International J.Math. Combin. Vol.1 (2011), 123-132

One-Mother Vertex Graphs

F. Salama

(Mathematics Department, Faculty of Science, Tanta University, Tanta, Egypt)

E-mail: fatma2salama@yahoo.com

Abstract: In this paper we will define a new type of graph. The idea of this definition is based on when we illustrate the cardiovascular system by a graph we find that not all vertices have the same important so we define this new graph and call it 1- mother vertex graph.

Key Words: Smarandache mother-father graph, 1-mother graph, matrices

AMS(2010): 97K30, 94C15

§1. Introduction

Unlike other areas in mathematics, graph theory traces its beginning to definite time and place: the problem of the seven bridges of Königsberg, which was solved in 1736 by Leonhard Euler. And in 1752 we find Euler's Theorem for planer graph. However, after this development, little was accomplished in this area for almost a century [4].

here are many physical systems whose performance depends not only on the characteristics of the components but also on the relative locations of the elements. An obvious example is an electrical network. One simple way of displaying a structure of a system is to draw a diagram consisting of points called vertices and line segments called edges which connect these vertices so that such vertices and edges indicate components and relationships between these components. Such a diagram is called linear graph. A graph G is a triple consisting of a vertex-set V(G), an edge-set E(G) and a relation that associated with each edge two vertices called its endpoints.

§2. Definitions and Background

Definition 2.1 *An abstract graph G is a diagram consisting of finite non empty set of elements called vertices denoted by V(G) together with a set of unordered pairs of these elements called edges denoted by E(G). The set of vertices of the graph G is called the vertex-set of G and the list of the edges is called the edge-list of G [1,5,9,10].*

Definition 2.2 *An oriented abstract graph is a pair (V,E) where V is finite non empty set of vertices and E is a set of ordered pairs of distinct elements of E with the property that if* $(v,w)\in E$ *then* $(w,v)\notin E$ *where the element (v,w) denote the edge from v to w [4,5].*

[1]Received December 22, 2010. Accepted March 3, 2011.

Definition 2.3 *An empty graph is a graph with no vertices and no edges [5].*

Definition 2.4 *A null graph is a graph containing no edges [9,10].*

Definition 2.5 *A multiple edges defined as two or more edges joining the same pair of vertices [1,8,9,10].*

Definition 2.6 *A loop is an edge joining a vertex to itself [1,8,9,10].*

Definition 2.7 *A simple graph is a graph with no loops or multiple edges [9].*

Definition 2.8 *A multiple graph is a graph with allows multiple edges and loops [1,8,9,10].*

Definition 2.9 *A complete graph is a graph in which every two distinct vertices are joined by exactly one edge [5,6,9,10].*

Definition 2.10 *A connected graph is a graph that in one piece, where as one which splits in to several pieces is disconnected [9].*

Definition 2.11 *Given a graph G, a graph H is called a subgraph of G if the vertices of H are vertices of G and the edges of H are edges of G [5,6,8].*

Definition 2.12 *Let v and w be two vertices of a graph. If v and w are joined by an edges, then v and w are said to be adjacent. Also, v and w are said to be incident with e then e is said to be incident with v and w [10].*

Definition 2.13 *Let G be a graph without loops, with n-vertices labeled* $1, 2, 3, ..., n$. *The adjacency matrix A(G) is the* $n \times n$ *matrix in which the entry in row i and column j is the number of edges joining the vertices i and j [10].*

Definition 2.14 *Let G be a graph without loops, with n- vertices labeled 1,2,3,. . . ., n and m edges labeled 1,2,3,. . . .,m. The incidence matrix I(G) is the* $n\times m$ *matrix in which the entry in row i and column j is l if vertex i is incident with edge j and 0 otherwise [10].*

§3. Main Results

In this article, we will define new types of graphs as follows:

Definition 3.1 *A Smarandache mother-father graph is a graph G in which there are vertices* $u_m^1, u_m^2, \cdots, u_m^n$, $v_m^1, v_m^2, \cdots, v_m^n$ *in G with a partition of* $V_1, v_2, \cdots, V_n$ *of* $V(G)$ *such that* v_m^i *is important than* v_1^i, v_1^i *is important than* $v_2^i \cdots$, *and* v_j^i *is important than* $v_{j+1}^i, \cdots$, *important than* u_m^i *for* $\forall 1 \leq i \leq n,\ j \geqslant 1$, *we call* $v_m^i, u_m^i, 1 \leq i \leq n$ *mother vertices and father vertices. Particularly, if* $n = 1$ *and there are no father vertices in a graph G, we call such a graph G 1-mother graph, seeing Figure 1.*

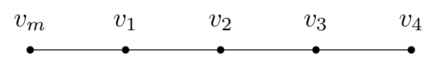
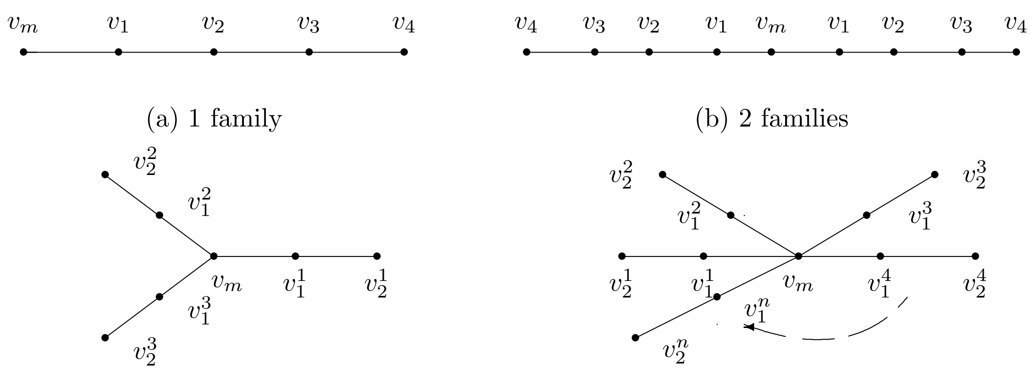


Figure 1

Now we will classify the 1-mother vertex graph with respect to the number of the family which contacts with the mother vertex as follows:

Definition 3.2 *A 1-mother vertex graph with n families of vertices is a graph G_m which it's vertex-set has the form $V\left\{v_m; v_1^1, v_2^1, v_3^1, \cdots; v_1^2, v_2^2, v_3^2, \cdots; \cdots; v_1^n, v_2^n, v_3^n, \cdots\right\}$, where $v_1^i, v_2^i, v_3^i, \cdots$ is the i-th family, seeing Figure 2.*



(a) 1 family (b) 2 families

(c) 3 families (d) n families

Figure 2

Definition 3.3 *Any edge has v_m as a vertex is called a mother edge.*

In Figure 2, there is a one mother edge in (a), two mother edges in (b), three mother edges in (c) and n mother edges in (d).

Note 1) The families of vertices in a 1-mother vertex graph with n families not necessary have the same number of vertices , seeing Figure 3.

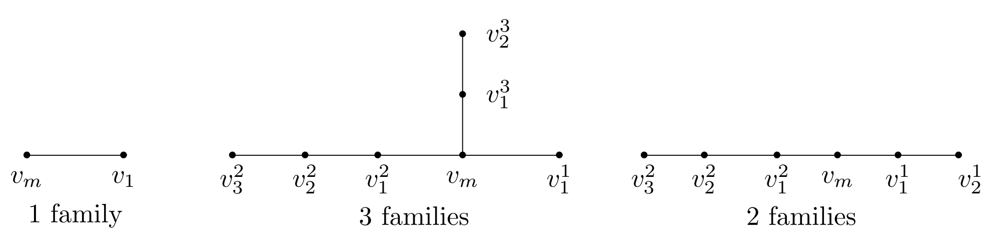


Figure 3

2) The following graph is not 1-mother vertex graph, seeing Figure 4.

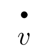


Figure 4

Definition 3.4 *An empty 1-mother vertex graph is an 1-mother vertex graph with no vertices and no edges.*

Definition 3.5 *A simple 1-mother vertex graph is an 1-mother vertex graph with no loops and no multiple edges, seeing Figure 3.*

Definition 3.6 *A multiple 1-mother vertex graph is an 1-mother vertex graph allows multiple edges and loops, seeing Figure 5.*

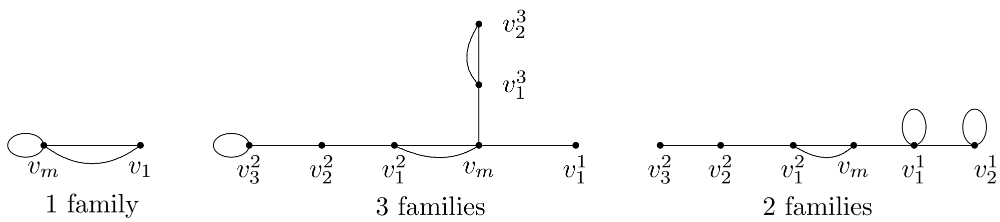
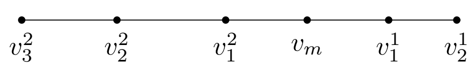
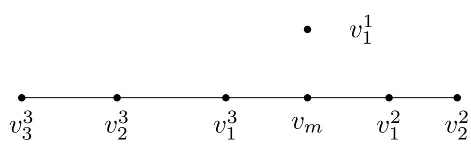


Figure 5

Definition 3.7 *A connected 1-mother vertex graph is 1-mother vertex graph that in one piece and the one which splits into several pieces is disconnected,seeing Figure 6.*



(a) A connected1-mother vertex graph with 2 families



(a) A connected1-mother vertex graph with 2 families

Figure 6

Note The following graph is not disconnected 1-mother vertex graph and also is not 1-mother vertex graph.

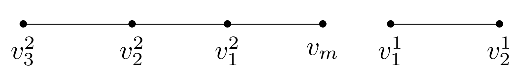


Figure 7

Definition 3.8 *A graph H_m^i is said to be main supgraph of G_m^n ,where $n, i \in z^+$ and $i \leq n$, if $V(H_m^i) \subseteq V(G_m^n)$, $E(H_m^i) \subseteq E(G_m^n)$ and $v_m \in V(H_m^i)$.*

Proposition 3.1 *The main supgraph H_m^i of G_m^n is 1-mother vertex graph.*

Definition 3.9 *A graph H is a supgraph of G_m^n if $V(H) \subseteq V(G_m^n)$, $E(H) \subseteq E(G_m^n)$ and $v_m \notin V(H_m^i)$.*

Proposition 3.2 *A supgraph H of G_m^n is not 1-mother vertex graph.*

Example 3.1 As shown in Figure 8, H_m^2 is a main supgraph of G_m^2 and H is a supgraph of G_m^2.

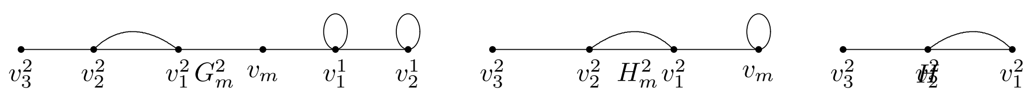


Figure 8

Definition 3.10 *An oriented 1-mother vertex graph is a pair (V, E) where V is finite non empty set of vertices and E is a set of ordered pairs of distinct elements of E with the property that if $(v, w) \in E$, then $(w, v) \notin E$, where the element (v, w) denote the edge from v to w, seeing Figure 9.*

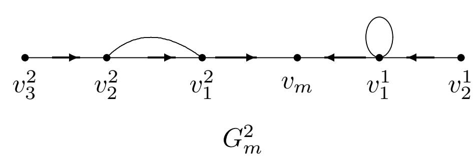


Figure 9

Definition 3.11 *Let G_m^n be a 1-mother vertex graph, with n-families of vertices. The adjacency matrix A(G_m^n) is the (n+1)x(n+1) matrix in which the entry in row i and column j is matrix its elements are the number of edges joining the families i and j.*

Definition 3.12 *Let G_m^n be a 1-mother vertex graph, with n-families. The incidence matrix I(G_m^n) is the (n+1) x n matrix in which the entry in row i and column j is matrix its elements is l if vertex in family i incident with edge in family j and 0 otherwise.*

Example 3.2 The adjacency matrix and the incidence matrix of a 1-mother vertex graph G_m^2

as shown in Figure 9 are given by

$$A(G_m^2) = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \qquad I(G_m^2) = \begin{bmatrix} 1 & 1^1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

where the symbol 1^1in the matrix in the row 1 and column 2 of the incidence matrix means that there exists a loop at the vertex $v_1^1$with the edge e_1^1.

Theorem *A* *A complete 1-mother vertex graph is not defined.*

Proof Let there exist a complete 1-mother vertex graph. Then this mean that every two distinct vertices are joined which is contradict with the definition of the 1-mother vertex graph. Hence the complete 1-mother vertex graph is not define. □

New we will define the union of any 1-mother vertex graphs as follows:

Definition 3.13 *The union of G_m^s and G_m^v , denoted $G_m^s \cup G_m^v$ is the graph with vertex set $V_1 \cup V_2$ and edge set $E_1 \cup E_2$.*

Proposition 3.3 *The union of any 1-mother vertex graphs is 1-mother vertex graph if $v_m \in V_1 \cap V_2$.*

Proof Let we have two 1-mother vertex graphs, the union of these graphs has one of two types.

1) If $v_m \in V_1 \cap V_2$, i.e. the new graph has one mother vertex, then the new graph is 1-mother vertex graph, seeing Figure 10.a.

2) If $v_m \notin V_1 \cap V_2$, i.e. the new graph has more than one mother vertex, then the new graph is not 1-mother vertex graph, seeing Figure 10-b. □

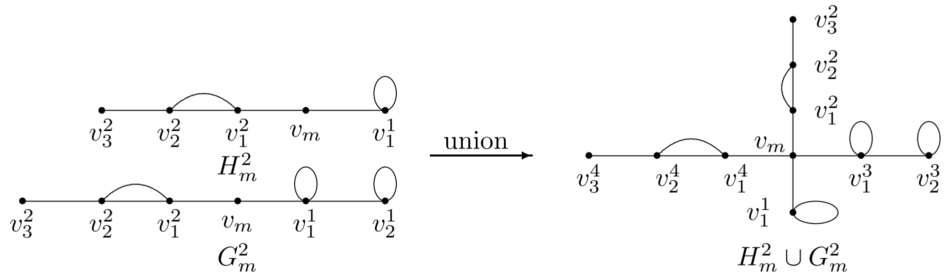


Figure 10-a

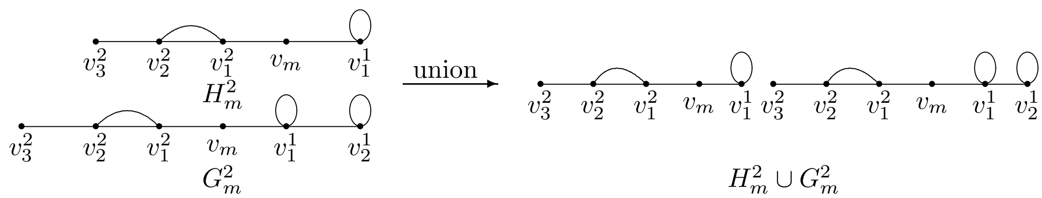


Figure 10-b

The intersection of 1-mother vertex graphs will be defined as follows:

Definition 3.14 *The intersection of G_m^s and G_m^v, denoted $G_m^s \cap G_m^v$ is the graph with vertex set $V_1 \cap V_2$ and edge set $E_1 \cap E_2$.*

Proposition 3.4 *The intersection of any number of 1-mother vertex graphs is 1-mother vertex graph if $v_m \in V_1 \cap V_2$ or $V_1 \cap V_2 = \phi$ and $E_1 \cap E_2$.*

Proof Let we have n number of 1-mother vertex graphs, the intersection of these graphs has one of two types.

1) If $v_m \in V_1 \cap V_2 \cdots \cap V_n$, i.e. the new graph has one mother vertex, then the new graph is 1-mother vertex graph, seeing Figure 11-a.

2) If $v_m \notin V_1 \cap V_2 \cdots \cap V_n = \phi$, i.e. the new graph is the empty 1-mother vertex graph.

3) If $v_m \notin V_1 \cap V_2 \cdots \cap V_n \neq \phi$, i.e. the new graph has more than one mother vertex, then the new graph is not 1-mother vertex graph, see Figure 11-b. □

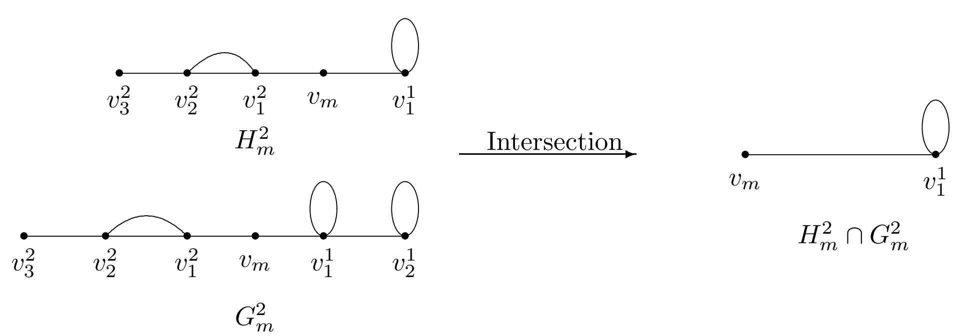


Figure 11-a

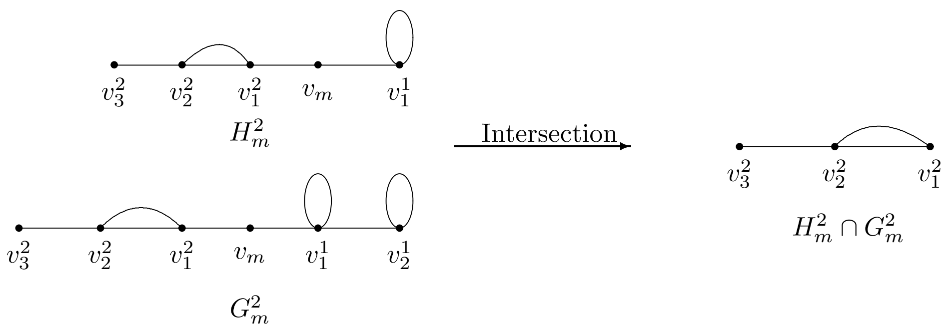
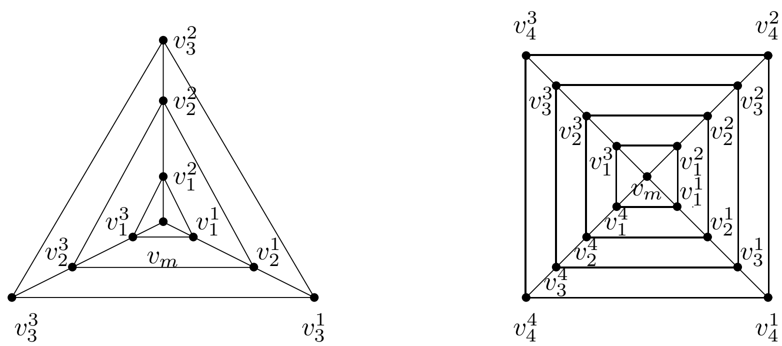


Figure 11-b

In this section we will define special types of 1-mother vertex graphs.

Definition 3.15 *A spider mother graph S_m^n is 1-mother vertex graph has the form as shown in Figure 12.*



(a) Spider mother graph with 3 families (b) Spider mother graph with 4 families

Figure 12

Note The least number of families which the spider graph has is three.

Definition 3.16 *A tree mother graph T_m^n is 1-mother vertex graph has the form as shown in Figure 13.*

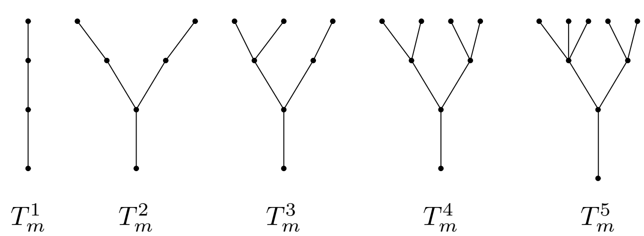


Figure 13

Example 3.3 The adjacency matrix of the spider graph as shown in Figure 12-a are given by

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

the existence of the unit matrix in column I and raw j means that the family in the column I and the family in the raw j have the relation between the vertices which have the same order.

Definition 3.17 *An orbit mother graph is 1-mother vertex graph is a 1-mother vertex graph containing no edges and the elements in the same family have the same distance from the mother vertex, seeing Figure 14.*

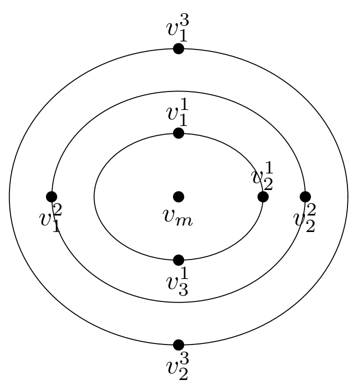


Figure 14

§4. Applications

(1) The solar system is orbit mother graph.

(2) If we illustrate the nervous system by using the graph we find that the nervous system is 1-mother vertex graph, seeing Figure 16.

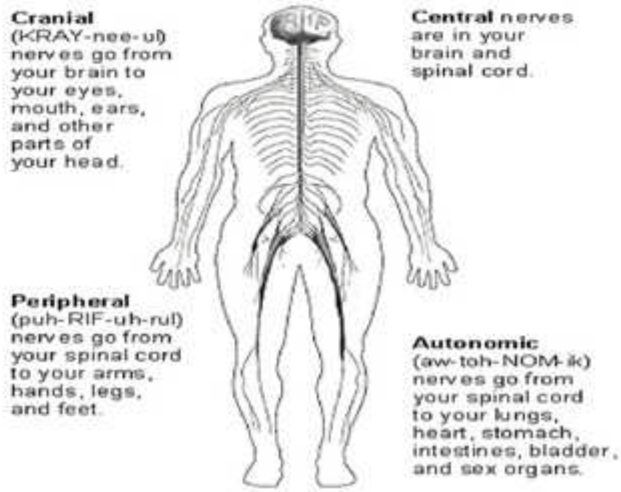


Figure 16

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