ON TWO SUBSETS OF GENERALIZED SMARANDACHE PALINDROMES

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Abstract Two subsets of generalized Smarandache palindromes are constructed to determine some of their properties. New sequences, conjectures, and unsolved questions are given.

Palindromes are positive integers that read the same way forward and backward. Example: 53135 is a palindrome.

Generalized Smarandache Palindromes (GSPs) are positive integers of the form

$$r_1 r_2 \cdots r_n r_n \cdots r_2 r_1 \quad \text{or} \quad r_1 r_2 \cdots r_{n-1} r_n r_{n-1} \cdots r_2 r_1$$

where all $r_1, r_2, \cdots, r_n$ are integers consisting of one or more decimal digits ([1], [2]). Example: 2145645621 is a GSP because we can split it into groups such as (21)(456)(456)(21). Note that it must be possible to split any GSP into at least two groups to avoid making every integer a GSP.

In this paper we will consider two simple subsets of GSPs. These subsets will involve numbers of the form $R_1R_2R_2R_1$ where $R_1 = n$ (with $n > 9$ in one case), and $R_2 = f(n)$, where f is a simple function, or $R_1 = f(n)$ and $R_2 = n$. The digital sum of n will be the first function used, which we will denote by ds(n). Note that regular palindromic numbers will not be considered, only "pure" GSPs.

Definition:A Smarandache digital sum GSP (SDG) is a number of the form $R_1R_2R_2R_1$ where $R_1 = n$, with $n > 9$, and $R_2 = ds(n)$. For example, $SDG(13) = 134413$ because $ds(13) = 1 + 3 = 4$, and thus we concatenate 13_4_4_13.

Because we are not concerned with regular palindromic GSPs we will exclude from our formula all integers n such that their digital sum is also palindromic, since these are the only numbers that yield regular palindromes from the concatenation: $n\_ds(n)\_ds(n)\_n$.

A computer program was written to construct SDG numbers and exclude any regular palindromes. The following sequence was produced.

Sequence 1: 101110, 123312, 134413, 145514, 156615, 167716, 178817, 189918, 19101019, 202220, 213321, 235523, 246624, 257725, 268826, 279927, 28101028, 29111129, 303330, 314431, 325532, 347734, 358835, 369936, 37101037, 38111138, ...

The most natural place to start our investigation of the properties of SDGs is by asking if any are prime numbers.

A computer search produced the following n such that SDG(n) is prime.

Sequence 2: 17, 23, 43, 61, 71, 157, 167, 169, 193, 199, 269, 283, 307, 377, 379, 409, 469, 491, 497, 509, 523, 553, 559, 563, 587, 617, 631, 637, 677, 709, 737, 767, 839, 869, 871, 913, 947, 971, 983, 1003, 1039, 1051, 1061, 1067, 1069, 1073, 1081, 1093, 1121, 1123, 1147, 1241, 1243, 1267, 1303, 1369, 1409, 1441, 1451, ...

SDG primes appear to be plentiful. One of the larger primes found was 99877404099877, which can be split like this (99877)(40)(40)(99877) – among other ways – to show that it is a GSP.

It is well known that a regular palindrome > 11 must consist of an odd number of digits for it to be prime. (For a proof, see [3].) This is not the case for SDGs, however. In fact, it is easy to see that due to the way our subset of GSPs are constructed, they will always consist of an even number of decimal digits.

Note that the majority of the values in Sequence 2 are primes themselves, although some are composites.

Conjecture: There are infinitely many prime and composite values n such that SDG(n) is prime.

Unsolved question: Will there be more composite n or more prime n such that SDG(n) is prime?

Three computer searches were conducted to determine whether there were any square, triangular, or Fibonacci SDG values. None were found for all $n < 10^5$.

Unsolved question: Are there any square, triangular, or Fibonacci numbers in Sequence 1?

We now note a fascinating curiosity concerning SDG numbers. But before doing so we need another definition. Peter Wallrodt defined "brilliant numbers" as numbers consisting of two prime factors of the same length in decimal representation [4]. For example, $99973 = 257 * 389$, is a brilliant number since both of its prime factors have 3 digits. Brilliant numbers play a crucial role in testing prime factoring programs.

A computer program was written to search for brilliant SDG numbers. Below are the first 37 values of n (out of 85 found) such that SDG(n) is brilliant.

Sequence 3: 13, 149, 253, 547, 881, 1177, 1247, 1271, 1987, 2359, 3053, 3251, 3371, 4831, 4867, 4937, 5551, 7099, 10187, 10351, 10861, 10883, 11579, 11639, 11717, 11963, 12241, 12347, 12589, 13199, 13871, 14339, 14699, 14861, 14963, 15149, 15287, ...

Conjecture: There are infinitely many brilliant SDG numbers.

It is somewhat surprising that there are so many brilliant SDG numbers. Here is one of the larger ones: $32677252532677 = 3401213 \times 9607529$.

Unsolved question: What is it about the form of SDGs which make them highly susceptible to being brilliant numbers?

Our second subset of GSPs is provided to illustrate the large region of unexplored territory they represent. It will also involve another function, which we will define as $H(n) = ld(n)sd(n)$, where $ld(n)$ is the largest digit of n, and $sd(n)$ is the smallest digit of n, respectively. For example, $H(345) = 125$ because $5^3 = 125$.

Definition: A Smarandache digital power GSP (SDPG) is a number of the form $R_1R_2R_2R_1$ where $R_1 = H(n)$, and $R_2 = n$. For example, SDGP$(24) =$ 16242416 because $H(24) = 42 = 16$, and thus we concatenate $16\_24\_24\_16$.

A computer program was written to construct SDPG numbers and exclude any regular palindromes. The following sequence was produced.

Sequence 4: 273327, 25644256, 3125553125, 466566646656, 82354377823543, 167772168816777216, 387420489993387420489, 110101, 212122, 313133, 414144, 515155, 616166, 717177, 818188, 919199, 120201, 221212, 923239, 16242416, 25252525, 36262636, 49272749, 64282864, 81292981, 130301, 331313, 932329, 27333327, ...

Sequence 4 is more erratic than Sequence 1. And it is interesting that even though we switched the order of concatenation of n and $f(n)$ with this subset of GSPs, we shall see that they still share many of the same properties with SDGs. We close with some data and conjectures concerning Sequence 4.

The sequence n such that SDGP(n) is prime begins: 13, 23, 40, 59, 70, 89, 90, 229, 292, 293, 329, 392, 529, 692, 796, 949, 964, 982, 1000, 1002, 1017, 1018, 1024, 1033, 1035, 1063, 1068, 1069, 1074, ...

Conjecture: Sequence is infinite.

Computer searches revealed that there are no square, triangular, or Fibonacci SDGPs for all $n < 10^4$.

Conjecture: None exist.

The sequence of n such that SDGP(n) is brilliant begins: 30, 1003, 1006, 1054, 1327, 1707, 2070, 2076, 2077, 2089, 2208, 2250, 2599, 2620, 2701, 3004, 3007, 3037, 3505, 3700, 3807, 3820, 3909, 4045, ...

Conjecture: Sequence is infinite.

What other functions would be interesting to introduce into the construction of GSPs?

References

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[2] F. Smarandache, Generalized Palindromes, Arizona State University Special Collections, Tempe.

[3] H. Ibstedt, Palindrome Studies (Part I), Smarandache Notions Journal, **14** (2004), 23-35.

[4] D. Alpern, Brilliant Numbers, http://www.alpertron.com.ar/BRILLIANT.HTM