SMARANDACHE REPLICATING DIGITAL FUNCTION NUMBERS

Jason Earls
R.R. 1-43-05 Fritch, TX 79036
jason_earls@hotmail.com

Abstract In 1987, Mike Keith introduced "repfigits" (replicating Fibonacci-like digits) [1]. In this paper two generalizations of repfigits are presented in which Smarandache-type functions are applied to the digits of n. Some conjectures and unsolved questions are then proposed.

Repfigits (replicating Fibonacci-like digits) are positive integers N such that in a sequence generated with the $n-$digits of N, and then continuing the sequence by summing the previous n terms, N eventually appears. For example, 3684 is a repfigit since it occurs in the sequence

$$3, 6, 8, 4, 21, 39, 72, 136, 268, 515, 991, 1910, 3684, \cdots$$

One generalization of repfigits is revrepfigits [2], in which the reversal of N occurs in a sequence generated in the same manner as given in the definition of repfigits. For example, 8166 is a revrepfigit since the sequence

$$8, 1, 6, 6, 21, 34, 67, 128, 250, 479, 924, 1781, 3434, 6618, \cdots$$

contains the reversal of 8166.

In this paper, two other generalizations of repfigits are made. These do away with the aesthetic aspect of the original repfigits, since we will not be concerned with all of the digits of a number to begin our sequences, only with three functions that operate on the base-10 representations of numbers.

SRDS Numbers. First, we will define some functions. Let $sd(n)$ denote the smallest digit of n, $ld(n)$ denote the largest digit of n, and $digsum(n)$ denote the digital sum of n, respectively. Examples: $sd(12345) = 1$ since 1 is the smallest digit of 12345, $ld(12345) = 5$ since 5 is the largest digit of 12345, and $digsum(12345) = 15$ since $1 + 2 + 3 + 4 + 5 = 15$. The digital sum function was mentioned in [3] and many papers in SNJ have dealt with it.

Definition: A Smarandache replicating digital sum number $SRDS$ is a number $N > 9$ such that when a sequence is formed by the recursion

$$SRDS(n) = SRDS(n-1) + SRDS(n-2) + SRDS(n-3),$$

where $SRDS(1) = sd(n), SRDS(2) = ld(n)$,and $SRDS(3) = digsum(n)$, then N occurs somewhere in the sequence.

For example, 8464 is a $SRDS$ number because it appears in the sequence

$$4, 8, 22, 34, 64, 120, 218, 402, 740, 1360, 2502, 4602, 8464, \cdots$$

Notice that the first term is the smallest digit of 8464, the second term is the largest digit of 8464, and the third term is the digital sum of 8464.

A computer program was written to search for $SRDS$ numbers, and the following were found.

18, 37, 53, 142, 284, 583, 4232, 4477, 5135, 7662, 8464, 9367, 15169, 22500, 24192,
28553, 40707, 46245, 49611, 59841, 199305, 213977, 228649, 232072, 302925, 398406,
771809, 1127617, 2280951, 2875059, 3174997, 7082341, 10217260, 14137273,...

Conjecture: There are infinitely many $SRDS$ numbers.

Unsolved questions: What is the level of algorithmic complexity for finding $SRDS$ numbers when using a brute-force method? What is the most efficient way to find these numbers? Are there infinitely many prime $SRDS$ numbers? Are $SRDS$ numbers more plentiful than repfigits?

SRDP Numbers.

Our second generalization is very similar to the $SRDS$ numbers. The only difference is that we will use the function $digprod(n)$ for the third term instead of $digsum(n)$, where $digprod(n)$ denotes the product of the nonzero digits of n. Example, $digprod(7605) = 210$ because $7 \times 6 \times 5 = 210$.

Definition: A Smarandache replicating digital product number (SRDP) is a number $N > 9$ such that when a sequence is formed by the recursion

$$SRDP(n) = SRDP(n-1) + SRDP(n-2) + SRDP(n-3),$$

where $SRDP(1) = sd(n), SRDP(2) = ld(n)$, and $SRDP(3) = digprod(n)$,

then N occurs somewhere in the sequence. For example, 1941 is a $SRDP$ number because it appears in the sequence

$$1, 9, 36, 46, 91, 173, 310, 574, 1057, 1941, \cdots$$

Notice that the first term is the smallest digit of 1941, the second term is the largest digit of 1941, and the third term is the digital product of 1941.

A computer program was written to search for $SRDP$ numbers, and the following sequence was found.

13, 19, 29, 39, 44, 49, 54, 59, 64, 69, 74, 79, 84, 89, 94, 99, 284, 996, 1908, 1941,
2588, 3374, 3489, 10856, 34088, 39756, 125519, 140490, 240424, 244035, 317422,
420742, 442204, 777994, 1759032,...

Conjecture: There are infinitely many $SRDP$ numbers.

Unsolved questions: What is the level of algorithmic complexity for finding $SRDP$ numbers using a brute-force method? What is the most efficient way to find these numbers? Are there infinitely many prime $SRDP$ numbers? Are there more $SRDP$ numbers than $SRDS$ numbers?

References

[1] M. Keith, Repfigit Numbers, Journal of Recreational Mathematics **19** (1987), No. 2, 41.

[2] N. J. A. Sloane, (2004), The On-Line Encyclopedia of Integer Sequences, Sequence #A097060, http://www.research.att.com/ njas/sequences/.

[3] F. Smarandache, Only Problems, Not Solutions, Xiquan Publishing House, Phoenix-Chicago, 1993.