

# Smarandache representation and its applications

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Here we for the first time define Smarandache representation of finite  $S$ -bisemigroup. We know every  $S$ -bisemigroup,  $S = S_1 \cup S_2$  contains a bigroup  $G = G_1 \cup G_2$ . The Smarandache representation  $S$ -bisemigroups depends on the  $S$ -bigroup  $G$  which we choose. Thus this method widens the Smarandache representations. We first define the notion of Smarandache pseudo neutrosophic bisemigroup.

**Definition 1.** Let  $S = S_1 \cup S_2$  be a neutrosophic bisemigroup. If  $S$  has only bi-group which is not a neutrosophic bigroup, then we call  $S$  a Smarandache pseudo neutrosophic bisemigroup ( $S$ -pseudo neutrosophic bisemigroup).

**Example 1.** Let  $S = S_1 \cup S_2$  where  $S_1 = Q(I) \times Q(I)$  and  $S_2 = \{2 \times 2 \text{ matrices with entries from } Q(I)\}$  both  $S_1$  and  $S_2$  under multiplication is a semigroup. Thus  $S$  is a neutrosophic bisemigroup. Take  $G = G_1 \cup G_2$  where  $G_1 = \{Q \setminus (0) \times Q \setminus (0)\}$  and  $G_2 = \{\text{set of all } 2 \times 2 \text{ matrices } A \text{ with entries from } Q \text{ such that } |A| \neq 0\}$ .  $G_1$  and  $G_2$  are groups under multiplication. So  $S$  is a pseudo Smarandache Neutrosophic bisemigroup.

Now we give the Smarandache representation of finite pseudo Smarandache neutrosophic bisemigroups.

**Definition 2.** Let  $G = G_1 \cup G_2$  be a Smarandache neutrosophic bisemigroup and  $V = V_1 \cup V_2$  be a bivector space. A Smarandache birepresentation of  $G$  on  $V$  is a mapping  $S_\rho = S_\rho^1 \cup S_\rho^2$  from  $H_1 \cup H_2$  ( $H_1 \cup H_2$  is a subbigroup of  $G$  which is not a neutrosophic bigroup) to invertible linear bitransformation on  $V = V_1 \cup V_2$  such that

$$S_{\rho_{xy}} = S_{\rho_{x_1 y_1}}^1 \cup S_{\rho_{x_2 y_2}}^2 = \left( S_{\rho_{x_1}}^1 \circ S_{\rho_{y_1}}^1 \right) \cup \left( S_{\rho_{x_2}}^2 \circ S_{\rho_{y_2}}^2 \right)$$

for all  $x_1, y_1 \in H_1$  and for all  $x_2, y_2 \in H_2$ ,  $H_1 \cup H_2 \subset G_1 \cup G_2$ . Here  $S_{\rho_x} = S_{\rho_{x_1}}^1 \cup S_{\rho_{x_2}}^2$  to denote the invertible linear bitransformation on  $V = V_1 \cup V_2$  associated to  $x = x_1 \cup x_2$  on  $H = H_1 \cup H_2$ , so that we may write

$$S_{\rho_x}(\nu) = S_{\rho_x}(\nu_1 \cup \nu_2) = S_{\rho_{x_1}}^1(\nu_1) \cup S_{\rho_{x_2}}^2(\nu_2)$$

for the image of the vector  $\nu = \nu_1 \cup \nu_2$  in  $V = V_1 \cup V_2$  under  $S_{\rho_x} = S_{\rho_{x_1}}^1 \cup S_{\rho_{x_2}}^2$ . As a result, we have that  $S_{\rho_e} = S_{\rho_{e_1}}^1 \cup S_{\rho_{e_2}}^2 = I^1 \cup I^2$  denotes the identity bitransformation on  $V = V_1 \cup V_2$  and  $S_{\rho_x}^{-1} = S_{\rho_{x_1}^{-1}}^1 \cup S_{\rho_{x_2}^{-1}}^2 = \left(S_{\rho_{x_1}}^1\right)^{-1} \cup \left(S_{\rho_{x_2}}^2\right)^{-1}$  for all  $x = x_1 \cup x_2 \in H_1 \cup H_2 \subset G_1 \cup G_2 = G$ .

In other words a birepresentation of  $H = H_1 \cup H_2$  on  $V = V_1 \cup V_2$  is a bihomomorphism from  $H$  into  $GL(V)$  i.e.  $(H_1 \text{ into } GL(V_1)) \cup (H_2 \text{ into } GL(V_2))$ . The bidimension of  $V = V_1 \cup V_2$  is called the bidegree of the representation.

Thus depending on the number of subbgroup of the  $S$ -neutrosophic bisemigroup we have several  $S$ -birepresentations of the finite  $S$ -neutrosophic bisemigroup.

Basic example of birepresentation would be Smarandache left regular birepresentation and Smarandache right regular birepresentation over a field  $K$  defined as follows.

We take  $V_H = V_{H_1} \cup V_{H_2}$  to be a bivector space of bifunctions on  $H_1 \cup H_2$  with values in  $K$  (where  $H = H_1 \cup H_2$  is a subbgroup of the  $S$ -neutrosophic bisemigroup where  $H$  is not a neutrosophic bigroup). For Smarandache left regular birepresentation ( $S$ -left regular biregular representative) relative to  $H = H_1 \cup H_2$  we define

$$SL_x = S^1 L_{x_1}^1 \cup S^2 L_{x_2}^2 = (S^1 \cup S^2) (L^1 \cup L_2)_{x_1 \cup x_2},$$

from  $V_{H_1} \cup V_{H_2} \rightarrow V_{H_1} \cup V_{H_2}$  for each  $x_1 \cup x_2 \in H = H_1 \cup H_2$  by for each  $x = x_1 \cup x_2$  in  $H = H_1 \cup H_2$  by  $SL_x(f)(z) = f(x^{-1}z)$  for each function  $f(z)$  in  $V_H = V_{H_1} \cup V_{H_2}$  i.e.  $S^1 L_{x_1}^1 f_1(z_1) \cup S^2 L_{x_2}^2 f_2(z_2) = f_1(x_1^{-1}z_1) \cup f_2(x_2^{-1}z_2)$ .

For the Smarandache right regular birepresentation ( $S$ -right regular birepresentation) we define  $SR_x = SR_{x_1 \cup x_2} : V_{H_1} \cup V_{H_2} \rightarrow V_{H_1} \cup V_{H_2}; H_1 \cup H_2 = H$  for each  $x = x_1 \cup x_2 \in H_1 \cup H_2$  by  $SR_x(f)(z) = f(zx)$ .

$$S^1 R_{x_1}^1 f_1(z_1) \cup S^2 R_{x_2}^2 (f_2(z_2)) = f_1(z_1 x_1) \cup f_2(z_2 x_2),$$

for each function  $f_1(z_1) \cup f_2(z_2) = f(z)$  in  $V_H = V_{H_1} \cup V_{H_2}$ .

Thus if  $x = x_1 \cup x_2$  and  $y = y_1 \cup y_2$  are elements  $H_1 \cup H_2 \subset G_1 \cup G_2$ .

Then

$$\begin{aligned} (SL_x \circ SL_y) (f(z)) &= SL_x(SL_y(f))(z) \\ &= (SL_y(f))x^{-1}z \\ &= f(y^{-1}x^{-1}z) \\ &= f_1(y_1^{-1}x_1^{-1}z_1) \cup f_2(y_2^{-1}x_2^{-1}z_2) \\ &= f_1((x_1 y_1)^{-1}z_1) \cup f_2((x_2 y_2)^{-1}z_2) \\ &= S^1 L_{x_1 y_1}^1 (f_1)(z_1) \cup S^2 L_{x_2 y_2}^2 f_2(z_2) \\ &= [(S^1 L_{x_1}^1 \cup S^2 L_{x_2}^2) (S^1 L_{y_1}^1 \cup S^2 L_{y_2}^2)] f(z_1 \cup z_2) \\ &= [(S^1 L_{x_1}^1 \cup S^2 L_{x_2}^2) \circ (S^1 L_{y_1}^1 \cup S^2 L_{y_2}^2)] (f_1(z_1) \cup f_2(z_2)). \end{aligned}$$

and

$$\begin{aligned}
(SR_x \circ SR_y)(f)(z) &= (S^1 R_{x_1}^1 \cup S^2 R_{x_2}^2) \circ (S^1 R_{y_1}^1 \cup S^2 R_{y_2}^2) (f)(z) \\
&= SR_x(SR_y(f))(z) \\
&= (S^1 R_{x_1}^1 \cup S^2 R_{x_2}^2) \circ (S^1 R_{y_1}^1 \cup S^2 R_{y_2}^2) (f)(z) \\
&= (S^1 R_{y_1}^1 (f_1) \cup S^2 R_{y_2}^2 (f_2)) (z_1 x_1 \cup z_2 x_2) \\
&= f_1(z_1 x_1 y_1) \cup f_2(z_2 x_2 y_2) \\
&= S^1 R_{x_1 y_1}^1 f_1(z_1) \cup S^2 R_{x_2 y_2}^2 f_2(z_2) \\
&= SR_{xy}(f)(z).
\end{aligned}$$

Thus for a given  $S$ -neutrosophic bisemigroup we will have several  $V$ 's associated with them i.e. bivector space functions on each  $H_1 \cup H_2 \subset G_1 \cup G_2$ ,  $H$  a subbigroup of the  $S$ -neutrosophic bisemigroup with values from  $K$ . This study in this direction is innovative.

We have yet another Smarandache birepresentation which can be convenient is the following. For each  $w = w_1 \cup w_2$  in  $H = H_1 \cup H_2$ ,  $H$  bisubgroups of the  $S$ -neutrosophic bisemigroup  $G = G_1 \cup G_2$ .

Define a bifunction

$$\phi_w(z) = \phi_{w_1}^1(z_1) \cup \phi_{w_2}^2(z_2)$$

on  $H_1 \cup H_2 = H$  by  $\phi_{w_1}^1(z_1) \cup \phi_{w_2}^2(z_2) = 1 \cup 1$ , where  $w = w_1 \cup w_2 = z = z_1 \cup z_2$ ,  $\phi_{w_1}^1(z_1) \cup \phi_{w_2}^2(z_2) = 0 \cup 0$  when  $z \neq w$ .

Thus the functions  $\phi_w = \phi_{w_1}^1 \cup \phi_{w_2}^2$  for  $w = w_1 \cup w_2$  in  $H = H_1 \cup H_2$  ( $H \subset G$ ) form a basis for the space of bifunctions on each  $H = H_1 \cup H_2$  contained in  $G = G_1 \cup G_2$ .

One can check that

$$SL_x(\phi_w) = (\phi_{xw}) \text{ i.e. } S^1 L_{x_1}^1(\phi_{w_1}) \cup S^2 L_{x_2}^2(\phi_{w_2}) = \phi_{x_1 w_1}^1 \cup \phi_{x_2 w_2}^2,$$

$$SR_x(\phi_w) = \phi_{xw} \text{ i.e. } S^1 R_{x_1}^1(\phi_{w_1}^1) \cup S^2 R_{x_2}^2(\phi_{w_2}^2) = \phi_{x_1 w_1}^1 \cup \phi_{x_2 w_2}^2,$$

for all  $x \in H_1 \cup H_2 \subset G$ .

Observe that

$$SL_x \circ SR_y = SR_y \circ SL_x \text{ i.e. } (S^1 L_{x_1}^1 \cup S^2 L_{x_2}^2) \circ (S^1 L_{y_1}^1 \cup S^2 L_{y_2}^2)$$

$$(S^1 L_{y_1}^1 \cup S^2 L_{y_2}^2) \circ (S^1 L_{x_1}^1 \cup S^2 L_{x_2}^2),$$

for all  $x = x_1 \cup x_2$  and  $y = y_1 \cup y_2$  in  $G = G_1 \cup G_2$ .

More generally suppose we have a bihomomorphism from the bigroups  $H = H_1 \cup H_2 \subset G = G_1 \cup G_2$  ( $G$  a  $S$ -neutrosophic bisemigroup) to the bigroup of permutations on a non empty finite biset.  $E^1 \cup E^2$ . That is suppose for each  $x_1$  in  $H_1 \subset G_1$  and  $x_2$  in  $H_2$ ,  $H_2 \subset G_2$ ,  $x$  in  $H_1 \cup H_2 \subset G_1 \cup G_2$  we have a bipermutation  $\pi_{x_1}^1 \cup \pi_{x_2}^1$  on  $E_1 \cup E_2$  i.e. one to one mapping of  $E_1$  on to  $E_1$  and  $E_2$  onto  $E_2$  such that

$$\pi_x \circ \pi_y = \pi_{x_1}^1 \circ \pi_{y_1}^1 \cup \pi_{x_2}^2 \circ \pi_{y_2}^2, \pi_e = \pi_{e_1}^1 \cup \pi_{e_2}^2$$

is the biidentity bimapping of  $E_1 \cup E_2$  and  $\pi_{x^{-1}} = \pi_{x_1^{-1}}^1 \cup \pi_{x_2^{-1}}^1$  is the inverse mapping of  $\pi_x = \pi_{x_1}^1 \cup \pi_{x_2}^2$  on  $E_1 \cup E_2$ . Let  $V_H = V_{H_1}^1 \cup V_{H_2}^2$  be the bivector space of  $K$ -valued bifunctions on  $E_1 \cup E_2$ .

Then we get the Smarandache birepresentation of  $H_1 \cup H_2$  on  $V_{H_1} \cup V_{H_2}$  by associating to each  $x = x_1 \cup x_2$  in  $H_1 \cup H_2$  the linear bimapping

$$\pi_x = \pi_{x_1}^1 \cup \pi_{x_2}^2 : V_{H_1} \cup V_{H_2} \rightarrow V_{H_1} \cup V_{H_2},$$

defined by

$$\pi_x(f)(a) = f(\pi_x(a)) \text{ i.e. } (\pi_{x_1} \cup \pi_{x_2})(f^1 \cup f^2)(a_1 \cup a_2) = f^1(\pi_{x_1}(a_1)) \cup f^2(\pi_{x_2}(a_2))$$

for every  $f^1(a_1) \cup f^2(a_2) = f(a)$  in  $V_{H_1} \cup V_{H_2}$ .

This is called the Smarandache bipermutation birepresentation corresponding to the bi-homomorphism  $x \mapsto \pi x$  i.e.  $x_1 \mapsto \pi_{x_1} \cup x_2 \mapsto \pi_{x_2}$  from  $H = H_1 \cup H_2$  to permutations on  $E = E_1 \cup E_2$ .

It is indeed a Smarandache birepresentation for we have several E's and  $V_H = V_{H_1}^1 \cup V_{H_2}^2$ 's depending on the number of proper subsets  $H = H_1 \cup H_2$  in  $G_1 \cup G_2$  ( $G$  the  $S$ -bisemigroup) which are bigroups under the operations of  $G = G_1 \cup G_2$  because for each  $x = x_1 \cup x_2$  and  $y = y_1 \cup y_2$  in  $H = H_1 \cup H_2$  and each function  $f(a) = f_1(a_1) \cup f_2(a_2)$  in  $V_H = V_{H_1}^1 \cup V_{H_2}^2$  we have

$$\begin{aligned} (\pi_x \circ \pi_y)(f)(a) &= (\pi_{x_1}^1 \cup \pi_{x_2}^2) \circ (\pi_{y_1}^1 \cup \pi_{y_2}^2)(f_1 \cup f_2)(a_1 \cup a_2) \\ &= (\pi_{x_1}^1 \circ \pi_{y_1}^1)(f_1)(a_1) \cup (\pi_{x_2}^2 \circ \pi_{y_2}^2)(f_2)(a_2) \\ &= \pi_{x_1}^1(\pi_{y_1}^1(f_1)(a_1)) \cup \pi_{x_2}^2(\pi_{y_2}^2(f_2)(a_2)) \\ &= \pi_{y_1}^1(f_1)(\pi_{x_1}^1(I^1(a_1))) \cup \pi_{y_2}^2(f_2)(\pi_{x_2}^2(I^2(a_2))) \\ &= f_1(\pi_{y_1}^1 1(\pi_{x_1}^1(1(a_1)))) \cup f_2(\pi_{y_2}^2 1(\pi_{x_2}^2(1(a_2)))) \\ &= f_1(\pi_{(x_1 y_1)}^1 1(a_1)) \cup f_2(\pi_{(x_2 y_2)}^2 1(a_2)). \end{aligned}$$

Alternatively for each  $b = b_1 \cup b_2 \in E_1 \cup E_2$  defined by

$$\psi_b(a) = \psi_{b_1}^1(a_1) \cup \psi_{b_2}^2(a_2)$$

be the function on  $E_1 \cup E_2$  defined by  $\psi_b(a) = 1$  i.e.,

$$\psi_{b_1}^1(a_1) \cup \psi_{b_2}^2(a_2) = 1 \cup 1.$$

When  $a = b$  i.e.  $a_1 \cup b_1 = a_2 \cup b_2$ ,  $\psi_b(a) = 0$  when  $a \neq b$ , i.e.  $\psi_{b_1}^1(a_1) \cup \psi_{b_2}^2(a_2) = 0 \cup 0$  when  $a_1 \cup b_1 \neq a_2 \cup b_2$ .

Then the collection of functions  $\psi_b$  for  $b \in E_1 \cup E_2$  is a basis for  $V_H = V_{H_1}^1 \cup V_{H_2}^2$  and  $\pi_x(\psi) = \psi_{\pi_x(b)} \forall x$  in  $H$  and  $b$  in  $E$  i.e.

$$\pi_{x_1}(\psi^1) \cup \pi_{x_2}(\psi^2) = \psi_{\pi_{x_1}(b_1)}^1 \cup \psi_{\pi_{x_2}(b_2)}^2,$$

for  $x = x_1 \cup x_2$  in  $H = H_1 \cup H_2$  and  $b_1 \cup b_2$  in  $E_1 \cup E_2$ . This is true for each proper subset  $H = H_1 \cup H_2$  in the  $S$ -neutrosophic semigroup  $G = G_1 \cup G_2$  and the bigroup  $H = H_1 \cup H_2$  associated with the bipermutations of the non empty finite set  $E = E_1 \cup E_2$ .

Next we shall discuss about Smarandache isomorphic bigroup representation. To this end we consider two bivector spaces  $V = V_1 \cup V_2$  and  $W = W_1 \cup W_2$  defined over the same field  $K$  and that  $T$  is a linear biisomorphism from  $V$  on to  $W$ .

Assume  $\rho H = \rho^1 H_1 \cup \rho^2 H_2$  and  $\rho' H = \rho'^1 H_1 \cup \rho'^2 H_2$  are Smarandache birepresentations of the subbigroup  $H = H_1 \cup H_2$  in  $G = G_1 \cup G_2$  ( $G$  a pseudo  $S$ -neutrosophic bisemigroup) on  $V$  and  $W$  respectively. To

$$(\rho H)_x = (\rho' H)_x \circ T \text{ for all } x = x_1 \cup x_2 \in H = H_1 \cup H_2,$$

i.e.

$$\begin{aligned} (T_1 \cup T_2) \circ (\rho^1 H_1 \cup \rho^2 H_2)_{x_1 \cup x_2} &= T_1 (\rho^1 H_1)_{x_1} \cup T_2 (\rho^2 H_2)_{x_2} \\ &= (\rho'^1 H_1)_{x_1}^1 \circ T_1 \cup (\rho'^2 H_2)_{x_2}^2 \circ T_2, \end{aligned}$$

then we say  $T = T_1 \cup T_2$  determines a Smarandache bi-isomorphism between the birepresentation  $\rho H$  and  $\rho' H$ . We may also say that  $\rho H$  and  $\rho' H$  are Smarandache biisomorphic  $S$ -bisemigroup birepresentations.

However it can be verified that Smarandache biisomorphic birepresentation have equal degree but the converse is not true in general.

Suppose  $V = W$  be the bivector space of  $K$ -valued functions on  $H = H_1 \cup H_2 \subset G_1 \cup G_2$  and define  $T$  on  $V = W$  by

$$T(f)(a) = f(a^{-1}) \text{ i.e. } T_1(f_1)(a_1) \cup T_2(f_2)(a_2) = f_1(a_1^{-1}) \cup f_2(a_2^{-1}).$$

This is one to one linear bimapping from the space of  $K$ -valued bifunctions  $H_1$  on to itself and

$$T \circ SR_x = SL_x \circ T,$$

i.e.

$$(T_1 \circ S^1 R_{x_1}^1) \cup (T_2 \circ S^2 R_{x_2}^2) = (S^1 L_{x_1}^1 \circ T_1) \cup (S^2 L_{x_2}^2 \circ T_2),$$

for all  $x = x_1 \cup x_2$  in  $H = H_1 \cup H_2$ .

For if  $f(a)$  is a bifunction on  $G = G_1 \cup G_2$  then

$$\begin{aligned} (T \circ SR_x)(f)(a) &= T(SR_x(f))(a) \\ &= SR_x(f)(a^{-1}) \\ &= f(a^{-1}x) \\ &= T(f)(x^{-1}a) \\ &= SL_x(T(f))(a) \\ &= (SL_x \circ T)(f)(a). \end{aligned}$$

Therefore we see that  $S$ -left and  $S$ -right birepresentations of  $H = H_1 \cup H_2$  are biisomorphic to each other.

Suppose now that  $H = H_1 \cup H_2$  is a subbigroup of the  $S$ -bisemigroup  $G$  and  $\rho H = \rho^1 H_1 \cup \rho^2 H_2$  is a birepresentation of  $H = H_1 \cup H_2$  on the bivector space  $V_H = V_{H_1}^1 \cup V_{H_2}^2$  over the field  $K$  and let  $\nu_1, \dots, \nu_n$  be a basis of  $V_H = V_{H_1}^1 \cup V_{H_2}^2$ . For each  $x = x_1 \cup x_2$  in  $H = H_1 \cup H_2$

we can associate to  $(\rho H)_x = (\rho^1 H_1)_{x_1} \cup (\rho^2 H_2)_{x_2}$  an invertible  $n \times n$  bimatrix with entries in  $K$  using this basis we denote this bimatrix by  $(M^1 H_1)_{x_1} \cup (M^2 H_2)_{x_2} = (MH)_x$  where  $M = M_1 \cup M_2$ .

The composition rule can be rewritten as

$$\begin{aligned} (MH)_{xy} &= (MH)_x (MH)_y \\ &= (M^1 H_1)_{x_1 y_1} \cup (M^2 H_2)_{x_2 y_2} \\ &= \left[ (M^1 H_1)_{x_1} \cup (M^2 H_2)_{x_2} \right] \left[ (M^1 H_1)_{y_1} \cup (M^2 H_2)_{y_2} \right] \\ &= (M^1 H_1)_{x_1} (M^1 H_1)_{y_1} \cup (M^2 H_2)_{x_2} (M^2 H_2)_{y_2}, \end{aligned}$$

where the bimatrix product is used on the right side of the equation. We see depending on each  $H = H_1 \cup H_2$  we can have different bimatrices  $MH = M^1 H_1 \cup M^2 H_2$ , and it need not in general be always a  $n \times n$  bimatrixes it can also be a  $m \times m$  bimatrix  $m \neq n$ . A different choice of basis for  $V = V_1 \cup V_2$  will lead to a different mapping  $x \mapsto Nx$  i.e.  $x_1 \cup x_2 \mapsto N_{x_1}^1 \cup N_{x_2}^2$  from  $H$  to invertible  $n \times n$  bimatrixes.

However the two mappings

$$\begin{aligned} x &\mapsto M_x = M_{x_1}^1 \cup M_{x_2}^2 \\ x &\mapsto N_x = N_{x_1}^1 \cup N_{x_2}^2, \end{aligned}$$

will be called as Smarandache similar relative to the subbigrup  $H = H_1 \cup H_2 \subset G = G_1 \cup G_2$  in the sense that there is an invertible  $n \times n$  bimatrix  $S = S^1 \cup S^2$  with entries in  $K$  such that  $N_x = S M_x S^{-1}$  i.e.  $N_{x_1}^1 \cup N_{x_2}^2 = S^1 M_{x_1}^1 (S^1)^{-1} \cup S^2 M_{x_2}^2 (S^2)^{-1}$  for all  $x = x_1 \cup x_2 \subset G = G_1 \cup G_2$ . It is pertinent to mention that when a different  $H'$  is taken  $H \neq H'$  i.e.  $H^1 \cup H^2 \neq (H')^1 \cup (H')^2$  then we may have a different  $m \times m$  bimatrix. Thus using a single  $S$ -neutrosophic bisemigroup we have very many such bimappings depending on each  $H \subset G$ . On the other hand one can begin with a bimapping  $x \mapsto M_x$  from  $H$  into invertible  $n \times n$  matrices with entries in  $K$  i.e.  $x_1 \mapsto M_{x_1}^1 \cup x_2 \mapsto M_{x_2}^2$  from  $H = H_1 \cup H_2$  into invertible  $n \times n$  matrices. Thus now one can reformulate the condition for two Smarandache birepresentations to be biisomorphic.

If one has two birepresentation of a fixed subbigrup  $H = H_1 \cup H_2$ ,  $H$  a subbigrup of the  $S$ -neutrosophic bisemigroup  $G$  on two bivector spaces  $V$  and  $W$  ( $V = V^1 \cup V^2$  and  $W = W^1 \cup W^2$ ) with the same scalar field  $K$  then these two Smarandache birepresentations are Smarandache biisomorphic if and only if the associated bimappings from  $H = H_1 \cup H_2$  to invertible bimatrixes as above, for any choice of basis on  $V = V^1 \cup V^2$  and  $W = W^1 \cup W^2$  are bisimilar with the bisimilarity bimatrix  $S$  having entries in  $K$ .

Now we proceed on to give a brief description of Smarandache biirreducible birepresentation, Smarandache biirreducible birepresentation and Smarandache bistable representation and so on. Now we proceed on to define Smarandache bireducibility of finite  $S$ -neutrosophic bisemigroups.

Let  $G$  be a finite neutrosophic  $S$ -bisemigroup when we say  $G$  is a  $S$ -finite bisemigroup or finite  $S$ -bisemigroup we only mean all proper subset in  $G$  which are subbigrups in  $G = G_1 \cup G_2$  are of finite order  $V_H$  be a bivector space over a field  $K$  and  $\rho H$  a birepresentation of  $H$  on  $V_H$ .

Suppose that there is a bivector space  $W_H$  of  $V_H$  such that  $(\rho H)_x W_H \subseteq W_H$  here  $W_H = W_{H_1}^1 \cup W_{H_2}^2$  where  $H = H_1 \cup H_2$  and  $V_H = V_{H_1}^1 \cup V_{H_2}^2$ ,  $H = H_1 \cup H_2$ ,  $\rho H = \rho^1 H_1 \cup \rho^2 H_2$  and  $x = x_1 \cup x_2 \in H$  i.e.  $x_1 \in H_1$  and  $x_2 \in H_2$ .

This is equivalent to saying that

$$(\rho H)_x (W_H) = W_H$$

i.e.

$$\left[ (\rho^1 H_1)_{x_1} \cup (\rho^2 H_2)_{x_2} \right] [W_{H_1}^1 \cup W_{H_2}^2] = W_{H_1}^1 \cup W_{H_2}^2,$$

for all  $x = x_1 \cup x_2 \in H_1 \cup H_2$  as  $(\rho H)_{x^{-1}} = [(\rho H)_x]^{-1}$ ,

i.e.

$$(\rho^1 H_1 \cup \rho^2 H_2)_{(x_1 \cup x_2)^{-1}} = \left[ (\rho^1 H_1 \cup \rho^2 H_2)_{(x_1 \cup x_2)} \right]^{-1},$$

$$(\rho^1 H_1)_{x_1^{-1}} \cup (\rho^2 H_2)_{x_2^{-1}} = \left[ (\rho^1 H_1)_{x_1} \right]^{-1} \cup \left[ (\rho^2 H_2)_{x_2} \right]^{-1}.$$

We say  $W_H = W_{H_1}^1 \cup W_{H_2}^2$  is Smarandache biinvariant or Smarandache bistable under the birepresentation  $\rho H = \rho^1 H_1 \cup \rho^2 H_2$ .

We say the bisubspace  $Z_H = Z_{H_1}^1 \cup Z_{H_2}^2$  of  $V_H = V_{H_1}^1 \cup V_{H_2}^2$  to be a Smarandache bicomplement of a subspace

$$W_H = W_{H_1}^1 \cup W_{H_2}^2 \text{ if } W_H \cap Z_H = \{0\}$$

and

$$W_H + Z_H = V_H \text{ i.e. } (W_{H_1}^1 \cap Z_{H_1}^1) \cup (W_{H_2}^2 \cap Z_{H_2}^2) = \{0\} \cup \{0\}$$

and

$$(W_{H_1}^1 + Z_{H_1}^1) \cup (W_{H_2}^2 + Z_{H_2}^2) = V_{H_1}^1 + V_{H_2}^2,$$

here  $W_{H_i}^i + Z_{H_i}^i$  ( $i = 1, 2$ ) denotes the bispan of  $W_H$  and  $Z_H$  which is a subspace of  $V_H$  consisting of bivectors of the form  $w + z = (w_1 + z_1) \cup (w_2 + z_2)$  where  $w \in W_H$  and  $z \in Z_H$ . These conditions are equivalent to saying that every bivector  $\nu = \nu_1 \cup \nu_2 \in V_{H_1}^1 \cup V_{H_2}^2$  can be written in a unique way as  $w + z = (w_1 + z_1) \cup (w_2 + z_2)$ ,  $w_i \in W_{H_i}^i$  and  $z_i \in Z_{H_i}^i$  ( $i = 1, 2$ ).

Complementary bispaces always exist because of basis for a bivector subspace of a bivector space can be enlarged to a basis of a whole bivector space. If  $Z_H = Z_{H_1}^1 \cup Z_{H_2}^2$  and  $W_H = W_{H_1}^1 \cup W_{H_2}^2$  are complementary bisubspaces (bisubspaces) of a bivector space  $V_H = V_{H_1}^1 \cup V_{H_2}^2$  then we get a linear bimapping  $P_H = P_{H_1}^1 \cup P_{H_2}^2$  on  $V_H = V_{H_1}^1 \cup V_{H_2}^2$  on to  $W_H = W_{H_1}^1 \cup W_{H_2}^2$  along  $Z_H = Z_{H_1}^1 \cup Z_{H_2}^2$  and is defined by  $P_H(w + z)w$  for all  $w \in W_H$  and  $z \in Z_H$ . Thus  $I_H - P_H$  is the biprojection of  $V_H$  on to  $Z_H$  along  $W_H$  where  $I_H$  denotes the identity bitransformation on  $V_H = V_{H_1}^1 \cup V_{H_2}^2$ .

**Note.**  $(P_H)^2 = (P_{H_1}^1 \cup P_{H_2}^2)^2 = (P_{H_1}^1)^2 \cup (P_{H_2}^2)^2 = P_{H_1}^1 \cup P_{H_2}^2$ , when  $P_H$  is a biprojection.

Conversely, if  $P_H$  is a linear bioperator on  $V_H$  such that  $(P_H)^2 = P_H$  then  $P_H$  is the biprojection of  $V_H$  on to the bisubspace of  $V_H$  which is the biimage of  $P_H = P_{H_1}^1 \cup P_{H_2}^2$  along the subspace of  $V_H$  which is the bikernel of  $\rho H = \rho^1 H_1 \cup \rho^2 H_2$ .

It is important to mention here unlike usual complements using a finite bigroup we see when we used pseudo  $S$ -neutrosophic bisemigroups. The situation is very varied. For each proper subset  $H$  of  $G(H_1 \cup H_2 \subset G_1 \cup G_2)$  where  $H$  is a subbigroup of  $G$  we get several important  $S$ -bicomplements and several  $S$ -biinvariant or  $S$ -bistable or  $S$ -birepresentative of  $\rho H = \rho^1 H_1 \cup \rho^2 H_2$ .

Now we proceed on to define Smarandache biirreducible birepresentation. Let  $G$  be a  $S$ -finite neutrosophic bisemigroup,  $V_H = V_{H_1}^1 \cup V_{H_2}^2$  be a bivector space over a field  $K$ ,  $\rho H = \rho^1 H_1 \cup \rho^2 H_2$  be a birepresentation of  $H$  on  $V_H$  and  $W_H$  is a subbivector space of  $V_H = V_{H_1}^1 \cup V_{H_2}^2$  which is invariant under  $\rho H = \rho^1 H_1 \cup \rho^2 H_2$ . Here we make an assumption that the field  $K$  has characteristic 0 or  $K$  has positive characteristic and the number of elements in each  $H = H_1 \cup H_2$  is not divisible by the characteristic  $K$ ,  $H_1 \cup H_2 \subset G_1 \cup G_2$  is a  $S$ -bisemigroup.

Let us show that there is a bisubspace  $Z_H = Z_{H_1}^1 \cup Z_{H_2}^2$  of  $V_{H_1}^1 \cup V_{H_2}^2 = V_H$  such that  $Z_H$  is a bicomplement of  $W_H = W_{H_1}^1 \cup W_{H_2}^2$  and  $Z_H$  is also biinvariant under the birepresentation  $\rho H$  of  $H$  i.e.  $\rho^1 H_1 \cup \rho^2 H_2$  of  $H_1 \cup H_2$  on  $V_H = V_{H_1}^1 \cup V_{H_2}^2$ . To do this we start with any bicomplements  $(Z_H)_o = (Z_{H_1}^1)_o \cup (Z_{H_2}^2)_o$  of  $W_H = W_{H_1}^1 \cup W_{H_2}^2$  of  $V_H = V_{H_1}^1 \cup V_{H_2}^2$  and let  $(P_H)_o = (P_{H_1}^1 \cup P_{H_2}^2)_o : V_H = V_{H_1}^1 \cup V_{H_2}^2 \rightarrow V_{H_1}^1 \cup V_{H_2}^2$  be the biprojection of  $V_H = V_{H_1}^1 \cup V_{H_2}^2$  on to  $W_{H_1}^1 \cup W_{H_2}^2 = W_H$  along  $(Z_H)_o$ . Thus  $(P_H)_o = (P_{H_1}^1 \cup P_{H_2}^2)_o$  maps  $V$  to  $W$  and  $(P_H)_o w = w$  for all  $w \in W$ .

Let  $m = m_1 \cup m_2$  denotes the number of elements in  $H = H_1 \cup H_2 \subset G_1 \cup G_2$  i.e.  $|H_i| = m_i$  ( $i = 1, 2$ ). Define a linear bimapping

$$P_H : V_H \rightarrow V_H$$

i.e.

$$P_{H_1}^1 \cup P_{H_2}^2 : V_{H_1}^1 \cup V_{H_2}^2 \rightarrow V_{H_1}^1 \cup V_{H_2}^2$$

by

$$\begin{aligned} P_H &= P_{H_1}^1 \cup P_{H_2}^2 \\ &= \frac{1}{m_1} \sum_{x_1 \in H_1} (\rho^1 H_1)_{x_1} \circ (P_{H_1}^1)_o \circ (\rho^1 H_1)_{x_1}^{-1} \cup \frac{1}{m_2} \sum_{x_2 \in H_2} (\rho^2 H_2)_{x_2} \circ (P_{H_2}^2)_o \circ (\rho^2 H_2)_{x_2}^{-1}, \end{aligned}$$

assumption on  $K$  implies that  $\frac{1}{m_i}$  ( $i = 1, 2$ ) makes sense as an element of  $K$  i.e. as the multiplicative inverse of a sum of  $m$  1's in  $K$  where 1 refers to the multiplicative identity element of  $K$ . This expression defines a linear bimapping on  $V_H = V_{H_1}^1 \cup V_{H_2}^2$  because  $(\rho H)'_x s$  and  $(P_H)_o$  are linear bimapping.

We actually have that  $P_H = P_{H_1}^1 \cup P_{H_2}^2$  bimaps  $V_H$  to  $W_H$  i.e.  $V_{H_1}^1 \cup V_{H_2}^2$  to  $W_{H_1}^1 \cup W_{H_2}^2$  and because the  $(P_H)_o = (P_{H_1}^1 \cup P_{H_2}^2)_o$  maps  $V_H = V_{H_1}^1 \cup V_{H_2}^2$  to  $W_H = W_{H_1}^1 \cup W_{H_2}^2$ , and because the  $(\rho H)'_x s = ((\rho^1 H_1)_{x_1} \cup (\rho^2 H_2)_{x_2})$  maps  $W_H = W_{H_1}^1 \cup W_{H_2}^2$  to  $W_{H_1}^1 \cup W_{H_2}^2$ . If  $w \in W_H$  then

$$\begin{aligned} [(\rho H)_x]^{-1} w &= \left[ (\rho^1 H_1)_{x_1} \cup (\rho^2 H_2)_{x_2} \right]^{-1} (w_1 \cup w_2) \\ &= (\rho^1 H_1)_{x_1}^{-1} (w_1) \cup (\rho^2 H_2)_{x_2}^{-1} (w_2) \in W_{H_1}^1 \cup W_{H_2}^2, \end{aligned}$$



for all  $x = x_1 \cup x_2$  in  $H = H_1 \cup H_2 \subset G = G_1 \cup G_2$  and then

$$\begin{aligned} (P_H)_o((\rho H)_x)^{-1} \omega &= (P_H)_o \left( (\rho^1 H_1)_{x_1} \right)^{-1} (w_1) \cup (P_{H_2}^2)_o \left( (\rho^2 H_2)_{x_2} \right)^{-1} (w_2) \\ &= \left( (\rho^1 H_1)_{x_1} \right)^{-1} (w_1) \cup \left( (\rho^2 H_2)_{x_2} \right)^{-1} (w_2). \end{aligned}$$

Thus we conclude that

$$(P_H)(w) = w \text{ i.e. } (P_{H_1}^1) (w_1) = w_1 ,$$

and

$$(P_{H_2}^2) (w_2) = w_2 \text{ i.e. } P_H = P_{H_1}^1 \cup P_{H_2}^2,$$

for all  $w = (w_1 \cup w_2)$  in  $W_H = W_{H_1}^1 \cup W_{H_2}^2$  by the very definition of  $P_H$ .

The definition of  $P_H$  also implies that

$$(\rho H)_y \circ P_H \circ [(\rho H)_y]^{-1} = P_H$$

i.e.

$$(\rho^1 H_1)_{y_1} \circ P_{H_1}^1 \circ \left( (\rho^1 H_1)_{y_1} \right)^{-1} \cup (\rho^2 H_2)_{y_2} \circ P_{H_2}^2 \circ \left( (\rho^2 H_2)_{y_2} \right)^{-1} = P_{H_1}^1 \cup P_{H_2}^2,$$

for all  $y \in H = H_1 \cup H_2$ .

The only case this does not occur is when  $W_H = \{0\}$  i.e.  $W_{H_1}^1 \cup W_{H_2}^2 = \{0\} \cup \{0\}$ . Because  $P_H(V_H) \subset W_H$  and  $P_H(w) = w$  for all  $w \in W_H = W_{H_1}^1 \cup W_{H_2}^2$ .  $P_H = P_{H_1}^1 \cup P_{H_2}^2$  is a biprojection of  $V_H$  onto  $W_H$  i.e.  $P_{H_i}^i$  is a projection of  $V_{H_i}^i$  onto  $W_{H_i}^i$ ,  $i = 1, 2$  along some bisubspace  $Z_H = Z_{H_1}^1 \cup Z_{H_2}^2$  of  $V_H = V_{H_1}^1 \cup V_{H_2}^2$ . Specifically one should take  $Z_H = Z_{H_1}^1 \cup Z_{H_2}^2$  to be the bikernel of  $P_H = P_{H_1}^1 \cup P_{H_2}^2$ . It is easy to see that  $W_H \cap Z_H = \{0\}$  i.e.  $W_{H_1}^1 \cap Z_{H_1}^1 = \{0\}$  and  $W_{H_2}^2 \cap Z_{H_2}^2 = \{0\}$  since  $P_{H_i}^i(w_i) = w_i$  for all  $w_i \in W_{H_i}^i$ ,  $i = 1, 2$ .

On the other hand if  $\nu = \nu_1 \cup \nu_2$  is any element of  $V_H = V_{H_1}^1 \cup V_{H_2}^2$  then we can write  $\nu = \nu_1 \cup \nu_2$  as  $P_H(\nu) = P_{H_1}^1(\nu_1) \cup P_{H_2}^2(\nu_2)$  so  $P_H(\nu) + (V - P_H(\nu))$ .

Thus  $\nu - P_H(\nu)$  lies in  $Z_H$ , the bikernel of  $P_H$ . This shows that  $W_H$  and  $Z_H$  satisfies the essential bicomplement of  $W_H$  in  $V_H$ . The biinvariance of  $Z_H$  under the birepresentation  $\rho H$  is evident.

Thus the Smarandache birepresentation  $\rho H$  of  $H$  on  $V_H$  is biisomorphic to the direct sum of  $H$  on  $W_H$  and  $Z_H$ , that are the birestrictions of  $\rho H$  to  $W_H$  and  $Z_H$ .

There can be smaller biinvariant bisubspaces within these biinvariant subspaces so that one can repeat the process for each  $H$ ,  $H \subset G$ . We say that the subspaces

$$(W_H)_1, (W_H)_2, \dots, (W_H)_t$$

of  $V_H$ , i.e.

$$(W_{H_1}^1 \cup W_{H_2}^2)_1, (W_{H_1}^1 \cup W_{H_2}^2)_2, \dots, (W_{H_1}^1 \cup W_{H_2}^2)_t,$$

of  $V_{H_1}^1 \cup V_{H_2}^2$  form an Smarandache biindependent system related to each subgroup  $H = H_1 \cup H_2 \subset G = G_1 \cup G_2$ . If  $(W_H)_j \neq (0)$  for each  $j$  and if  $w_j \in (W_H)_j$ ,  $1 \leq j \leq t$  and

$$\sum_{j=1}^t w_j = \sum_{j=1}^t w_j^1 \cup \sum_{j=1}^t w_j^2 = 0 \cup 0,$$

where  $w_j = w_j^1 \cup w_j^2$ ,  $w_j^1 \in W_{H_1}^1$  and  $w_j^2 \in W_{H_2}^2$  imply  $w_j^i = 0$  ( $i = 1, 2$ ;  $j = 1, 2, \dots, t$ ). If in addition it spans  $(W_H)_1, (W_H)_2, \dots, (W_H)_t = V_{H_1}^1 \cup V_{H_2}^2 = V_H$ , then every bivector  $\nu = \nu^1 \cup \nu^2$  on  $V_{H_1}^1 \cup V_{H_2}^2$  can be written in a unique way as  $\sum_{j=1}^t u_j$  with  $u_j = u_j^1 \cup u_j^2 \in (W_{H_1}^1 \cup W_{H_2}^2)$  for each  $j$ .

Next we proceed on to give two applications to Smarandache Markov bichains and Smarandache Leontief economic bimodels.

Suppose a physical or a mathematical system is such that at any movement it can occupy one of a finite number of states when we view them as stochastic bioprocess or Markov bichains we make an assumption that the system moves with time from one state to another so that a schedule of observation times keep the states of the system at these times. But when we tackle real world problems say even for simplicity the emotions of a person it need not fall under the category of sad, cold, happy, angry, affectionate, disinterested, disgusting, many times the emotions of a person may be very unpredictable depending largely on the situation, and the mood of the person and its relation with another, so such study cannot fall under Markov chains, for at a time more than one emotion may be in a person and also such states cannot be included and given as next pair of observation, these changes and several feelings at least two at a time will largely affect the very transition bimatrix

$$P = P_1 \cup P_2 = [p_{ij}^1] \cup [p_{ij}^2],$$

with non negative entries for which each of the column sums are one and all of whose entries are positive. This has relevance as even the policy makers are humans and their view is ultimate and this rules the situation. Here it is still pertinent to note that all decisions are not always possible at times certain of the views may be indeterminate at that period of time and may change after a period of time but all our present theory have no place for the indeterminacy only the neutrosophy gives the place for the concept of indeterminacy, based on which we have built neutrosophic vector spaces, neutrosophic bivector spaces, then now the notion of Smarandache -neutrosophic bivector spaces and so on.

So to overcome the problem we have indecisive situations we give negative values and indeterminate situations we give negative values so that our transition neutrosophic bimatrices individual columns sums do not add to one and all entries may not be positive.

Thus we call the new transition neutrosophic bimatrix which is a square bimatrix which can have negative entries and  $I$  the indeterminate also falling in the set  $[-1, 1] \cup \{I\}$  and whose column sums can also be less than 1 and  $I$  as the Smarandache neutrosophic transition bimatrix.

Further the Smarandache neutrosophic probability bivector will be a bicolumn vector which can take entries from  $[-1, 1] \cup [-I, I]$  whose sum can lie in the biinterval  $[-1, 1] \cup [-I, I]$ . The Smarandache neutrosophic probability bivectors  $x^{(n)}$  for  $n = 0, 1, 2, \dots$  are said to be the Smarandache state neutrosophic bivectors of a Smarandache neutrosophic Markov bioprocess. Clearly if  $P$  is a  $S$ -transition bimatrix of a Smarandache Markov bioprocess and  $x^{(n)} = x_1^{(n_1)} \cup x_2^{(n_2)}$  is the Smarandache state neutrosophic bivectors at the  $n^{th}$  pair of observation then

$$x^{(n+1)} \neq px^{(n)}$$

$$\text{i.e. } x_1^{(n+1)} \cup x_2^{(n+1)} \neq p_1 x_1^{(n_1)} \cup p_2 x_2^{(n_2)}.$$

Further research in this direction is innovative and interesting.

Matrix theory has been very successful in describing the inter relation between prices outputs and demands in an economic model. Here we just discuss some simple bimodels based on the ideals of the Nobel laureate Massily Leontief. We have used not only bimodel structure based on bimatrices also we have used the factor indeterminacy. So our matrices would be only Neutrosophic bimatrices. Two types of models which we wish to discuss are the closed or input-output model and the open or production model each of which assumes some economics parameter which describe the inter relations between the industries in the economy under considerations. Using neutrosophic bimatrix theory we can combine and study the effect of price bivector. Before the basic equations of the input-output model are built we just recall the definition of fuzzy neutrosophic bimatrix. For we need this type of matrix in our bimodel.

**Definition 3.** Let  $M_{n \times m} = \{(a_{ij})/a_{ij} \in K(I)\}$ , where  $K(I)$ , is a neutrosophic field. We call  $M_{n \times m}$  to be the neutrosophic rectangular matrix.

**Example 1.** Let  $Q(I) = \langle Q \cup I \rangle$  be the neutrosophic field.

$$M_{4 \times 3} = \begin{pmatrix} 0 & 1 & I \\ -2 & 4I & 0 \\ 1 & -I & 2 \\ 3I & 1 & 0 \end{pmatrix},$$

is the neutrosophic matrix, with entries from rationals and the indeterminacy  $I$ .

We define product of two neutrosophic matrices and the product is defined as follows: let

$$A = \begin{pmatrix} -1 & 2 & -I \\ 3 & I & 0 \end{pmatrix}_{2 \times 3} \quad \text{and} \quad B = \begin{pmatrix} I & 1 & 2 & 4 \\ 1 & I & 0 & 2 \\ 5 & -2 & 3I & -I \end{pmatrix}_{3 \times 4}$$

$$AB = \begin{pmatrix} -6I + 2 & -1 + 4I & -2 - 3I & I \\ -4I & 3 + I & 6 & 12 + 2I \end{pmatrix}_{2 \times 4}$$

(we use the fact  $I^2 = I$ ).

Let  $M_{n \times n} = \{(a_{ij}) | (a_{ij}) \in Q(I)\}$ ,  $M_{n \times n}$  is a neutrosophic vector space over  $Q$  and a strong neutrosophic vector space over  $Q(I)$ .

Now we proceed onto define the notion of fuzzy integral neutrosophic matrices and operations on them, for more about these refer [43].

**Definition 4.** Let  $N = [0, 1] \cup I$ , where  $I$  is the indeterminacy. The  $m \times n$  matrices  $M_{m \times n} = \{(a_{ij})/a_{ij} \in [0, 1] \cup I\}$  is called the fuzzy integral neutrosophic matrices. Clearly the class of  $m \times n$  matrices is contained in the class of fuzzy integral neutrosophic matrices.

**Example 2.** Let

$$A = \begin{pmatrix} I & 0.1 & 0 \\ 0.9 & 1 & I \end{pmatrix},$$

$A$  is a  $2 \times 3$  integral fuzzy neutrosophic matrix.

We define operation on these matrices. An integral fuzzy neutrosophic row vector is  $1 \times n$  integral fuzzy neutrosophic matrix. Similarly an integral fuzzy neutrosophic column vector is a  $m \times 1$  integral fuzzy neutrosophic matrix.

**Example 3.**  $A = (0.1, 0.3, 1, 0, 0, 0.7, I, 0.002, 0.01, I, 0.12)$  is a integral row vector or a  $1 \times 11$ , integral fuzzy neutrosophic matrix.

**Example 4.**  $B = (1, 0.2, 0.111, I, 0.32, 0.001, I, 0, 1)^T$  is an integral neutrosophic column vector or  $B$  is a  $9 \times 1$  integral fuzzy neutrosophic matrix.

We would be using the concept of fuzzy neutrosophic column or row vector in our study.

**Definition 5.** Let  $P = (p_{ij})$  be a  $m \times n$  integral fuzzy neutrosophic matrix and  $Q = (q_{ij})$  be a  $n \times p$  integral fuzzy neutrosophic matrix. The composition map  $P \bullet Q$  is defined by  $R = (r_{ij})$  which is a  $m \times p$  matrix where  $r_{ij} = \max_k \min(p_{ik}q_{kj})$  with the assumption  $\max(p_{ij}, I) = I$  and  $\min(p_{ij}, I) = I$  where  $p_{ij} \in [0, 1]$ .  $\min(0, I) = 0$  and  $\max(1, I) = 1$ .

**Example 5.** Let

$$P = \begin{bmatrix} 0.3 & I & 1 \\ 0 & 0.9 & 0.2 \\ 0.7 & 0 & 0.4 \end{bmatrix}, Q = (0.1, I, 0)^T$$

be two integral fuzzy neutrosophic matrices.

$$P \bullet Q = \begin{bmatrix} 0.3 & I & 1 \\ 0 & 0.9 & 0.2 \\ 0.7 & 0 & 0.4 \end{bmatrix} \bullet \begin{bmatrix} 0.1 \\ I \\ 0 \end{bmatrix} = (I, I, 0.1).$$

**Example 6.** Let

$$P = \begin{bmatrix} 0 & I \\ 0.3 & 1 \\ 0.8 & 0.4 \end{bmatrix} \text{ and } Q = \begin{bmatrix} 0.1 & 0.2 & 1 & 0 & I \\ 0 & 0.9 & 0.2 & 1 & 0 \end{bmatrix}.$$

One can define the max-min operation for any pair of integral fuzzy neutrosophic matrices with compatible operation.

Now we proceed onto define the notion of fuzzy neutrosophic matrices.

Let  $N_s = [0, 1] \cup \{nI/n \in (0, 1]\}$ , we call the set  $N_s$  to be the fuzzy neutrosophic set.

**Definition 6.** Let  $N_s$  be the fuzzy neutrosophic set.  $M_{n \times m} = \{(a_{ij})/a_{ij} \in N_s\}$ , we call the matrices with entries from  $N_s$  to be the fuzzy neutrosophic matrices.

**Example 7.** Let  $N_s = [0, 1] \cup \{nI/n \in (0, 1]\}$  be the set

$$P = \begin{bmatrix} 0 & 0.2I & 0.31 & I \\ I & 0.01 & 0.7I & 0 \\ 0.31I & 0.53I & 1 & 0.1 \end{bmatrix}.$$

$P$  is a  $3 \times 4$  fuzzy neutrosophic matrix.

**Example 8.** Let  $N_s = [0, 1] \cup \{nI/n \in (0, 1]\}$  be the fuzzy neutrosophic matrix.  $A = [0, 0.12I, I, 1, 0.31]$  is the fuzzy neutrosophic row vector:

$$B = \begin{bmatrix} 0.5I \\ 0.11 \\ I \\ 0 \\ -1 \end{bmatrix},$$

is the fuzzy neutrosophic column vector.

Now we proceed on to define operations on these fuzzy neutrosophic matrices.

Let  $M = (m_{ij})$  and  $N = (n_{ij})$  be two  $m \times n$  and  $n \times p$  fuzzy neutrosophic matrices.

$$M \bullet N = R = (r_{ij})$$

where the entries in the fuzzy neutrosophic matrices are fuzzy indeterminates i.e. the indeterminates have degrees from 0 to 1 i.e. even if some factor is an indeterminate we try to give it a degree to which it is indeterminate for instance  $0.9I$  denotes the indeterminacy rate; it is high where as  $0.01I$  denotes the low indeterminacy rate. Thus neutrosophic matrices have only the notion of degrees of indeterminacy. Any other type of operations can be defined on the neutrosophic matrices and fuzzy neutrosophic matrices. The notion of these matrices have been used to define neutrosophic relational equations and fuzzy neutrosophic relational equations.

Here we give define the notion of neutrosophic bimatrix and illustrate them with examples. Also we define fuzzy neutrosophic matrices.

**Definition 7.** Let  $A = A_1 \cup A_2$ , where  $A_1$  and  $A_2$  are two distinct neutrosophic matrices with entries from a neutrosophic field. Then  $A = A_1 \cup A_2$  is called the neutrosophic bimatrix.

It is important to note the following:

- (1) If both  $A_1$  and  $A_2$  are neutrosophic matrices we call  $A$  a neutrosophic bimatrix.
- (2) If only one of  $A_1$  or  $A_2$  is a neutrosophic matrix and other is not a neutrosophic matrix then we call  $A = A_1 \cup A_2$  as the semi neutrosophic bimatrix. (It is clear all neutrosophic bimatrices are trivially semi neutrosophic bimatrices).

If both  $A_1$  and  $A_2$  are  $m \times n$  neutrosophic matrices then we call  $A = A_1 \cup A_2$  a  $m \times n$  neutrosophic bimatrix or a rectangular neutrosophic bimatrix.

If  $A = A_1 \cup A_2$  be such that  $A_1$  and  $A_2$  are both  $n \times n$  neutrosophic matrices then we call  $A = A_1 \cup A_2$  a square or a  $n \times n$  neutrosophic bimatrix. If in the neutrosophic bimatrix  $A = A_1 \cup A_2$  both  $A_1$  and  $A_2$  are square matrices but of different order say  $A_1$  is a  $n \times n$  matrix and  $A_2$  a  $s \times s$  matrix then we call  $A = A_1 \cup A_2$  a mixed neutrosophic square bimatrix. (Similarly one can define mixed square semi neutrosophic bimatrix).

Likewise in  $A = A_1 \cup A_2$ , if both  $A_1$  and  $A_2$  are rectangular matrices say  $A_1$  is a  $m \times n$  matrix and  $A_2$  is a  $p \times q$  matrix then we call  $A = A_1 \cup A_2$  a mixed neutrosophic rectangular bimatrix. (If  $A = A_1 \cup A_2$  is a semi neutrosophic bimatrix then we call  $A$  the mixed rectangular semi neutrosophic bimatrix).

Just for the sake of clarity we give some illustration.

**Notation.** We denote a neutrosophic bimatrix by  $A_N = A_1 \cup A_2$ .

**Example 9.** Let

$$A_N = \begin{bmatrix} 0 & I & 0 \\ 1 & 2 & -1 \\ 3 & 2 & I \end{bmatrix} \cup \begin{bmatrix} 2 & I & 1 \\ I & 0 & I \\ 1 & 1 & 2 \end{bmatrix},$$

$A_N$  is the  $3 \times 3$  square neutrosophic bimatrix.

At times one may be interested to study the problem at hand (i.e. the present situation) and a situation at the  $r^{th}$  time period the predicted model.

All notion and concept at all times is not determinable. For at time a situation may exist for a industry that it cannot say the monetary value of the output of the  $i^{th}$  industry needed to satisfy the outside demand at one time, this notion may become an indeterminate (For instance with the advent of globalization the electronic goods manufacturing industries are facing a problem for in the Indian serenio when an exported goods is sold at a cheaper rate than manufactured Indian goods will not be sold for every one will prefer only an exported good, so in situation like this the industry faces only a indeterminacy for it cannot fully say anything about the movements of the manufactured goods in turn this will affect the  $\sigma_{ij}$ .  $\sigma_{ij}$  may also tend to become an indeterminate. So to study such situation simultaneously the neutrosophic bimatrix would be ideal we may have the newly redefined production vector which we choose to call as Smarandache neutrosophic production bivector which will have its values taken from +ve value or -ve value or an indeterminacy.

So Smarandache neutrosophic Leontief open model is got by permitting.

$$x \geq 0, d \geq 0, c \geq 0$$

$$x \leq 0, d \leq 0, c \leq 0$$

and  $x$  can be  $I$ ,  $d$  can take any value and  $c$  can be a neutrosophic bimatrix. We can say  $(1 - c)^{-1} \geq 0$  productive  $(1 - c)^{-1} < 0$  non productive or not up to satisfaction and  $(1 - c^{-1}) = nI$ ,  $I$  the indeterminacy i.e. the productivity cannot be determined i.e. one cannot say productive or non productive but cannot be determined.  $c = c_1 \cup c_2$  is the consumption neutrosophic bimatrix.

$c_1$  at time of study and  $c_2$  after a stipulated time period.  $x, d, c$  can be greater than or equal to zero less than zero or can be an indeterminate.

$$x = \begin{bmatrix} x_1^1 \\ \vdots \\ x_k^1 \end{bmatrix} \cup \begin{bmatrix} x_1^2 \\ \vdots \\ x_k^2 \end{bmatrix},$$

production neutrosophic bivector at the times  $t_1$  and  $t_2$  the demand neutrosophic bivector  $d = d^1 \cup d^2$

$$d = \begin{bmatrix} d_1^1 \\ \vdots \\ d_k^1 \end{bmatrix} \cup \begin{bmatrix} d_1^2 \\ \vdots \\ d_k^2 \end{bmatrix},$$

at time  $t_1$  and  $t_2$  respectively. Consumption neutrosophic bimatrix  $c = c_1 \cup c_2$

$$c_1 = \begin{bmatrix} \sigma_{11}^1 & \cdots & \sigma_{1k}^1 \\ \sigma_{21}^1 & \cdots & \sigma_{2k}^1 \\ \vdots & & \\ \sigma_{k1}^1 & \cdots & \sigma_{kk}^1 \end{bmatrix}, c_2 = \begin{bmatrix} \sigma_{11}^2 & \cdots & \sigma_{1k}^2 \\ \sigma_{21}^2 & \cdots & \sigma_{2k}^2 \\ \vdots & & \\ \sigma_{k1}^2 & \cdots & \sigma_{kk}^2 \end{bmatrix}$$

at times  $t_1$  and  $t_2$  respectively.

$$\begin{aligned} & \sigma_{i1} x_1 + \sigma_{i2} x_2 + \dots + \sigma_{ik} x_k \\ = & (\sigma_{i1}^1 x_1^1 + \sigma_{i2}^1 x_2^1 + \dots + \sigma_{ik}^1 x_k^1) \cup (\sigma_{i1}^2 x_1^2 + \sigma_{i2}^2 x_2^2 + \dots + \sigma_{ik}^2 x_k^2) \end{aligned}$$

is the value of the output of the  $i^{th}$  industry needed by all  $k$  industries at the time periods  $t_1$  and  $t_2$  to produce a total output specified by the production neutrosophic bivector  $x = x^1 \cup x^2$ . Consumption neutrosophic bimatrix  $c$  is such that; production if  $(1-c)^{-1}$  exists and  $(1-c)^{-1} \geq 0$ , i.e.  $c = c_1 \cup c_2$  and  $(1-c_1)^{-1} \cup (1-c_2)^{-1}$  exists and each of  $(1-c_1)^{-1}$  and  $(1-c_2)^{-1}$  is greater than or equal to zero. A consumption neutrosophic bimatrix  $c$  is productive if and only if there is some production bivector  $x \geq 0$  such that

$$x > cx \text{ i.e. } x^1 \cup x^2 > c^1 x^1 \cup c^2 x^2.$$

A consumption bimatrix  $c$  is productive if each of its birow sum is less than one. A consumption bimatrix  $c$  is productive if each of its bicolumn sums is less the one. Non productive if bivector  $x < 0$  such that  $x < cx$ .

Now quasi productive if one of  $x^1 \geq 0$  and  $x^1 > c^1 x^1$  or  $x^2 \geq 0$  and  $x^2 > c^2 x^2$ .

Now production is indeterminate if  $x$  is indeterminate  $x$  and  $cx$  are indeterminates or  $x$  is indeterminate and  $c x$  is determinate. Production is quasi indeterminate if at  $t_1$  or  $t_2$ ,  $x^i \geq 0$  and  $x^i > c^i x^i$  are indeterminates quasi non productive and indeterminate if one of  $x^i < 0$ ,  $c^i x^i < 0$  and one of  $x^i$  and  $I^i x^i$  are indeterminate. Quasi production if one of  $c^i x^i > 0$  and  $x^i > 0$  and  $x^i < 0$  and  $I^i x^i < 0$ . Thus 6 possibilities can occur at anytime of study say at times  $t_1$  and  $t_2$  for it is but very natural as in any industrial problem the occurrences of any factor like demand or production is very much dependent on the people and the government policy and other external factors.

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