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# Supermagic Coverings of Some Simple Graphs

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**Abstract**: A simple graph $G = (V, E)$ admits an $H$-covering if every edge in $E$ belongs to a subgraph of $G$ isomorphic to $H$. We say that $G$ is Smarandachely pair $\{s, l\}$ $H$-magic if there is a total labeling $f : V \cup E \to \{1, 2, 3, \cdots, |V| + |E|\}$ such that there are subgraphs $H_1 = (V_1, E_1)$ and $H_2 = (V_2, E_2)$ of $G$ isomorphic to $H$, the sum $\sum\limits_{v\in V_1} f(v) + \sum\limits_{e\in E_1} f(e) = s$ and $\sum\limits_{v\in V_2} f(v) + \sum\limits_{e\in E_2} f(e) = l$. Particularly, if $s = l$, such a Smarandachely pair $\{s, l\}$ $H$-magic is called $H$-magic and if $f(V) = \{1, 2, \cdots, |V|\}$, $G$ is said to be a $H$-supermagic. In this paper we show that edge amalgamation of a finite collection of graphs isomorphic to any 2-connected simple graph $H$ is $H$-supermagic.

**Key Words**: $H$-covering, Smarandachely pair $\{s, l\}$ $H$-magic, $H$-magic, $H$-supermagic.

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## §1. Introduction

The concept of $H$-magic graphs was introduced in [3]. An edge-covering of a graph $G$ is a family of different subgraphs $H_1, H_2, \ldots, H_k$ such that each edge of $E$ belongs to at least one of the subgraphs $H_i, 1 \leq i \leq k$. Then, it is said that $G$ admits an $(H_1, H_2, \ldots, H_k)$ - edge covering. If every $H_i$ is isomorphic to a given graph $H$, then we say that $G$ admits an $H$-covering.

Suppose that $G = (V, E)$ admits an $H$-covering. We say that a bijective function $f : V \cup E \to \{1, 2, 3, \cdots, |V| + |E|\}$ is an $H$-magic labeling of $G$ if there is a positive integer $m(f)$, which we call magic sum, such that for each subgraph $H' = (V', E')$ of $G$ isomorphic to $H$, we have,$f(H') = \sum_{v\in V'} f(v) + \sum_{e\in E'} f(e) = m(f)$. In this case we say that the graph $G$ is $H$-magic. When $f(V) = \{1, 2, \ |V|\}$, we say that $G$ is $H$-supermagic and we denote its supermagic-sum by $s(f)$.

We use the following notations. For any two integers $n < m$, we denote by $[n, m]$, the set of all consecutive integers from $n$ to $m$. For any set $\mathbb{I} \subset \mathbb{N}$ we write, $\sum \mathbb{I} = \sum\limits_{x\in\mathbb{I}} x$ and for any integers $k$, $\mathbb{I}+k = \{x+k : x \in \mathbb{I}\}$. Thus $k+[n, m]$ is the set of consecutive integers from $k+n$ to

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$k+m$. It can be easily verified that $\sum(\mathbb{I}+k) = \sum \mathbb{I}+k|\mathbb{I}|$. If $\mathbb{P} = \{X_1, X_2, \cdots, X_n\}$ is a partition of a set $X$ of integers with the same cardinality then we say $\mathbb{P}$ is an $n$-equipartition of $X$. Also we denote the set of subsets sums of the parts of $\mathbb{P}$ by $\sum\mathbb{P} = \{\sum X1, \sum X_2, \cdots, \sum X_n\}$.Finally, given a graph $G = (V, E)$ and a total labeling $f$ on it we denote by $f(G) = \sum f(V) + \sum f(E)$.

## §2. Preliminary Results

In this section we give some lemmas which are used to prove the main results in Section 3.

**Lemma** 2.1 *Let $h$ and $k$ be two positive integers and $h$ is odd. Then there exists a $k$-equipartition $\mathbb{P} = \{X_1, X_2, \cdots, X_k\}$ of $X = [1, hk]$ such that $\sum X_r = \dfrac{(h-1)(hk+k+1)}{2} + r$ for $1 \le r \le k$. Thus, $\sum\mathbb{P}$ is a set of consecutive integers given by $\sum\mathbb{P} = \dfrac{(h-1)(hk+k+1)}{2} + [1, k]$.*

*Proof* Let us arrange the set of integers $X = [1, hk]$ in a $h \times k$ matrix $\mathcal{A}$ as given below.

$$\mathcal{A} = \begin{pmatrix} 1 & 2 & \cdots & k-1 & k \\ n+1 & n+2 & \cdots & 2k-1 & 2k \\ 2n+1 & 2n+2 & \cdots & 3k-1 & 3k \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ (h-1)k+1 & (h-1)k+2 & \cdots & hk-1 & hk \end{pmatrix}_{h\times k}$$

That is, $\mathcal{A} = (a_{i,j})_{h\times k}$ where $a_{i,j} = (i-1)k + j$ for $1 \le i \le h$ and $1 \le j \le k$. For $1 \le r \le k$, define $X_r = \{a_{i,r}/1 \le i \le \frac{h+1}{2}\} \cup \{a_{i,k-r+1}/\frac{h+3}{2} \le i \le h\}$. Then

$$\begin{aligned} \sum X_r &= \sum_{i=1}^{\frac{h+1}{2}} a_{i,r} + \sum_{i=\frac{h+3}{2}}^{h} a_{i,k-r+1} \\ &= \sum_{i=1}^{\frac{h+1}{2}} (i-1)k + r \sum_{i=\frac{h+3}{2}}^{h} (i-1)k + k - r + 1 \\ &= \frac{h^2k + h - k - 1}{2} + r \\ &= \frac{(h-1)(hk+k+1)}{2} + r \quad \text{for} \quad 1 \le r \le k. \end{aligned}$$

Hence, $\sum\mathbb{P} = \dfrac{(h-1)(hk+k+1)}{2} + [1, k]$. □

**Example** 2.2 Let $h = 9$, $k = 6$ and $X = [1, 54]$. Then the partition subsets are $X_1 = \{1, 7, 13, 19, 25, 36, 42, 48, 54\}$, $X_2 = \{2, 8, 14, 20, 26, 35, 41, 47, 53\}$, $X_3 = \{3, 9, 15, 21, 27, 34, 40, 46, 52\}$, $X_4 = \{4, 10, 16, 22, 28, 33, 39, 45, 51\}$, $X_5 = \{5, 11, 17, 23, 29, 32, 38, 44, 50\}$ and $X_6 = \{6, 12, 18, 24, 30, 31, 37, 43, 49\}$. $\sum X_r = \dfrac{(h-1)(hk+k+1)}{2} + r = 244 + r$ for $1 \le r \le 6$.

**Lemma** 2.3 *Let $h$ and $k$ be two positive integers such that $h$ is even and $k \geq 3$ is odd. Then there exists a $k$-equipartition $\mathbb{P} = \{X_1, X_2, \cdots, X_k\}$ of $X = [1, hk]$ such that $\sum X_r = \frac{(h-1)(hk+k+1)}{2} + r$ for $1 \leq r \leq k$. Thus, $\sum \mathbb{P}$ is a set of consecutive integers given by $\sum \mathbb{P} = \frac{(h-1)(hk+k+1)}{2} + [1, k]$.*

*Proof* Let us arrange the set of integers $X = \{1, 2, 3, \cdots, hk\}$in a $h \times k$ matrix $\mathcal{A}$ as given below.

$$\mathcal{A} = \begin{pmatrix} 1 & 2 & \cdots & k-1 & k \\ n+1 & n+2 & \cdots & 2k-1 & 2k \\ 2n+1 & 2n+2 & \cdots & 3k-1 & 3k \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ (h-1)k+1 & (h-1)k+2 & \cdots & hk-1 & hk \end{pmatrix}_{h \times k}$$

That is, $\mathcal{A} = (a_{i,j})_{h \times k}$ where $a_{i,j} = (i-1)k + j$ for $1 \leq i \leq h$ and $1 \leq j \leq k$. For $1 \leq r \leq k$, define $Y_r = \{a_{i,r}/1 \leq i \leq \frac{h}{2}\} \cup \{a_{i,k-r+1}/\frac{h}{2} + 1 \leq i \leq h-1\}$. Then

$$\begin{aligned} \sum Y_r &= \sum_{i=1}^{\frac{h}{2}} a_{i,r} + \sum_{i=\frac{h}{2}+1}^{h-1} a_{i,k-r+1} \\ &= \sum_{i=1}^{\frac{h}{2}} \{(i-1)k + r\} + \sum_{i=\frac{h}{2}+1}^{h-1} \{(i-1)k + k - r + 1\} \\ &= \frac{k(h-1)^2 + h - k - 2}{2} + r \end{aligned}$$

For $1 \leq r \leq k$, define $X_r = Y_{\sigma(r)} \cup \{(h-1)k + \pi(r)\}$, where $\sigma$ and $\pi$ denote the permutations of $\{1, 2, \cdots, k\}$ given by $\sigma(r) = \begin{cases} \frac{k-2r+1}{2} & \text{for} \quad 1 \leq r \leq \frac{k-1}{2} \\ \frac{3k-2r+1}{2} & \text{for} \quad \frac{k+1}{2} \leq r \leq k \end{cases}$ and $\pi(r) = \begin{cases} 2r & \text{for} \quad 1 \leq r \leq \frac{k-1}{2} \\ 2r-k & \text{for} \quad \frac{k+1}{2} \leq r \leq k \end{cases}$. Then

$$\begin{aligned} \sum X_r &= \sum Y_{\sigma(r)} + (h-1)k + \pi(r) \\ &= \frac{k(h-1)^2 + h - k - 2}{2} + \sigma(r) + (h-1)k + \pi(r) \end{aligned}$$

$\sum X_r = \begin{cases} \frac{k(h-1)^2 + h - k - 2}{2} + \frac{k-2r+1}{2} + (h-1)k + 2r & \text{for} \quad 1 \leq r \leq \frac{k-1}{2} \\ \frac{k(h-1)^2 + h - k - 2}{2} + \frac{3k-2r+1}{2} + (h-1)k + 2r - k & \text{for} \quad \frac{k+1}{2} \leq r \leq k \end{cases}$. On simplification we get $\sum X_r = \frac{(h-1)(hk+k+1)}{2} + r$ for $1 \leq r \leq k$. Hence, $\sum \mathbb{P} = \frac{(h-1)(hk+k+1)}{2} + [1, k]$. $\square$

**Example** 2.4 Let $h = 6$,$k = 5$ and $X = [1, 30]$. $Y_1 = \{1, 6, 11, 20, 25\}$, $Y_2 = \{2, 7, 12, 19, 24\}$, $Y_3 = \{3, 8, 13, 18, 23\}$, $Y_4 = \{4, 9, 14, 17, 22\}$and $Y_5 = \{5, 10, 15, 16, 21\}$. By definition the partition subsets are, $X_r = Y_{\sigma(r)} \cup \{(h-1)k + \pi(r)$ for $1 \leq r \leq 5$. $X_1 = \{2, 7, 12, 19, 24, 27\}$, $X_2 = \{1, 6, 11, 20, 25, 29\}$, $X_3 = \{5, 10, 15, 16, 21, 26\}$ $X_4 = \{4, 9, 14, 17, 22, 28\}$ $X_5 = \{3, 8, 13, 18, 23, 30\}$, Now, $\sum X_r = \dfrac{(h-1)(hk+k+1)}{2} + r = 90 + r$ for $1 \leq r \leq 5$.

**Lemma** 2.5 *If $h$ is even, then there exists a $k$-equipartition $\mathbb{P} = \{X_1, X_2, \cdots, X_k\}$ of $X = [1, hk]$ such that $\sum X_r = \dfrac{h(hk+1)}{2}$ for $1 \leq r \leq k$. Thus, the subsets sum are equal and is equal to $\dfrac{h(hk+1)}{2}$.*

*Proof* Let us arrange the set of integers $X = \{1, 2, 3, \cdots, hk\}$in a $h \times k$ matrix $\mathcal{A}$ as given below.

$$\mathcal{A} = \begin{pmatrix} 1 & 2 & \cdots & k-1 & k \\ n+1 & n+2 & \cdots & 2k-1 & 2k \\ 2n+1 & 2n+2 & \cdots & 3k-1 & 3k \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ (h-1)k+1 & (h-1)k+2 & \cdots & hk-1 & hk \end{pmatrix}_{h \times k}$$

That is, $\mathcal{A} = (a_{i,j})_{h \times k}$ where $a_{i,j} = (i-1)k + j$ for $1 \leq i \leq h$ and $1 \leq j \leq k$. For $1 \leq r \leq k$, define $X_r = \{a_{i,r}/1 \leq i \leq \frac{h}{2}\} \cup \{a_{i,k-r+1}/\frac{h}{2} + 1 \leq i \leq h-1\}$. Then

$$\begin{aligned} \sum X_r &= \sum_{i=1}^{\frac{h}{2}} a_{i,r} + \sum_{i=\frac{h}{2}+1}^{h} a_{i,k-r+1} \\ &= \sum_{i=1}^{\frac{h}{2}} \{(i-1)k + r\} + \sum_{i=\frac{h}{2}+1}^{h} \{(i-1)k + k - r + 1\} = \frac{h(hk+1)}{2} \end{aligned}$$

Thus, the subsets sum are equal and is equal to $\dfrac{h(hk+1)}{2}$. □

**Example** 2.6 Let $h = 6$, $k = 5$ and $X = [1, 30]$. Then the partition subsets are $X_1 = \{1, 6, 11, 20, 25, 30\}$, $X_2 = \{2, 7, 12, 19, 24, 29\}$, $X_3 = \{3, 8, 13, 18, 23, 28\}$, $X_4 = \{4, 9, 14, 17, 22, 27\}$ and $X_5 = \{5, 10, 15, 16, 21, 26\}$. Now, $\sum X_r = \dfrac{h(hk+1)}{2} = 93$ for $1 \leq r \leq 5$.

**Lemma** 2.7 *Let $h$ and $k$ be two even positive integers and $h \geq 4$. If $X = [1, hk+1] - \{\dfrac{k}{2} + 1\}$, there exists a $k$-equipartition $\mathbb{P} = \{X_1, X_2, \cdots, X_k\}$ of $X$ such that $\sum X_r = \dfrac{h^2k + 3h - k - 2}{2} + r$ for $1 \leq r \leq k$. Thus $\sum \mathbb{P}$ is a set of consecutive integers $\dfrac{h^2k + 3h - k - 2}{2} + [1, k]$.*

*Proof* First we prove this lemma for $h = 2$ and we generalize for any even integer $h \geq 4$.

**Case 1:** $h = 2$.

$X = [1, 2k+1] - \{\frac{k}{2}+1\}$. For $1 \le r \le k$, define

$$X_r = \begin{cases} \{\frac{k}{2}+1-r, k+1+2r\} & \text{for} \quad 1 \le r \le \frac{k}{2} \\ \{\frac{3k}{2}+2-r, 2r\} & \text{for} \quad \frac{k}{2}+1 \le r \le k \end{cases}.$$

Hence, $\sum X_r = \frac{3k}{2} + 2 + r$ for $1 \le r \le k$.

**Case 2:** $h \ge 4$

Let $Y = [1, 2k+1] - \{\dfrac{k}{2}+1\}$ and $Z = [2k+2, hk+1]$. Then $X = Y \cup Z$. By Case 1, there exists a $k$-equipartition $\mathbb{P}_1 = \{Y_1, Y_2, \cdots, Y_k\}$ of Y such that

$$\sum Y_r = \frac{3k}{2} + 2 + r \quad \text{for} \quad 1 \le r \le k \tag{1}$$

Since $h-2$ is even, by Lemma 2.5, there exists a $k$-equipartition $\mathbb{P}_2' = \{Z_1', Z_2', \cdots, Z_k'\}$ of $[1, (h-2)k]$ such that $\sum Z_r' = \dfrac{(h-2)(hk-2k+1)}{2}$ for $1 \le r \le k$. Adding $2k+1$ to $[1, (h-2)k]$, we get a $k$-equipartition $\mathbb{P}_2 = \{Z_1, Z_2, \cdots, Z_k\}$ of $Z = [2k+2, hk+1]$ such that $\sum Z_r = (h-2)(2k+1) + \dfrac{(h-2)(hk-2k+1)}{2}$ for $1 \le r \le k$. Let $X_r = Y_r \cup Z_r$ for $1 \le r \le k$. Then,

$$\begin{aligned} \sum X_r &= \sum Y_r \cup \sum Z_r \\ &= \frac{h^2k + 3h - k - 2}{2} + r \quad \text{for} \quad 1 \le r \le k. \end{aligned}$$

Hence, $\sum \mathbb{P}$ is a set of consecutive integers $\dfrac{h^2k+3h-k-2}{2} + [1, k]$. □

**Example** 2.8 Let $h = 6$, $k = 6$ and $X = [1, 37] - \{4\}$.Then the partition subsets are $X_1 = \{3, 9, 14, 20, 31, 37\}$, $X_2 = \{2, 11, 15, 21, 30, 36\}$, $X_3 = \{1, 13, 16, 22, 29, 35\}$, $X_4 = \{7, 8, 17, 23, 28, 34\}$, $X_5 = \{6, 10, 18, 24, 27, 33\}$ and $X_6 = \{5, 12, 19, 25, 26, 32\}$. Now,

$$\sum X_r = \frac{h^2k + 3h - k - 2}{2} + r = 113 + r$$

for $1 \le r \le 6$.

**Lemma** 2.9 *Let $h$ and $k$ be two even positive integers. If $X = [1, hk+2] - \{1, \dfrac{k}{2}+2\}$, there exists a $k$-equipartition $\mathbb{P} = \{X_1, X_2, \cdots, X_k\}$ of $X$ such that $\sum X_r = \dfrac{h^2k+5h-k-2}{2} + r$ for $1 \le r \le k$. Thus $\sum \mathbb{P}$ is a set of consecutive integers $\dfrac{h^2k+5h-k-2}{2} + [1, k]$.*

*Proof* First we prove this lemma for $h = 2$ and we generalize for any even integer $h \ge 4$.

**Case 1:** $h = 2$

$X = [1, 2k+2] - \{1, \dfrac{k}{2}+2\}$. For $1 \le r \le k$, define

$$X_r = \begin{cases} \{\dfrac{k}{2}+1-r, k+2+2r\} & \text{for} \quad 1 \le r \le \dfrac{k}{2}, \\ \{\dfrac{3k}{2}+3-r, 2r+1\} & \text{for} \quad \dfrac{k}{2}+1 \le r \le k. \end{cases}$$

Hence, $\sum X_r = \dfrac{3k}{2} + 4 + r$ for $1 \leq r \leq k$.

**Case 2:** $h \geq 4$

Let $Y = [1, 2k+2] - \{1, \dfrac{k}{2}+2\}$ and $Z = [2k+3, hk+2]$. Then $X = Y \cup Z$. By Case 1, there exists a $k$-equipartition $\mathbb{P}_1 = \{Y_1, Y_2, \cdots, Y_k\}$ of Y such that

$$\sum Y_r = \frac{3k}{2} + 4 + r \quad \text{for} \quad 1 \leq r \leq k \tag{2}$$

Since $h-2$ is even, by Lemma 2.5, there exists a $k$-equipartition $\mathbb{P}'_2 = \{Z'_1, Z'_2, \cdots, Z'_k\}$ of $[1, (h-2)k]$ such that $\sum Z'_r = \dfrac{(h-2)(hk-2k+1)}{2}$ for $1 \leq r \leq k$. Adding $2k+2$ to $[1, (h-2)k]$, we get a $k$-equipartition $\mathbb{P}_2 = \{Z_1, Z_2, \cdots, Z_k\}$ of $Z = [2k+3, hk+2]$ such that $\sum Z_r = (h-2)(2k+2) + \dfrac{(h-2)(hk-2k+1)}{2}$ for $1 \leq r \leq k$. Let $X_r = Y_r \cup Z_r$ for $1 \leq r \leq k$. Then,

$$\begin{aligned} \sum X_r &= \sum Y_r \cup \sum Z_r \\ &= \frac{h^2k + 5h - k - 2}{2} + r \quad \text{for} \quad 1 \leq r \leq k. \end{aligned}$$

Hence, $\sum \mathbb{P}$ is a set of consecutive integers $\dfrac{h^2k+5h-k-2}{2} + [1, k]$. □

**Example** 2.10 Let $h = 6$, $k = 6$ and $X = [1, 38] - \{1, 5\}$.Then the partition subsets are $X_1 = \{4, 10, 15, 21, 32, 38\}$, $X_2 = \{3, 12, 16, 22, 31, 37\}$, $X_3 = \{2, 14, 17, 23, 30, 36\}$, $X_4 = \{8, 9, 18, 24, 29, 35\}$, $X_5 = \{7, 11, 19, 25, 28, 34\}$ and $X_6 = \{6, 13, 20, 26, 27, 33\}$. Now, $\sum X_r = \dfrac{h^2k+5h-k-2}{2} + r = 119 + r$ for $1 \leq r \leq 6$.

## §3. Main Results

**Definition** 3.1(Edge amalgamation of a finite collection of graphs, [1]) *For any finite collection $(G_i, u_iv_i)$ of graphs $G_i$, each with a fixed edge $u_iv_i$, Carlson [1] defined the edge amalgamation $\mathcal{E}dgeamal\{(G_i, u_iv_i)\}$ as the graph obtained by taking the union of all the $G_i$'s and identifying their fixed edges.*

**Definition** 3.2( Generalized Book) *If all the $G_i$'s are cycles then $\mathcal{E}dgeamal\{(G_i, u_iv_i)\}$ is called a generalized book.*

**Theorem** 3.3 *Let $H$ be a 2-connected $(p, q)$ simple graph. Then the edge amalgamation $\mathcal{E}dgeamal\{(H_i, u_iv_i)\}$ of any finite collection $\{H_i, u_iv_i\}$ of graphs $H_i$, each with a fixed edge $u_iv_i$ isomorphic to $H$ is $H$-supermagic for all values of $p$ and $q$.*

*Proof* Let $\{H_i, u_iv_i\}$ be a collection of $n$ graphs $H_i$, each with a fixed edge $u_iv_i$ and isomorphic to a 2-connected simple graph $H$.
Let $G = \mathcal{E}dgeamal\{(H_i, u_iv_i)\}$ with vertex set $V$ and edge set $E$. Note that $|V| = n(p-2)+2$ and $|E| = n(q-1)+1$. Let $H_i = (V_i, E_i)$ for $1 \leq i \leq n$. Label the common edge of $G$ as $e = w_1w_2$. Let $V'_i = V_i - \{w_1, w_2\}$ and $E'_i = E - \{e\}$ for $1 \leq i \leq n$.

**Case 1:** $n$ is odd

**Subcase (i):** $p$ is even and $q$ is odd

Since $p-2$ and $q-1$ are even by Lemma 2.5 there exists $n$-equipartitions $\mathbb{P}'_1 = \{X'_1, X'_2, \cdots, X'_n\}$ of $[1,(p-2)n]$ and $\mathbb{P}'_2 = \{Y'_1, Y'_2, \cdots, Y'_n\}$ of $[1,(q-1)n]$ such that

$$\sum X'_i = \frac{(p-2)(pn-2n+1)}{2}, \quad \sum Y'_i = \frac{(q-1)(qn-n+1)}{2}.$$

Add 2 to each element of the set $[1,(p-2)n]$ and $(p-2)n+3$ to each element of the set $[1,(q-1)n]$. We get $n$-equipartitions $\mathbb{P}_1 = \{X_1, X_2, \cdots, X_n\}$ of $[3, pn-2n+3]$ and $\mathbb{P}_2 = \{Y_1, Y_2, \cdots, Y_n\}$ of $[pn-2n+4, (p+q-3)n+3]$ such that

$$\sum X_i = (p-2)2 + \frac{(p-2)(pn-2n+1)}{2}, \quad \sum Y_i = (q-1)(pn-2n+3) + \frac{(q-1)(qn-n+1)}{2}.$$

Define a total labeling $f : V \cup E \to [1,(p+q-3)n+3]$ as follows:

$$\begin{aligned} f(w_1) &= 1 \quad \text{and} \quad f(w_2) = 2. \\ f(e) &= pn-2n+3. \\ f(V'_i) &= X_i \quad \text{for} \quad 1 \le i \le n. \\ f(E'_i) &= Y_{n-i+1} \quad \text{for} \quad 1 \le i \le n. \end{aligned}$$

Then for $1 \le i \le n$,

$$\begin{aligned} f(H_i) &= f(w_1) + f(w_2) + f(e) + \sum f(V'_i) + \sum f(E'_i) \\ &= f(w_1) + f(w_2) + f(e) + \sum X'_i + \sum Y'_{n-i+1} \\ &= \frac{n(p+q)^2 + p + q + 5(n-1)}{2} - (n-1)(2p+3q) \\ &= constant. \end{aligned}$$

Since $H_i \cong H$ for $1 \le i \le n$, $G$ is $H$-supermagic.

**Subcase (ii):** $p$ is odd and $q$ is even

Since $p-2$ and $q-1$ are odd, by Lemma 2.1 there exists $n$-equipartitions $\mathbb{P}'_1 = \{X'_1, X'_2, \cdots, X'_n\}$ of $[1,(p-2)n]$ and $\mathbb{P}'_2 = \{Y'_1, Y'_2, \cdots, Y'_n\}$ of $[1,(q-1)n]$ such that

$$\sum X'_i = \frac{(p-3)(pn-n+1)}{2} + i, \quad \sum Y'_i = \frac{(q-2)(qn+1)}{2} + i$$

for $1 \le i \le n$. Add 2 to each element of the set $[1,(p-2)n]$ and $(p-2)n+3$ to each element of the set $[1,(q-1)n]$. We get $n$-equipartitions $\mathbb{P}_1 = \{X_1, X_2, \cdots, X_n\}$ of $[3, pn-2n+3]$ and $\mathbb{P}_2 = \{Y_1, Y_2, \cdots, Y_n\}$ of $[pn-2n+4, (p+q-3)n+3]$ such that

$$\sum X_i = (p-2)2 + \frac{(p-3)(pn-n+1)}{2} + i, \quad \sum Y_i = (q-1)(pn-2n+3) + \frac{(q-2)(nq+1)}{2} + i$$

for $1 \le i \le n$. Define a total labeling $f : V \cup E \to [1,(p+q-3)n+3]$ as follows:

$$\begin{aligned} f(w_1) &= 1 \quad \text{and} \quad f(w_2) = 2. \\ f(e) &= pn-2n+3. \\ f(V'_i) &= X_i \quad \text{for} \quad 1 \le i \le n. \\ f(E'_i) &= Y_{n-i+1} \quad \text{for} \quad 1 \le i \le n. \end{aligned}$$

Then for $1 \leq i \leq n$,

$$\begin{aligned} f(H_i) &= f(w_1)+f(w_2)+f(e)+\sum f(V_i')+\sum f(E_i') \\ &= f(w_1)+f(w_2)+f(e)+\sum X_i'+\sum Y_{n-i+1}' \\ &= \frac{n(p+q)^2+p+q+5(n-1)}{2}-(n-1)(2p+3q) \\ &= constant. \end{aligned}$$

Since $H_i \cong H$ for $1 \leq i \leq n$, $G$ is $H$-supermagic.

**Subcase (iii):** $p$ and $q$ are odd

Since $p-2$ is odd, by Lemma 2.1 there exists an $n$-equipartition $\mathbb{P}_1' = \{X_1', X_2', \cdots, X_n'\}$ of $[1,(p-2)n]$ such that $\sum X_i' = \frac{(p-3)(pn-n+1)}{2}+i$ for $1 \leq i \leq n$. Since $q-1$ is even and $n$ is odd, by Lemma 2.3 there exists an $n$-equipartition $\mathbb{P}_2' = \{Y_1', Y_2', \cdots, Y_n'\}$ of $[1,(q-1)n]$ such that $\sum Y_i' = \frac{(q-2)(qn+1)}{2}+i$ for $1 \leq i \leq n$. Add 2 to each element of the set $[1,(p-2)n]$ and $(p-2)n+3$ to each element of the set $[1,(q-1)n]$. We get $n$-equipartitions $\mathbb{P}_1 = \{X_1, X_2, \cdots, X_n\}$ of $[3, pn-2n+3]$ and $\mathbb{P}_2 = \{Y_1, Y_2, \cdots, Y_n\}$ of $[pn-2n+4, (p+q-3)n+3]$ such that

$$\begin{aligned} \sum X_i &= (p-2)2+\frac{(p-3)(np-n+1)}{2}+i, \\ \sum Y_i &= (q-1)(pn-2n+3)+\frac{(q-2)(qn+1)}{2}+i \end{aligned}$$

for $1 \leq i \leq n$. Define a total labeling $f : V \cup E \to [1,(p+q-3)n+3]$ as follows:

$$\begin{aligned} f(w_1) &= 1 \quad \text{and} \quad f(w_2)=2. \\ f(e) &= pn-2n+3. \\ f(V_i') &= X_i \quad \text{for} \quad 1 \leq i \leq n. \\ f(E_i') &= Y_{n-i+1} \quad \text{for} \quad 1 \leq i \leq n. \end{aligned}$$

Then for $1 \leq i \leq n$,

$$\begin{aligned} f(H_i) &= f(w_1)+f(w_2)+f(e)+\sum f(V_i')+\sum f(E_i') \\ &= f(w_1)+f(w_2)+f(e)+\sum X_i'+\sum Y_{n-i+1}' \\ &= \frac{n(p+q)^2+p+q+5(n-1)}{2}-(n-1)(2p+3q) \\ &= constant. \end{aligned}$$

Since $H_i \cong H$ for $1 \leq i \leq n$, $G$ is $H$-supermagic.

**Subcase (iv):** $p$ and $q$ are even

Since $p-2$ is even and $n$ is odd, by Lemma 2.3 there exists an $n$-equipartition $\mathbb{P}_1' = \{X_1', X_2', \cdots, X_n'\}$ of $[1,(p-2)n]$ such that $\sum X_i' = \frac{(p-3)(pn-n+1)}{2}+i$ for $1 \leq i \leq n$. Since $q-1$ is odd, by Lemma 2.1 there exists an $n$-equipartition $\mathbb{P}_2' = \{Y_1', Y_2', \cdots, Y_n'\}$ of

$[1,(q-1)n]$ such that $\sum Y_i' = \frac{(q-2)(qn+1)}{2} + i$ for $1 \le i \le n$. Add 2 to each element of the set $[1,(p-2)n]$ and $(p-2)n+3$ to each element of the set $[1,(q-1)n]$. We get $n$-equipartitions $\mathbb{P}_1 = \{X_1, X_2, \cdots, X_n\}$ of $[3, pn-2n+3]$ and $\mathbb{P}_2 = \{Y_1, Y_2, \cdots, Y_n\}$ of $[pn-2n+4, (p+q-3)n+3]$ such that

$$\sum X_i = (p-2)2 + \frac{(p-3)(pn-n+1)}{2} + i, \quad \sum Y_i = (q-1)(pn-2n+3) + \frac{(q-2)(qn+1)}{2} + i$$

for $1 \le i \le n$. Define a total labeling $f : V \cup E \to [1, (p+q-3)n+3]$ as follows:

$$\begin{aligned} f(w_1) &= 1 \quad \text{and} \quad f(w_2) = 2. \\ f(e) &= pn - 2n + 3. \\ f(V_i') &= X_i \quad \text{for} \quad 1 \le i \le n. \\ f(E_i') &= Y_{n-i+1} \quad \text{for} \quad 1 \le i \le n. \end{aligned}$$

Then for $1 \le i \le n$,

$$\begin{aligned} f(H_i) &= f(w_1) + f(w_2) + f(e) + \sum f(V_i') + \sum f(E_i') \\ &= f(w_1) + f(w_2) + f(e) + \sum X_i' + \sum Y_{n-i+1}' \\ &= \frac{n(p+q)^2 + p + q + 5(n-1)}{2} - (n-1)(2p+3q) \\ &= \textit{constant}. \end{aligned}$$

Since $H_i \cong H$ for $1 \le i \le n$, $G$ is $H$-supermagic.

**Case 2:** $n$ is even

**Subcase (i):** $p$ is even and $q$ is odd

The argument in Subcase(i) of Case (1) is independent of the nature of $n$. Hence we get $G$ is $H$-supermagic.

**Subcase (ii):** $p$ is odd and $q$ is even

The argument in Subcase(ii) of Case (1) is independent of the nature of $n$. Hence we get $G$ is $H$-supermagic.

**Subcase (iii):** $p$ and $q$ are odd

Since $p-2$ is odd, by Lemma 2.1 there exists an $n$-equipartition $\mathbb{P}_1' = \{X_1', X_2', \cdots, X_n'\}$ of $[1,(p-2)n]$ such that $\sum X_i' = \frac{(p-3)(pn-n+1)}{2} + i$ for $1 \le i \le n$. Since $q-1$ and $n$ are even, by Lemma 2.7 there exists an $n$-equipartition $\mathbb{P}_2' = \{Y_1', Y_2', \cdots, Y_n'\}$ of $[1,(q-1)n+1] - \{\frac{n}{2}+1\}$ such that $\sum Y_i' = \frac{(q-1)^2 n + 3(q-1) - n - 2}{2} + i$ for $1 \le i \le n$. Add 2 to each element of the set $[1,(p-2)n]$ and $(p-2)n+2$ to each element of the set $[1,(q-1)n]$. We get $n$-equipartitions $\mathbb{P}_1 = \{X_1, X_2, \cdots, X_n\}$ of $[3, pn-2n+3]$ and $\mathbb{P}_2 = \{Y_1, Y_2, \cdots, Y_n\}$ of $[pn-2n+3, (p+q-3)n+3] - \{(p-2)n + \frac{n}{2} + 3\}$ such that

$$\sum X_i = (p-2)2 + \frac{(p-3)(pn-n+1)}{2} + i,$$

$$\sum Y_i = (q-1)(pn-2n+2) + \frac{(q-1)^2 n + 3(q-1) - n - 2}{2} + i$$

for $1 \leq i \leq n$. Define a total labeling $f : V \cup E \rightarrow [1, (p+q-3)n+3]$ as follows:

$$\begin{aligned} f(w_1) &= 1 \quad \text{and} \quad f(w_2) = 2. \\ f(e) &= (p-2)n + \frac{n}{2} + 3. \\ f(V_i') &= X_i \quad \text{for} \quad 1 \leq i \leq n. \\ f(E_i') &= Y_{n-i+1} \quad \text{for} \quad 1 \leq i \leq n. \end{aligned}$$

Then for $1 \leq i \leq n$,

$$\begin{aligned} f(H_i) &= f(w_1) + f(w_2) + f(e) + \sum f(V_i') + \sum f(E_i') \\ &= f(w_1) + f(w_2) + f(e) + \sum X_i' + \sum Y_{n-i+1}' \\ &= \frac{n(p+q)^2 + p + q}{2} - (n-1)(2p+3q-3) \\ &= constant. \end{aligned}$$

Since $H_i \cong H$ for $1 \leq i \leq n$, $G$ is $H$-supermagic.

**Subcase (iv):** $p$ and $q$ are even

Since $p-2$ and $n$ are even, by Lemma 2.9 there exists an $n$-equipartition $\mathbb{P}_1 = \{X_1, X_2, \cdots, X_n\}$ of $[1, (p-2)n+2] - \{1, \frac{n}{2}+2\}$ such that $\sum X_i = \frac{(p-2)^2 n + 5(p-2) - n - 2}{2} + i$ for $1 \leq i \leq n$.

Since $q-1$ is odd, by Lemma 2.1 there exists an $n$-equipartition $\mathbb{P}_2' = \{Y_1', Y_2', \cdots, Y_n'\}$ of $[1, (q-1)n]$ and $\sum Y_i' = \frac{(q-2)(qn+1)}{2} + i$ for $1 \leq i \leq n$. Add $(p-2)n+3$ to each element of the set $[1, (q-1)n]$. We get an $n$-equipartition $\mathbb{P}_2 = \{Y_1, Y_2, \cdots, Y_n\}$ of $[pn-2n+4, (p+q-3)n+3]$ such that $\sum Y_i = (q-1)(pn-2n+3) + \frac{(q-2)(qn+1)}{2} + i$ for $1 \leq i \leq n$. Define a total labeling $f : V \cup E \rightarrow [1, (p+q-3)n+3]$ as follows:

$$\begin{aligned} f(w_1) &= 1 \quad \text{and} \quad f(w_2) = \frac{n}{2} + 2. \\ f(e) &= pn - 2n + 3. \\ f(V_i') &= X_i \quad \text{for} \quad 1 \leq i \leq n. \\ f(E_i') &= Y_{n-i+1} \quad \text{for} \quad 1 \leq i \leq n. \end{aligned}$$

Then for $1 \leq i \leq n$,

$$\begin{aligned} f(H_i) &= f(w_1) + f(w_2) + f(e) + \sum f(V_i') + \sum f(E_i') \\ &= f(w_1) + f(w_2) + f(e) + \sum X_i' + \sum Y_{n-i+1}' \\ &= \frac{n(p+q)^2 + p + q}{2} - (n-1)(2p+3q-3) \\ &= constant. \end{aligned}$$

Since $H_i \cong H$ for $1 \leq i \leq n$, $G$ is $H$-supermagic.

Hence, the edge amalgamation $\mathcal{E}dgeamal\{(H_i, u_iv_i)\}$ of any finite collection $\{H_i, u_iv_i\}$ of graphs $H_i$, each with a fixed edge $u_iv_i$ and isomorphic to $H$ is $H$-supermagic for all values of $p$ and $q$. □

**Illustration** 3.4 Let $H_1$,$H_2$,$H_3$,$H_4$ and $H_5$ be five graphs isomorphic to the wheel $W_4 = C_4 + K_1$ and their fixed edges given by dotted lines. Then the Edge amalgamation graph $\mathcal{E}dgeamal\{(H_i, u_iv_i)\}$ of the given collection is $W_4$-supermagic with supermagic sum 303.

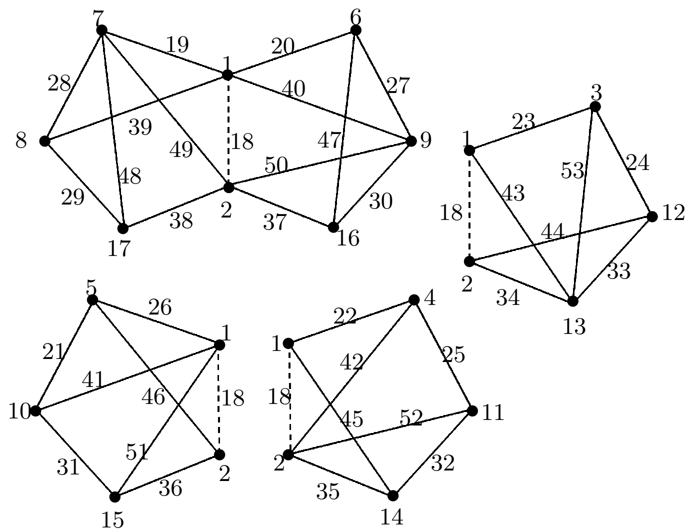


**Fig.**1

**Illustration** 3.5 Let $H_1$,$H_2$,$H_3$ and $H_4$ be four graphs isomorphic to $H$ and their fixed edges given by dotted lines. Then the Edge amalgamation graph $\mathcal{E}dgeamal\{(H_i, u_iv_i)\}$ of the given collection is $H$-supermagic with supermagic sum 300.

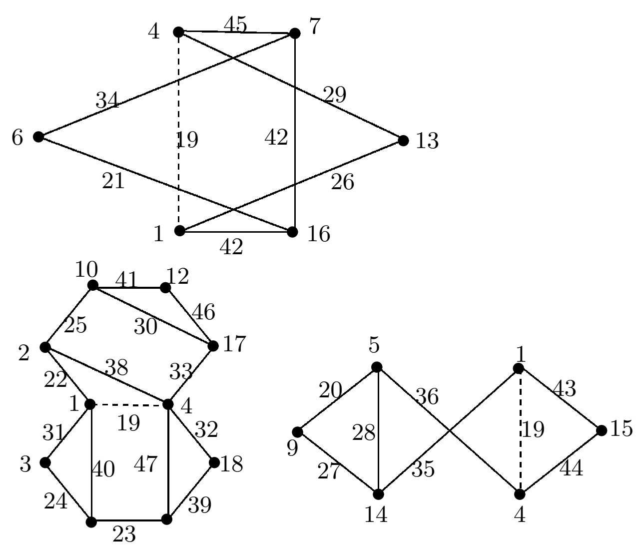


**Fig.**2

**Definition** 3.6(Book with $m$-gon pages) *Let $n$ and $m$ be any positive integers with $n \geq 1$ and $m \geq 3$ . Then, $n$ copies of the cycle $C_m$ with an edge in common is called a book with $n$ $m$-gon pages. That is, if $\{Gi, u_iv_i\}$ is a collection of $n$ copies of the cycle $C_m$ each with a fixed edge $u_iv_i$ then $\mathcal{E}dgeamal\{(G_i, u_iv_i)\}$ is called a book with $n$ $m$-gon pages.*

A book with 3 pentagon pages is given below.

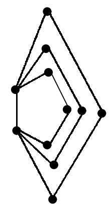

**Fig.**3

**Corollary** 3.7 *Books with $n$ $m$-gon pages are $C_m$ -supermagic for every positive integers $n \geq 1$and $m \geq 3$.*

**Illustration** 3.8 $C_5$-supermagic covering of a book with 3 hexagon pages is given below. The supermagic sum is 167.

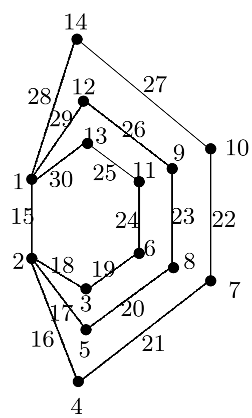


**Fig.**4

**Theorem** 3.9 *Let $H_i = K_{1,k}$ with vertex set $V(H_i) = \{v_i, v_{ir} : 1 \leq r \leq k\}$ and the edge set $E(H_i) = \{v_iv_{ir} : 1 \leq r \leq k\}$ where $1 \leq i \leq k$ and $G$ be a graph obtained by joining a new vertex $w$ with $v_{11}, v_{21}, \cdots, v_{k1}$. Then $G$ is $K_{1,k}$-supermagic.*

*Proof* Let $V_i = \{v_i, v_{ir} : 1 \leq r \leq k\}$ and $E_i = \{V_iv_{jr} : 1 \leq r \leq k\}$ for $1 \leq i \leq k$. Then the vertex and edge set of $G = (V, E)$ are given by $V = \cup_{i=1}^{k} V_i \cup \{v\}$ and $E = \cup_{i=1}^{k} E_i \cup \{vv_1, vv_2, \cdots, vv_k\}$. Also $|V| = k^2 + k + 1$ and $|E| = k^2 + k$. Let $V_{k+1} = \{w, v_1, v_2, \cdots, v_k\}$ and $E_{k+1} = \{wv_1, wv_2, \cdots, wv_k\}$ and $H_{k+1} = (V_{k+1}, E_{k+1})$ be the graph with vertex set $V_{k+1}$ and edge set $E_{k+1}$. Note that any edge of $E$ belongs to at least one of the subgraphs $H_i$ for $1 \leq i \leq k + 1$. Since $H_i \cong K_{1,k}$ for $1 \leq i \leq k + 1$, $G$ admits a $K_{1,k}$-covering.

**Case 1:** $k$ is odd

Since $k+1$ is even, by Lemma 2.3, there exists a $k$-equipartition $\mathbb{P} = \{X_1, X_2, \cdots, X_k\}$ of $X = [1, (k+1)k]$ such that

$$\sum X_i = \frac{k(k+1)^2}{2} + i \quad \text{for} \quad 1 \le i \le k \tag{3}$$

It can be easily verified from the definition of $X_r$ in Lemma 2.3 that $\left(\frac{k+1}{2} - 1\right) k + \sigma(r) \in X_r$ for $1 \le r \le k$, where $\sigma$ denotes the permutation of $\{1, 2, \cdots, k\}$ given by

$$\sigma(r) = \begin{cases} \frac{k-2r+1}{2} & \text{for} \quad 1 \le r \le \frac{k-1}{2} \\ \frac{3k-2r+1}{2} & \text{for} \quad \frac{k+1}{2} \le r \le k \end{cases}.$$

Construct $X_{k+1} = \{\left(\frac{k+1}{2} - 1\right) k + \sigma(r) : 1 \le r \le k\} \cup \{k^2 + k + 1\}$.

$$\begin{aligned} \sum X_{k+1} &= \sum_{r=1}^{k} \left[\left(\frac{k+1}{2} - 1\right) k + \sigma(r)\right] + k^2 + k + 1 \\ &= \frac{k^2(k-1)}{2} + \frac{k(k+1)}{2} + k^2 + k + 1 \\ &= \frac{k(k+1)^2}{2} + k + 1 \end{aligned} \tag{4}$$

From (1) and (2) we have

$$\sum X_i = \frac{k(k+1)^2}{2} + i \quad \text{for} \quad 1 \le i \le k+1 \tag{5}$$

As $k$ is odd, by Lemma 1, there exists a $k+1$-equipartition $\mathbb{Q}' = \{Y_1', Y_2', \cdots, Y_{k+1}'\}$ of the set $Y = [1, k(k+1)]$ such that $\sum Y_i' = \frac{(k-1)[(k+1)^2+1]}{2} + i$ for $1 \le i \le k+1$.

Adding $k^2+k+1$ to $[1, k(k+1)]$, we get a $k+1$-equipartition $\mathbb{Q} = \{Y_1, Y_2, \cdots, Y_{k+1}\}$ of the set $Y = [k^2+k+2, 2k^2+2k+1]$ such that

$$\sum Y_i = k(k^2+k+1) + \frac{(k-1)[(k+1)^2+1]}{2} + i \quad \text{for} \quad 1 \le i \le k+1 \tag{6}$$

Define a total labeling $f : V \cup E \to [1, 2k^2+2k+1]$ as follows:

(i) $f(w) = k^2 + k + 1$.

(ii) $f(V_i) = X_i$ with $f(v_{i1}) = \left(\frac{k+1}{2} - 1\right) k + \sigma(r)$ for $1 \le i \le k+1$.

(iii) $f(E_i) = Y_{k+2-i}$ for $1 \le i \le k+1$.

Then for $1 \le i \le k+1$,

$$\begin{aligned} f(H_i) &= \sum f(V_i) + \sum f(E_i) = \sum X_i + \sum Y_{k+2-i} \\ &= \frac{4k^3 + 5k^2 + 5k + 2}{2}, \end{aligned}$$

which is a constant. Since $H_i \cong K_{1,k}$ for $1 \leq i \leq k+1$, $G$ is $K_{1,k}$-supermagic.

**Case 2:** $k$ is even

Since $k+1$ is odd, by Lemma 1,there exists a $k$-equipartition $\mathbb{P} = \{X_1, X_2, \cdots, X_k\}$ of $X = [1, (k+1)k]$ such that

$$\sum X_i = \frac{k(k+1)^2}{2} + i \quad \text{for} \quad 1 \leq i \leq k \tag{7}$$

It can be easily verified from the definition of $X_r$ in Lemma 2.3 that $\left(\frac{k+2}{2} - 1\right) k + r \in X_r$ for $1 \leq r \leq \frac{k}{2}$, and $\left(\frac{k}{2} - 1\right) k + r \in X_r$ for $\frac{k}{2} + 1 \leq r \leq k$. Construct $X_{k+1} = \{\left(\frac{k+2}{2} - 1\right) k + r : 1 \leq r \leq \frac{k}{2}\} \cup \{\left(\frac{k}{2} - 1\right) k + r : \frac{k}{2} + 1 \leq r \leq k\} \cup \{k^2 + k + 1\}$.

$$\begin{aligned}\sum X_{k+1} &= \sum_{r=1}^{\frac{k}{2}} \left[\left(\frac{k+2}{2} - 1\right) k + r\right] + \sum_{\frac{k}{2}+1}^{k} \left[\left(\frac{k}{2} - 1\right) k + r\right] + k^2 + k + 1 \\ &= \frac{k^2(k-1)}{2} + \frac{k(k+1)}{2} + k^2 + k + 1 \\ &= \frac{k(k+1)^2}{2} + k + 1 \end{aligned} \tag{8}$$

From (5) and (6) we have

$$\sum X_i = \frac{k(k+1)^2}{2} + i \quad \text{for} \quad 1 \leq i \leq k+1 \tag{9}$$

As $k$ is even, by Lemma 2.3, there exists a $k+1$-equipartition $\mathbb{Q}' = \{Y_1', Y_2', \cdots, Y_{k+1}'\}$ of the set $Y = [1, k(k+1)]$ such that $\sum Y_i' = \frac{(k-1)[(k+1)^2+1]}{2} + i$ for $1 \leq i \leq k+1$. Adding $k^2 + k + 1$ to $[1, k(k+1)]$, we get a $k+1$-equipartition $\mathbb{Q} = \{Y_1, Y_2, \cdots, Y_{k+1}\}$ of the set $Y = [k^2 + k + 2, 2k^2 + 2k + 1]$ such that

$$\sum Y_i = k(k^2 + k + 1) + \frac{(k-1)[(k+1)^2 + 1]}{2} + i \quad \text{for} \quad 1 \leq i \leq k+1 \tag{10}$$

Define a total labeling $f : V \cup E \to [1, 2k^2 + 2k + 1]$ as follows:

(i) $f(w) = k^2 + k + 1$.

(ii) $f(V_i) = X_i$ with $f(v_{i1}) = \left(\frac{k+2}{2} - 1\right) k + r$ for $1 \leq i \leq \frac{k}{2}$ and $f(v_{i1}) = \left(\frac{k}{2} - 1\right) k + r$ for $\frac{k}{2} + 1 \leq i \leq k$.

(iii) $f(E_i) = Y_{k+2-i}$ for $1 \leq i \leq k+1$.

Then for $1 \leq i \leq k+1$,

$$\begin{aligned} f(H_i) &= \sum f(V_i) + \sum f(E_i) \\ &= \sum X_i + \sum Y_{k+2-i} \\ &= \frac{4k^3 + 5k^2 + 5k + 2}{2}, \end{aligned}$$

which is a constant. Since $H_i \cong K_{1,k}$ for $1 \leq i \leq k+1$, $G$ is $K_{1,k}$-supermagic. Thus, in both the cases $G$ is $K_{1,k}$-supermagic with supermagic sum $s(f) = \dfrac{4k^3 + 5k^2 + 5k + 2}{2}$. □

**Illustration** 3.10

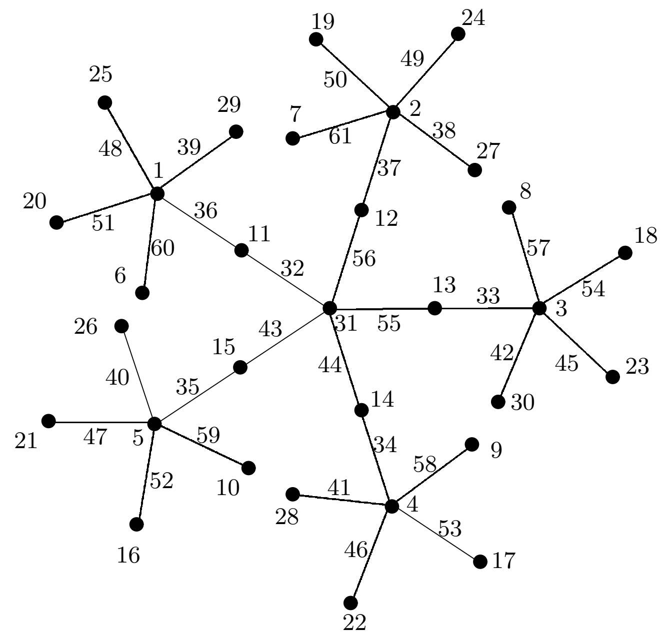


**Fig.1.** $G$**- is** $K_{1,5}$**-supermagic with supermagic sum 236.**

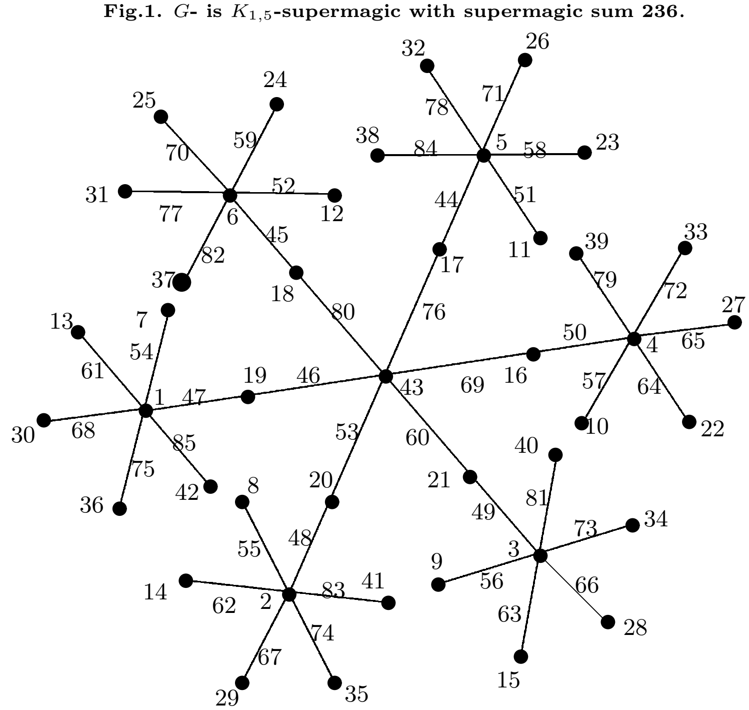


**Fig.2.** $G$**- is** $K_{1,6}$**-supermagic with supermagic sum 538.**

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