SMARANDACHE REVERSE POWER SUMMATION NUMBERS

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Abstract A computer program was written and a search through the first $1000 SRPS$ numbers yielded several useful results.

Consider the sequence: $1^1 = 1, 1^2 + 2^1 = 3, 1^3 + 2^2 + 3^1 = 8, 1^4 + 2^3 + 3^2 + 4^1 = 22, \cdots$

The formula for these numbers is

$$\sum_{k=1}^{n} (n-k+1)^k$$

which produces the sequence:
$1, 3, 8, 22, 65, 209, 732, 2780, 11377, 49863, 232768, 1151914, 6018785, 33087205, 190780212, 1150653920, 7241710929, 47454745803, 323154696184, 2282779990494, 16700904488705, 126356632390297, 987303454928972, \cdots$

We shall call these values the Smarandache Reverse Power Summation numbers ($SRPS$), since the symmetry in their definition is reminiscent of other Smarandache classes of numbers, such as the sequences listed in [1], [2], and [3].

The purpose of this note is to define the $SRPS$ sequence, and to make an attempt at determining what types of numbers it contains.

A computer program was written and a search through the first $1000 SRPS$ numbers yielded the following results:

Only the trivial square $SRPS(1) = 1$was found. Are there any nontrivial square $SRPS$ numbers? The author conjectures: no.

Two primes, $SRPS(2) = 3$, and

$$SRPS(34) = 4065902334349345653147857 9$$

were found. However, the author conjectures that there are more prime SRPS numbers, but probably not infinitely many.

The trivial triangular numbers $SRPS(1) = 1$ and $SRPS(2) = 3$ were found. Are there any nontrivial triangular $SRPS$ numbers?

When $n = 1, 2, 3, 6, 7, 16, 33$, and 99, $SRPS(n)$ is a Harshad number (a number that is divisible by the sum of its own digits). For example,

$$SRPS(16) = 1150653920$$

has a digital sum of 32, and $1150653920/32 = 35957935$. The author conjectures that there are infinitely many $SRPS$ Harshad numbers.

When $n = 1, 2, 3$, and 4, $SRPS(n)$ is a palindrome. Will there ever be any more palindromic $SRPS$ numbers?

When $n = 4, 5, 6, 9, 12, 13$, and 62, $SRPS(n)$ is a semiprime (a number that is the product of exactly two primes). For example, $SRPS(13) = 6018785 = 5 \times 1203757$. The author conjectures that there are infinitely many semiprime $SRPS$ numbers. (Note that due to the difficulty of factorization, only the first $67 SRPS$ numbers were checked instead of the first 1000.)

References

1. Smarandaches Sequences, Vol. 1, http://www.gallup.unm.edu/ smarandache/SNAQINT.txt

2. Smarandache Sequences, Vol. 2, http://www.gallup.unm.edu/ smarandache/SNAQINT2.txt

3. Smarandache Sequences, Vol. 3, http://www.gallup.unm.edu/ smarandache/SNAQINT3.txt