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About Smarandache prime additive complement

Yanchun Guo[†] [‡]

[†] Department of Mathematics, Northwest University, Xi'an, Shaanxi, P.R.China

[‡] Department of Mathematics, Xianyang Normal College, Xianyang, Shaanxi, P.R.China

Abstract For any positive integer n, the Smarandache prime additive complement function $SPAC(n)$ is defined as the smallest integer $k \geq 0$ such that $n + k$ is a prime. The main purpose of this paper is using the elementary method to prove that it is possible to have k as large as we want $k,\ k-1,\ k-2,\ \cdots,\ 2,\ 1,\ 0$ included in the sequence $\{SPAC(n)\}$.

Keywords The Smarandache prime additive complement, sequence.

§1. Introduction and Results

For any positive integer n, the famous Smarandache prime additive complement function $SPAC(n)$ is defined as the smallest integer $k \geq 0$ such that $n + k$ is a prime. The first few value of this sequence are:

$$1,\ 0,\ 0,\ 1,\ 0,\ 1,\ 0,\ 3,\ 2,\ 1,\ 0,\ 1,\ 0,\ 3,\ 2,\ 1,\ 0,\ 1,\ 0,\ 3,\ 2,\ 1,\ 0,\ 5,\ 4,\ 3,\ 2,\ 1,\ 0, \cdots.$$

In the book "Only problems, not solutions", Professor F.Smarandache asked us to study the properties of the sequence $\{SPAC(n)\}$. He also proposed the following problem:

Problem A. If it is possible to have k as large as we want

$$k,\ k-1,\ k-2,\ k-3,\ \cdots,\ 2,\ 1,\ 0\ (\text{ odd } k)$$

included in the sequence $\{SPAC(n)\}$.In reference [6], Kenichiro Kashihara proposed another problem as follows:

Problem B. Is it possible to have k as large as we want

$$k,\ k-1,\ k-2,\ k-3,\ \cdots,\ 2,\ 1,\ 0\ (\text{ even } k\ )$$

included in this sequence.

About these two problems, it seems that none had studied them yet, at least we have not seen such a paper before. The problems are important and interesting, because there are close relationship between the sequence $\{SPAC(n)\}$ and the prime distribution. The main purpose of this paper is using the elementary method to study these two problems, and proved that they are true. That is, we shall prove the following:

Theorem. It is possible to have the positive integer k as large as we want

$$k,\ k-1,\ k-2,\ k-3,\ \cdots,\ 2,\ 1,\ 0$$

included in the sequence $\{SPAC(n)\}$.

§2. Proof of the theorem

In this section, we shall prove our theorem directly. Let k and n are positive integers with $n > k+1$, here k as large as we want. Let P be the smallest prime such that $P > n! + n$. It is clear that $P-1$, $P-2$, $\cdots$, $P-k$, $\cdots$, $n!+n$, $\cdots$, $n!+2$ are all composite numbers. Now we consider $k+1$ positive integers:

$$p-k,\ p-k+1,\ p-k+2,\ \cdots,\ p-1,\ p.$$

The Smarandache prime additive complements of these numbers are

$$SPAC(p-k) = k,\ SPAC(p-k-1) = k-1,\ \cdots,\ SPAC(p-1) = 1,\ SPAC(p) = 0.$$

Note that the numbers $k,\ k-1,\ k-2,\ \cdots,\ 1,\ 0$ are included in the sequence $\{SPAC(n)\}$. So the Problem A and Problem B are true.

This completes the proof of the theorem.

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