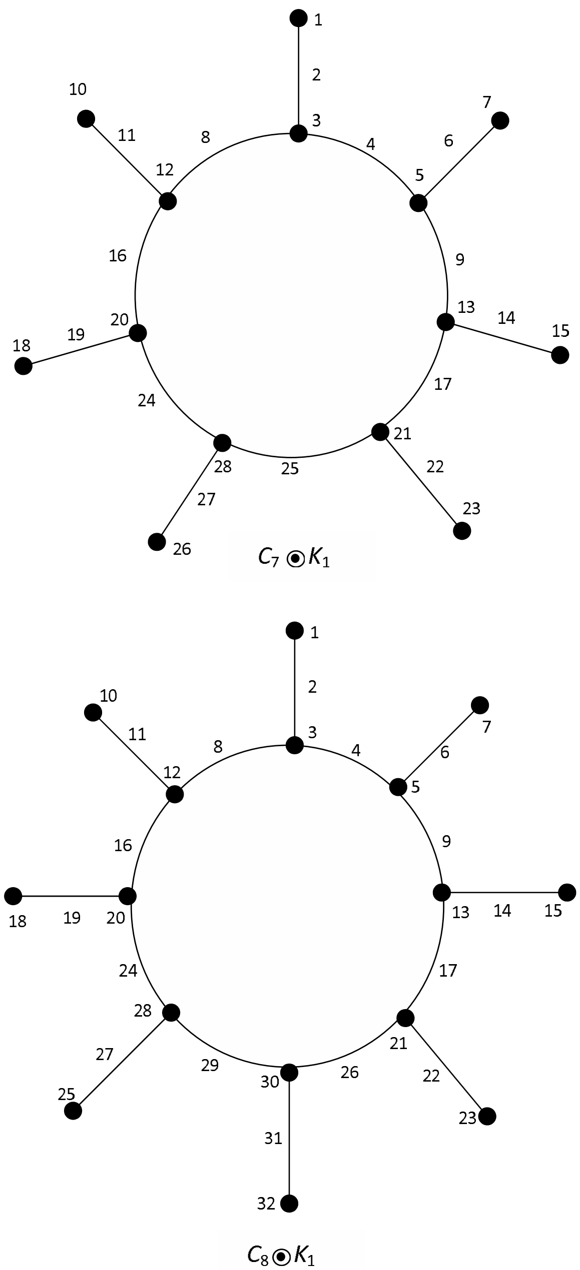


Fig.2.5

Remark 2.7 C_4 is not a super mean graph, but $C_4 \odot K_1$ is a super mean graph.

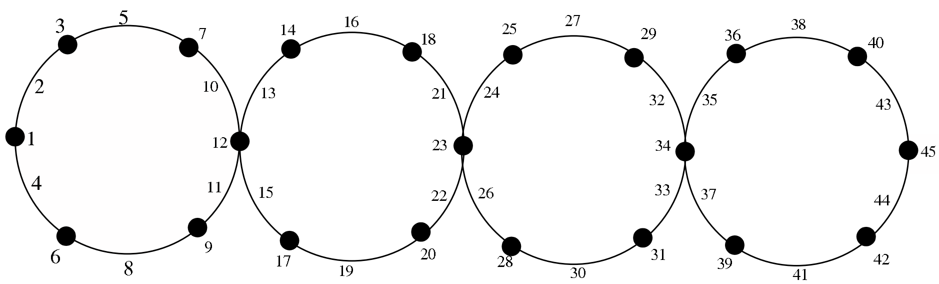
Theorem 2.8 *The graph mC_n- snake, $m \geq 1, n \geq 3$ and $n \neq 4$ has a super mean labeling.*

Proof We prove this result by induction on m.

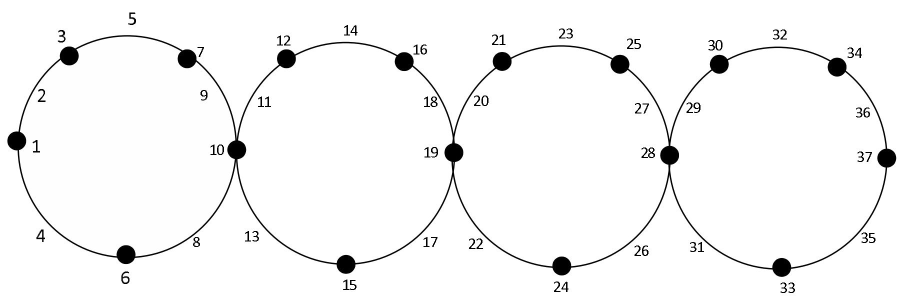
Let $v_{1_j}, v_{2_j}, \ldots, v_{n_j}$ be the vertices and $e_{1_j}, e_{2_j}, \ldots, e_{n_j}$ be the edges of mC_n for $1 \leq j \leq m$. Let f be a super mean labeling of the cycle C_n.

When $m = 1$, by Remark 1.5, C_n is a super mean graph, $n \geq 3, n \neq 4$. Hence the result is true when $m = 1$.

Let $m = 2$. The cyclic snake $2C_n$ is the graph obtained from 2 copies of C_n by identifying the vertex $v_{(k+2)_1}$ in the first copy of C_n at a vertex v_{1_2} in the second copy of C_n when $n = 2k+1$ and identifying the vertex $v_{(k+1)_1}$ in the first copy of C_n at a vertex v_{1_2} in the second copy of C_n when $n = 2k$.



$4C_6$-snake



$4C_5$-snake

Fig.2.6

Define a super mean labeling g of $2C_n$ as follows:

For $1 \leq i \leq n$,

$$
\begin{aligned}
g(v_{i_1}) &= f(v_{i_1}) \\
g(v_{i_2}) &= f(v_{i_1}) + 2n - 1 \\
g^*(e_{i_1}) &= f^*(e_{i_1}) \\
g^*(e_{i_2}) &= f^*(e_{i_1}) + 2n - 1.
\end{aligned}
$$

Thus, $2C_n$-snake is a super mean graph.

Assume that mC_n-snake is a super mean graph for any $m \geq 1$. We will prove that $(m+1)C_n$-snake is a super mean graph. Super mean labeling g of $(m+1)C_n$ is defined as follows:

$$
\begin{aligned}
g(v_{i_j}) &= f(v_{i_1}) + (j-1)(2n-1), && 1 \leq i \leq n, 2 \leq j \leq m \\
g(v_{i_{m+1}}) &= f(v_{i_1}) + m(2n-1), && 1 \leq i \leq n
\end{aligned}
$$

For the vertex labeling g, the induced edge labeling g^* is defined as follows:

$$\begin{aligned} g^*(e_{i_j}) &= f^*(e_{i_1}) + (j-1)(2n-1), && 1 \le i \le n, 2 \le j \le m \\ g^*(e_{i_{m+1}}) &= f^*(e_{i_1}) + m(2n-1), && 1 \le i \le n \end{aligned}$$

Then it is easy to check the resultant labeling g is a super mean labeling of $(m+1)C_n$-snake. For example the super mean labelings of $4C_6$-snake and $4C_5$- snake are shown in Fig.2.6. □

Theorem 2.9 *If G is a super mean graph then $P_n(G)$ is also a super mean graph.*

Proof Let f be a super mean labeling of G. Let $v_1, v_2, \ldots, v_p$ be the vertices and $e_1, e_2, \ldots, e_q$ be the edges of G and let $u_1, u_2, \ldots, u_n$ and $E_1, E_2, \ldots, E_{n-1}$ be the vertices and edge of P_n respectively.

We define g on $P_n(G)$ as follows:

$$\begin{aligned} g(v_i) &= f(v_i), && 1 \le i \le p. \\ g(u_j) &= p + q + 2j - 2, && 1 \le j \le n. \end{aligned}$$

For the vertex labeling g, the induced edge labeling g^* is defined as follows:

$$\begin{aligned} g^*(e_i) &= f(e_i) && 1 \le i \le p. \\ g^*(E_j) &= p + q + 2j - 1, && 1 \le j \le n-1. \end{aligned}$$

Then g is a super mean labeling of $P_n(G)$. □

Corollary 1.10 *Dragon $P_n(C_m)$ is a super mean graph for $m \ge 3$ and $m \ne 4$.*

Proof Since C_m is a super mean graph for $m \ge 3$ and $m \ne 4$, by using the above theorem, $P_n(C_m)$ for $m \ge 3$ and $m \ne 4$ is also a super mean graph. For example, the super mean labeling of $P_5(C_6)$ is shown in Fig.2.7. □

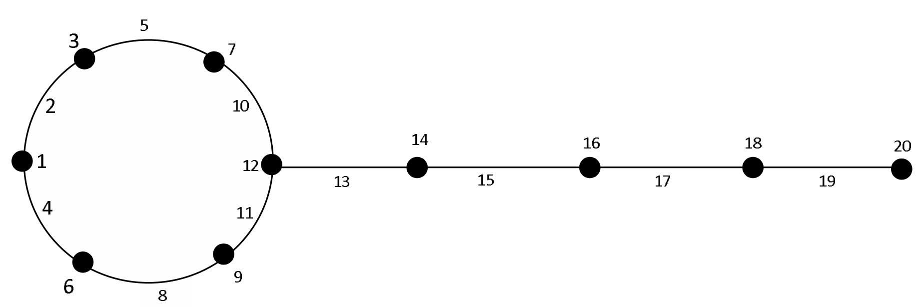


Fig.2.7

Remark 2.11 The converse of the above theorem need not be true. For example consider the graph C_4. $P_n(C_4)$ for $n \ge 3$ is a super mean graph but C_4 is not a super mean graph. The super mean labeling of the graph $P_4(C_4)$ is shown in Fig.2.8

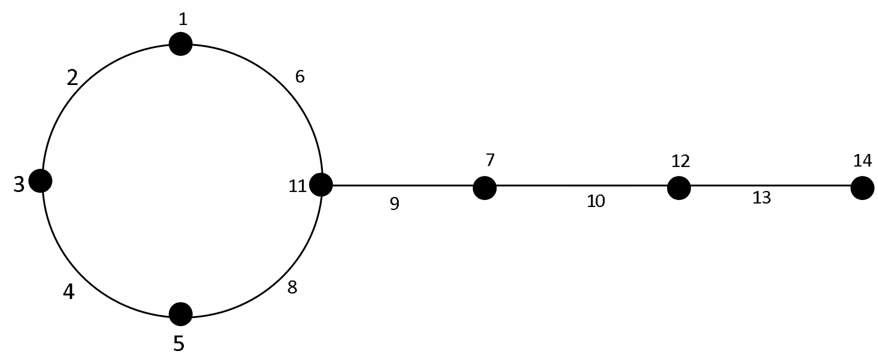


Fig.2.8

Theorem 2.12 *$C_m \times P_n$ for $n \geq 1, m = 3, 5$ are super mean graphs.*

Proof Let $V(C_m \times P_n) = \{v_{ij} : 1 \leq i \leq m, 1 \leq j \leq n\}$ and $E(C_m \times P_n) = \{e_{i_j} : e_{i_j} = v_{i_j} v_{(i+1)_j}, 1 \leq j \leq n, 1 \leq i \leq m\} \cup \{E_{i_j} : E_{i_j} = v_{i_j} v_{i_{j+1}}, 1 \leq j \leq n-1, 1 \leq i \leq m\}$ where $i+1$ is taken modulo m.

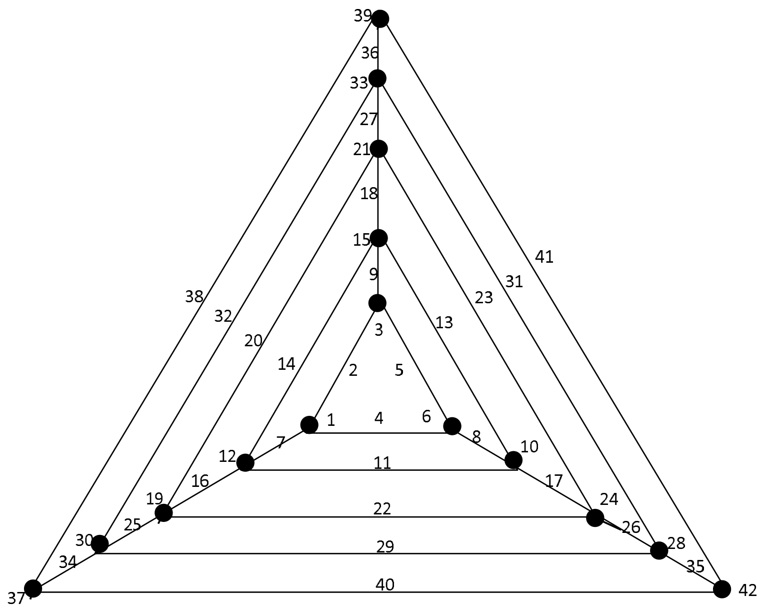
Case i $m = 3$

First we label the vertices of C_3^1 and C_3^2 as follows:

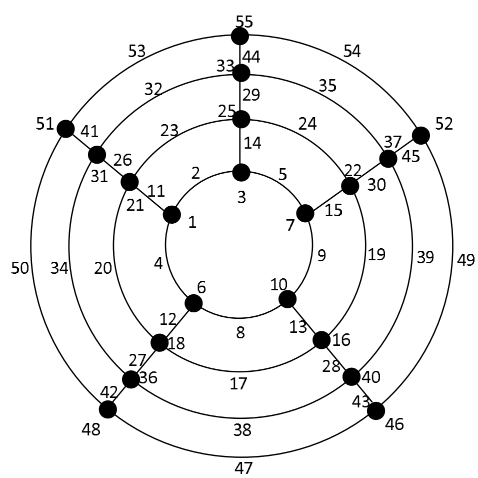
$$
\begin{aligned}
f(v_{1_1}) &= 1 \\
f(v_{i_1}) &= 3i - 3, && 2 \leq i \leq 3 \\
f(v_{i_2}) &= 12 + 3(i-1), && 1 \leq i \leq 2 \\
f(v_{3_2}) &= 10
\end{aligned}
$$

For the vertex labeling f, the induced edge labeling f^* is defined as follows:

$$
\begin{aligned}
f^*(e_{i_1}) &= 2 + 3(i-1), && 1 \leq i \leq 2 \\
f^*(e_{3_1}) &= 4 \\
f^*(e_{1_2}) &= 14 \\
f^*(e_{i_2}) &= 13 - 2(i-2), && 2 \leq i \leq 3 \\
f^*(E_{i_1}) &= 7 + 2(i-1), && 1 \leq i \leq 2 \\
f^*(E_{3_1}) &= 8
\end{aligned}
$$



$C_3 \times P_5$



$C_5 \times P_4$

Fig.2.9

If the vertices and edges of C_3^{2j-1} and C_3^{2j} are labeled then the vertices and edges of C_3^{2j+1} and C_3^{2j+2} are labeled as follows:

$$\begin{aligned}
f(v_{i_{2j+1}}) &= f(v_{i_{2j-1}}) + 18, && 1 \le i \le 3, 1 \le j \le \tfrac{n-1}{2} \text{ if } n \text{ is odd and}\\
& && 1 \le j \le \tfrac{n-2}{2} \text{ if } n \text{ is even}\\
f(v_{i_{2j+2}}) &= f(v_{i_{2j}}) + 18, && 1 \le i \le 3, 1 \le j \le \tfrac{n-3}{2} \text{ if } n \text{ is odd and}\\
& && 1 \le j \le \tfrac{n-2}{2} \text{ if } n \text{ is even.}\\
f^*(e_{i_{2j+1}}) &= f^*(e_{i_{2j-1}}) + 18, && 1 \le i \le 3, 1 \le j \le \tfrac{n-1}{2} \text{ if } n \text{ is odd and}\\
& && 1 \le j \le \tfrac{n-2}{2} \text{ if } n \text{ is even}\\
f^*(e_{j_{2j+2}}) &= f^*(e_{i_{2j}}) + 18, && 1 \le i \le 3, 1 \le j \le \tfrac{n-3}{2} \text{ if } n \text{ is odd and}\\
& && 1 \le j \le \tfrac{n-2}{2} \text{ if } n \text{ is even}\\
f^*(E_{i_{2j+1}}) &= f^*(E_{i_{2j-1}}) + 18, && 1 \le i \le 3, 1 \le j \le \tfrac{n-3}{2} \text{ if } n \text{ is odd and}\\
& && 1 \le j \le \tfrac{n-2}{2} \text{ if } n \text{ is even}
\end{aligned}$$

$$\begin{aligned}
f^*(E_{i_{2j+2}}) &= f^*(E_{i_{2j}}) + 18, && 1 \le i \le 3, 1 \le j \le \tfrac{n-3}{2} \text{ if } n \text{ is odd and}\\
& && 1 \le j \le \tfrac{n-4}{2} \text{ if } n \text{ is even}
\end{aligned}$$

Case ii $m = 5$.

First we Label the vertices of C_5^1 and C_5^2 as follows:

$f(v_{1_1}) = 1$

$$f(v_{i_1}) = \begin{cases} 4i-5, & 2 \le i \le 3\\ 10-4(i-4), & 4 \le i \le 5 \end{cases}$$

$f(v_{1_2}) = 21$

$$f(v_{i_2}) = \begin{cases} 25-3(i-2), & 2 \le i \le 3\\ 16+2(i-4) & 4 \le i \le 5 \end{cases}$$

For the vertex labeling f, the induced edge labeling f^* is defined as follows:

$f^*(e_{i_1}) = 2 + 3(i-1), \qquad 1 \le i \le 2$

$f^*(e_{3_1}) = 9$

$f^*(e_{i_1}) = 8 - 4(i-4), \qquad 4 \le i \le 5$

$$f^*(e_{i_2}) = \begin{cases} 23+(i-1), & 1 \le i \le 2\\ 19-2(i-3), & 3 \le i \le 4 \end{cases}$$

$f^*(e_{5_2}) = 20,$

$f^*(E_{1_1}) = 11$

$$f^*(E_{i_1}) = \begin{cases} 14+(i-2), & 2 \le i \le 3\\ 13-(i-4), & 4 \le i \le 5 \end{cases}$$

If the vertices and edges of C_5^{2j-1} and C_5^{2j} are labeled then the vertices and edges of C_5^{2j+1} and C_5^{2j+2} are labeled as follows:

$$
\begin{aligned}
f(v_{i_{2j+1}}) &= l(v_{i_{2j-1}}) + 30, 1 \le i \le 5, && 1 \le j \le \tfrac{n-2}{2} \text{ if } n \text{ is even and} \\
& && 1 \le j \le \tfrac{n-1}{2} \text{ if } n \text{ is odd} \\
f(v_{i_{2j+2}}) &= l(v_{i_{2j}}) + 30, 1 \le i \le 5, && 1 \le j \le \tfrac{n-2}{2} \text{ if } n \text{ is even and} \\
& && 1 \le j \le \tfrac{n-3}{2} \text{ if } n \text{ is odd.} \\
f^*(E_{i_{2j+1}}) &= f^*(E_{i_{2j-1}}) + 30, 1 \le i \le 5, && 1 \le j \le \tfrac{n-3}{2} \text{ if } n \text{ is odd and} \\
& && 1 \le j \le \tfrac{n-2}{2} \text{ if } n \text{ is even} \\
f^*(E_{i_{2j+2}}) &= f^*(E_{i_{2j}}) + 30, 1 \le i \le 5, && 1 \le j \le \tfrac{n-3}{2} \text{ if } n \text{ is odd and} \\
& && 1 \le j \le \tfrac{n-4}{2} \text{ if } n \text{ is even} \\
f^*(e_{i_{2j+1}}) &= f^*(e_{i_{2j-1}}) + 30, 1 \le i \le 5, && 1 \le j \le \tfrac{n-2}{2} \text{ if } n \text{ is even and} \\
& && 1 \le j \le \tfrac{n-1}{2} \text{ if } n \text{ is odd} \\
f^*(e_{i_{2j+2}}) &= f^*(e_{i_{2j}}) + 30, 1 \le i \le 5, && 1 \le j \le \tfrac{n-2}{2} \text{ if } n \text{ is even and} \\
& && 1 \le j \le \tfrac{n-3}{2} \text{ if } n \text{ is odd.}
\end{aligned}
$$

Then it is easy to check that the labeling f is a super mean labeling of $C_3 \times P_n$ and $C_5 \times P_n$. For example the super mean labeling of $C_3 \times P_5$ and $C_5 \times P_4$ are shown in Fig.2.9. □

§3. Open Problems

We present the following open problem for further research.

Open Problem. *For what values of m (except 3,5) the graph $C_m \times P_n$ is super mean graph.*

References

[1] R. Balakrishnan and N. Renganathan, *A Text Book on Graph Theory*, Springer Verlag, 2000.

[2] R. Ponraj and D. Ramya, Super Mean labeling of graphs, Preprint.

[3] R. Ponraj and D. Ramya, On super mean graphs of order ≤ 5, *Bulletin of Pure and Applied Sciences,* (Section E Maths and Statistics) **25E**(2006), 143-148.

[4] R. Ponraj and S. Somasundaram, *Further results on mean graphs*, *Proceedings of Sacoeference*, August 2005. 443 - 448.

[5] R. Ponraj and S. Somasundaram, Mean graphs obtained from mean graphs, *Journal of Discrete Mathematical Sciences & Cryptography*, **11**(2)(2008), 239-252.

[6] Selvam Avadayappan and R. Vasuki, Some results on mean graphs, *Ultra Scientist of Physical Sciences,* **21**(1)M (2009), 273-284.

[7] S. Somasundaram and R. Ponraj, Mean labelings of graphs, *National Academy Science letter,* **26** (2003), 210 - 213.

[8] S. Somasundaram and R. Ponraj, On Mean graphs of order ≤ 5, *Journal of Decision and Mathematical Sciences* **9**(1-3) (2004), 48-58.

International J.Math. Combin. Vol.3 (2009), 97-102

Chromatic Polynomial of Smarandache ν_E-Product of Graphs

Khalil Paryab

(Department of Mathematics of Iran University of Science and Technology, Tehran, Iran.)

Ebrahim Zare

(Department of Mathematics of Imamali University, Tehran, Iran.)

Email: paryab@iust.ac.ir, zare.math@gmail.com

Abstract: Let $G_1 = (V_1, E_1)$, $G_2 = (V_2, E_2)$ be two graphs. For a chosen edge set $E \subset E_2$, the *Smarandache ν_E-product* $G_1 \times_{\nu_E} G_2$ of $G_1,\ G_2$ is defined by

$$V(G_1 \times_{\nu_E} G_2) = V_1 \times V_2,$$

$$E(G_1 \times_{\nu_E} G_2) = \{(a,b)(a',b') | a = a', (b,b') \in E_2, \text{or } b = b', (a,a') \in E_1\}$$
$$\cup\{(a,b)(a',b') | (a,a') \in E_1 \text{ and } (b,b') \in E\}.$$

Particularly, if $E = \emptyset$ or E_2, then $G_1 \times_{\nu_E} G_2$ is the *Cartesian product* $G_1 \times G_2$ or *strong product* $G_1 * G_2$ of G_1 and G_2 in graph theory. Finding the chromatic polynomial of Smarandache ν_E-product of two graphs is an unsolved problem in general, even for the Cartesian product and strong product of two graphs. In this paper we determine the chromatic polynomial in the case of the Cartesian and strong product of a tree and a complete graph.

Keywords: Coloring graph, Smarandache ν_E-product graph, strong product graph, Cartesian product graph, chromatic polynomial.

AMS(2000): 05C15

§1. Introduction

Sabidussi and Vizing defined Graph products first time in [4] [5]. A lot of works has been done on various topics related to graph products, however there are still many open problems [3]. Generally, we can construct Smarandache ν_E-product of graphs G_1 and G_2 for $E \subset E(G_2)$ as follows.

Let $G_1 = (V_1, E_1)$, $G_2 = (V_2, E_2)$ be two graphs. For a chosen edge set $E \subset E_2$, the *Smarandache ν_E-product* $G_1 \times_{\nu_E} G_2$ of $G_1,\ G_2$ is defined by

$$V(G_1 \times_{\nu_E} G_2) = V_1 \times V_2,$$

$$E(G_1 \times_{\nu_E} G_2) = \{(a,b)(a',b') | a = a', (b,b') \in E_2, \text{or } b = b', (a,a') \in E_1\}$$
$$\cup\{(a,b)(a',b') | (a,a') \in E_1 \text{ and } (b,b') \in E\}.$$

[1]Received July 28, 2009. Accepted Aug.30, 2009.