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# On the Smarandache Pseudo-number Sequences

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**Abstract** The main purpose of this paper is using elementary method to study the main value of the $m$-th power mean of the sum of all digits in the Smarandache pseudo-number sequence, and give some interesting asymptotic formulae for them.
**Keywords** Smarandache Pseudo-multiple of 5, pseudo-even, pseudo-odd sequence number; Sum of digits; Asymptotic formulae.

## §1. Introduction

A number is called Smarandache pseudo-multiple of 5 if some permutation of the digits is a multiple of 5, including the identity permutation. For example: $51, 52, 53, 54, 56, 57, 58, 59, 101, 102, 103, 104, 106 \cdots$ are Smarandache pseudo-multiple of 5 numbers. Similarly we can define the Smarandache pseudo-even numbers and the Smarandache pseudo-odd numbers. In reference [1], Professor F.Smarandache asked us to study the properties of the pseudo-multiple of 5, pseudo-even, pseudo-odd sequence. Let $A$ denote the set of all Smarandache Pseudo-multiple of 5 numbers; Let $B$ denote the set of all Smarandache Pseudo-even numbers and Let $C$ denote the set of all Smarandache Pseudo-odd numbers. For convenience, denoted by $A(n)$, the sum of all the digits of the base 10 digits of $n$. That is

$$A(n) = \sum_{i=0}^{k} a_i$$

if $n = a_k 10^k + a_{k-1} 10^{k-1} + \cdots + a_1 10 + a_0$. In this paper, we shall use the element method to study the mean value of the $m$-power of the sum of all digits in the pseudo-number sequence, and give some interesting formulae for them. That is, we shall prove the following results:

**Theorem 1.** For any integer number $x \geq 1$, we have the asymptotic formula

$$\sum_{\substack{n \in A \\ n \leq x}} A^m(n) = x \left( \frac{9}{2} \log x \right)^m + O\left( x (\log x)^{m-1} \right).$$

**Theorem 2** For any integer number $x \geq 1$, we have the asymptotic formula

$$\sum_{\substack{n \in B \\ n \leq x}} A^m(n) = x \left( \frac{9}{2} \log x \right)^m + O(x (\log x)^{m-1}).$$

**Theorem 3** For any integer number $x \geq 1$, we have the asymptotic formula

$$\sum_{\substack{n\in C\\ n\leq x}} A^m(n) = x\left(\frac{9}{2}\log x\right)^m + O(x(\log x)^{m-1}).$$

## §2. Some lemmas

To complete the proof of the theorem, we need the following lemmas.

**Lemma 1.** For any integer number $x \geq 1$, we have the asymptotic formula

$$\sum_{n\leq x} A^m(n) = x\left(\frac{9}{2}\log x\right)^m + O\left(x(\log x)^{m-1}\right).$$

**Proof.** See reference [1].

**Lemma 2.** For any integer number $x \geq 1$. Let $D$ denotes the complementary set of $A$, then we have the asymptotic formula

$$\sum_{\substack{n\in D\\ n\leq x}} A^m(n) = O\left(x\frac{(\log x)^m}{(\frac{5}{4})^{\log x}}\right).$$

**Proof.** From the definition of the set $D$, we know that the base 10 digits of the numbers in $D$ are $1, 2, 3, 4, 6, 7, 8, 9$, not including $0, 5$. So, there are $8^m$ m-digit number in $D$. Hence, for any integer $n$, there is a $k$ such that $10^{k-1} \leq x < 10^k$. Then we have

$$\sum_{\substack{n\in D\\ n\leq x}} A^m(n) \leq \sum_{t=1}^{k} \sum_{\substack{10^{t-1}\leq n<10^t\\ n\in D}} A^m(n)$$

Noting that

$$\sum_{\substack{10^{t-1}\leq n<10^t\\ n\in D}} A^m(n) < (9t)^m \times 8^t,$$

we can write

$$\sum_{t=1}^{k} \sum_{\substack{10^{t-1}\leq n<10^t\\ n\in D}} A^m(n) < \sum_{t=1}^{k}(9t)^m \times 8^t < 9^m \times k^m \times 8^{k+1}.$$

Since $k \leq (\log x) + 1 < k + 1$, we have

$$\sum_{\substack{n\in D\\ n\leq x}} A^m(n) = O\left((\log x)^m \times 8^{\log x}\right) = O\left(x\frac{(\log x)^m}{(\frac{5}{4})^{\log x}}\right).$$

This proves Lemma 2.

**Lemma 3.** For any integer number $x \geq 1$. Let $E$ denote the complementary set of $B$, then we have the asymptotic formula

$$\sum_{\substack{n\in E\\ n\leq x}} A^m(n) = O\left(x\frac{(\log x)^m}{2^{\log x}}\right).$$

**Proof.** By use the same method of proving Lemma 2, we can also get this Lemma.

## §3. Proof of the theorems

Now we complete the proof of the theorems. First we prove Theorem 1. From the definition of Smarandache pseudo-multiple of 5 numbers, Lemma 1 and Lemma 2, we can get

$$\begin{aligned}
\sum_{\substack{n\in A\\ n\leq x}} A^m(n) &= \sum_{n\leq x} A^m(n) - \sum_{\substack{n\in D\\ n\leq x}} A^m(n)\\
&= x\left(\frac{9}{2}\log x\right)^m + O(x(\log x)^{m-1}) - O\left(x\frac{(\log x)^m}{(\frac{5}{4})^{\log x}}\right)\\
&= x\left(\frac{9}{2}\log x\right)^m + O(x(\log x)^{m-1}).
\end{aligned}$$

This completes the proof of Theorem 1. Using the same method of proving Theorem 1, we can also deduce the other Theorems.

## References

[1] F. Smarandache, Only problem, Not Solution, Chicago, Xiquan Publ. House, 1993.

[2] Harald Riede, Asymptotic estimation of a sum of digits, Fibonacci Quarterly, **36(1)**(1998), 72-75.